Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.4-Cotangent/113-4.4.7-d-trig-^m-a+b-c-cot-^n-^p

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [64]. This is test number [113].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (64)	0.00 (0)
Mathematica	100.00 (64)	0.00 (0)
Fricas	100.00 (64)	0.00 (0)
Maple	98.44 (63)	1.56 (1)
Giac	82.81 (53)	17.19 (11)
Mupad	60.94 (39)	39.06 (25)
Maxima	32.81 (21)	67.19 (43)
Sympy	17.19 (11)	82.81 (53)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

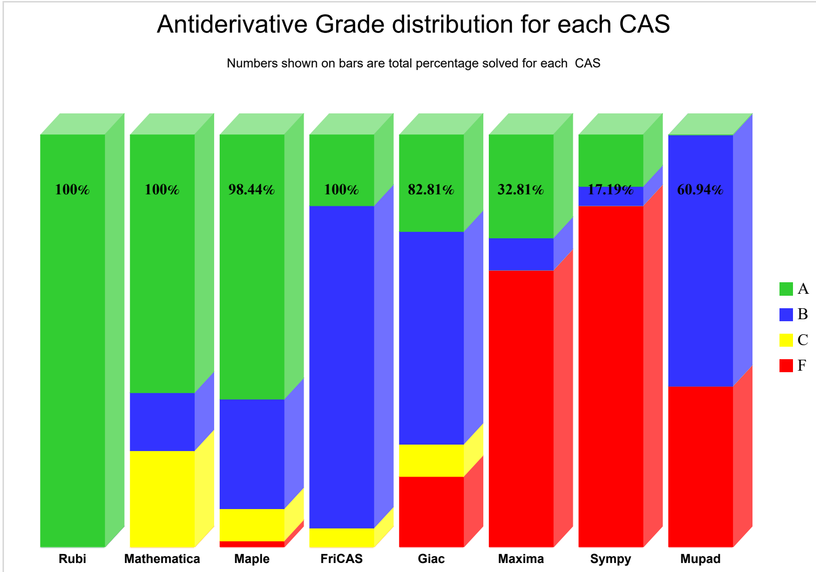
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

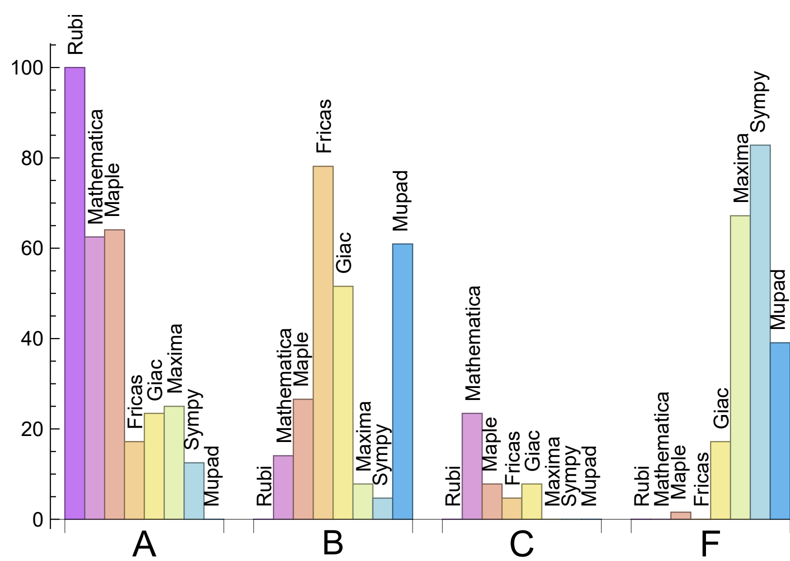
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Maple	64.06	26.56	7.81	1.56
Mathematica	62.50	14.06	23.44	0.00
Maxima	25.00	7.81	0.00	67.19
Giac	23.44	51.56	7.81	17.19
Fricas	17.19	78.12	4.69	0.00
Sympy	12.50	4.69	0.00	82.81
Mupad	N/A	60.94	0.00	39.06

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	1	100.00 %	0.00 %	0.00 %
Fricas	0	0.00 %	0.00 %	0.00 %
Giac	11	0.00 %	18.18 %	81.82 %
Maxima	43	53.49 %	0.00 %	46.51 %
Sympy	53	100.00 %	0.00 %	0.00 %
Mupad	25	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

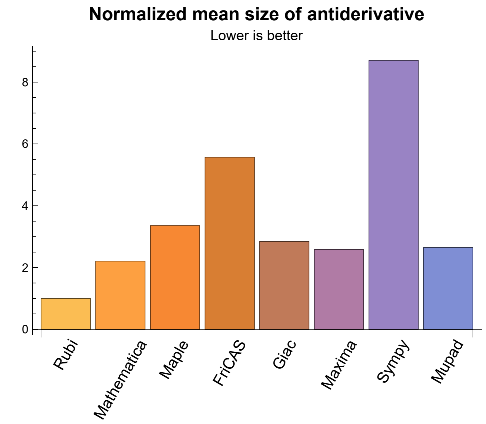
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

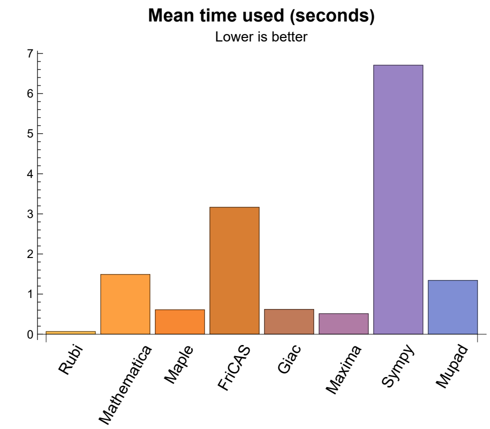
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.07	70.42	1.00	60.50	1.00
Mathematica	1.49	189.98	2.21	72.00	1.15
Maple	0.61	280.94	3.36	90.00	1.45
Maxima	0.51	86.71	2.58	48.00	1.19
Fricas	3.17	466.34	5.57	337.00	5.38
Sympy	6.71	1065.55	8.70	68.00	1.20
Giac	0.62	241.58	2.85	161.00	2.27
Mupad	1.34	243.08	2.65	47.00	1.00

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
2
4
6
8
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
1
2
3
4
5
6
7
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{\}$

Mathematica $\{22, 25, 35, 36, 37, 48, 53, 55, 58\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

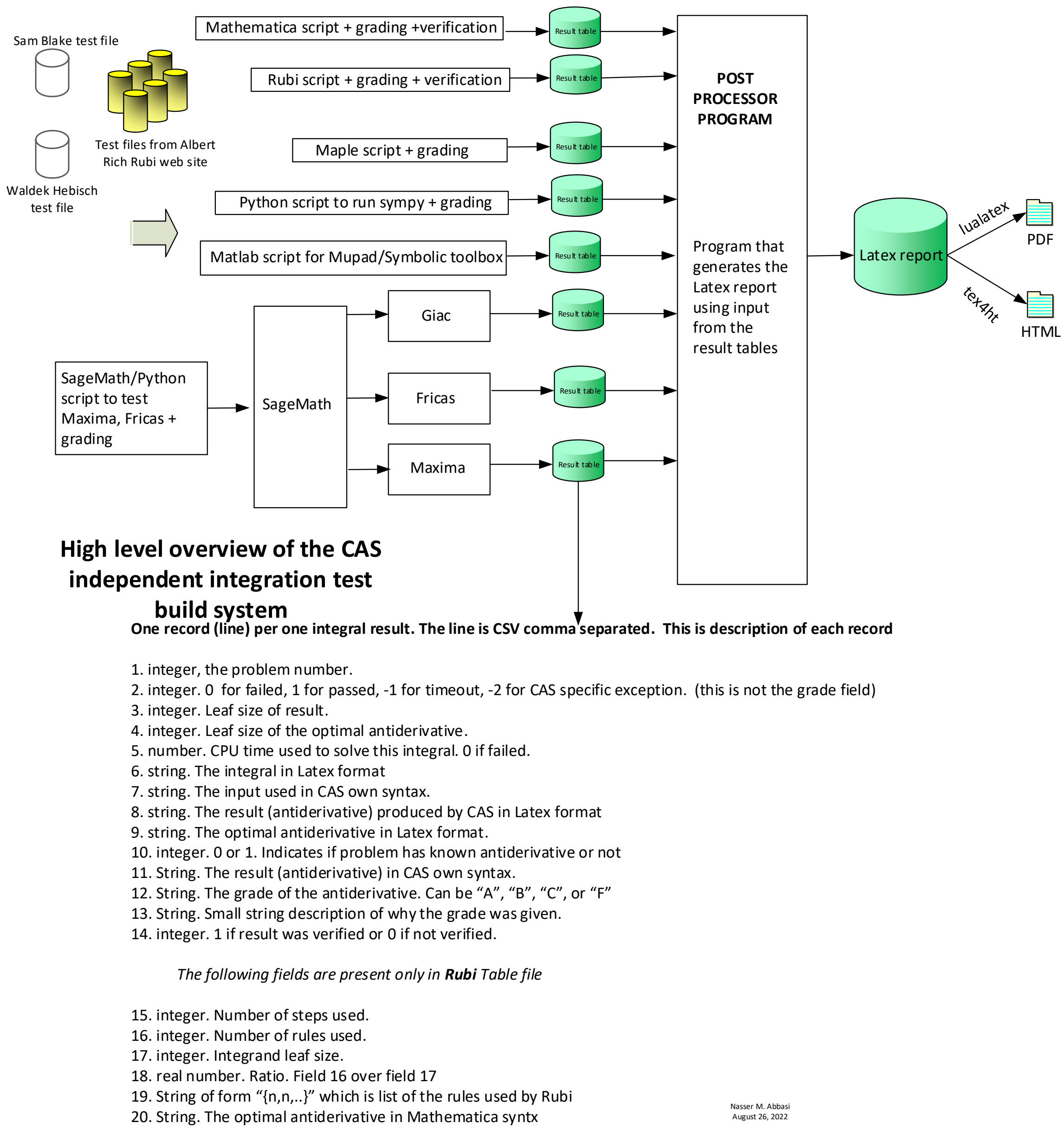
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 3, 4, 5, 6, 7, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 26, 27, 28, 29, 31, 32, 33, 39, 40, 41, 42, 43, 44, 46, 47, 49, 59, 60, 61, 62, 63, 64 }

B grade: { 8, 9, 12, 22, 30, 34, 38, 45, 50 }

C grade: { 2, 24, 25, 35, 36, 37, 48, 51, 52, 53, 54, 55, 56, 57, 58 }

F grade: { }

2.1.3 Maple

A grade: { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 49, 50, 51, 54, 55, 56, 59, 60, 62 }

B grade: { 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 48, 53, 58, 61, 63, 64 }

C grade: { 1, 21, 29, 47, 52 }

F grade: { 57 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 10, 12, 13, 14, 15, 16, 17, 18, 59 }

B grade: { 8, 9, 11, 40, 43 }

C grade: { }

F grade: { 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64 }

2.1.5 FriCAS

A grade: { 5, 14, 18, 21, 24, 25, 29, 30, 47, 48, 59 }

B grade: { 1, 2, 3, 4, 6, 7, 8, 9, 10, 15, 16, 17, 19, 20, 22, 23, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64 }

C grade: { 11, 12, 13 }

F grade: { }

2.1.6 Sympy

A grade: { 2, 3, 4, 10, 16, 51, 56, 59 }

B grade: { 5, 6, 7 }

C grade: { }

F grade: { 1, 8, 9, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 57, 58, 60, 61, 62, 63, 64 }

2.1.7 Giac

A grade: { 1, 2, 5, 6, 7, 8, 9, 14, 15, 16, 17, 18, 59, 62, 63 }

B grade: { 3, 4, 10, 20, 21, 23, 24, 25, 34, 35, 36, 37, 38, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 64 }

C grade: { 11, 12, 13, 39, 40 }

F grade: { 19, 22, 26, 27, 28, 29, 30, 31, 32, 33, 42 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 26, 28, 29, 34, 38, 39, 40, 42, 43, 44, 46, 47, 49, 51, 52, 54, 56, 57, 59 }

C grade: { }

F grade: { 15, 22, 23, 24, 25, 27, 30, 31, 32, 33, 35, 36, 37, 41, 45, 48, 50, 53, 55, 58, 60, 61, 62, 63, 64 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	148	2454	179	1236	0	248	828
N.S.	1	1.00	0.64	10.53	0.77	5.30	0.00	1.06	3.55
time (sec)	N/A	0.214	0.896	12.088	0.530	2.746	0.000	0.646	0.925

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	34	31	23	48	22	40	20
N.S.	1	1.00	1.70	1.55	1.15	2.40	1.10	2.00	1.00
time (sec)	N/A	0.010	0.028	0.077	0.524	2.723	0.050	0.463	0.336

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	71	68	63	127	68	114	45
N.S.	1	1.00	1.51	1.45	1.34	2.70	1.45	2.43	0.96
time (sec)	N/A	0.024	1.250	0.087	0.526	2.940	0.087	0.440	0.118

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	111	116	112	253	126	229	76
N.S.	1	1.00	1.42	1.49	1.44	3.24	1.62	2.94	0.97
time (sec)	N/A	0.035	2.904	0.098	0.495	3.259	0.133	0.534	0.454

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	56	48	252	238	65	41
N.S.	1	1.00	1.00	1.14	0.98	5.14	4.86	1.33	0.84
time (sec)	N/A	0.051	0.061	0.257	0.493	3.485	0.695	0.491	0.124

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	90	99	115	534	2125	123	119
N.S.	1	1.00	0.93	1.02	1.19	5.51	21.91	1.27	1.23
time (sec)	N/A	0.075	0.963	0.328	0.506	3.060	9.355	0.472	0.786

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	138	148	228	1068	8964	206	2500
N.S.	1	1.00	0.92	0.99	1.52	7.12	59.76	1.37	16.67
time (sec)	N/A	0.118	0.330	0.453	0.525	3.176	49.587	0.549	3.287

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	51	19	300	91	0	32	18
N.S.	1	1.00	2.32	0.86	13.64	4.14	0.00	1.45	0.82
time (sec)	N/A	0.013	0.111	0.184	0.545	3.020	0.000	0.425	0.375

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	28	6	35	53	0	10	5
N.S.	1	1.00	5.60	1.20	7.00	10.60	0.00	2.00	1.00
time (sec)	N/A	0.011	0.015	0.199	0.600	2.257	0.000	0.454	0.318

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	10	21	14	28	12
N.S.	1	1.00	1.00	1.08	0.83	1.75	1.17	2.33	1.00
time (sec)	N/A	0.013	0.012	0.128	0.503	2.659	0.158	0.425	0.392

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	C	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	48	32	284	73	0	34	31
N.S.	1	1.00	1.37	0.91	8.11	2.09	0.00	0.97	0.89
time (sec)	N/A	0.018	0.088	0.125	0.619	2.280	0.000	0.434	0.368

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	C	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	30	15	17	19	0	11	14
N.S.	1	1.00	2.14	1.07	1.21	1.36	0.00	0.79	1.00
time (sec)	N/A	0.015	0.021	0.126	0.530	2.697	0.000	0.433	0.394

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	12	14	0	28	13
N.S.	1	1.00	1.00	1.07	0.86	1.00	0.00	2.00	0.93
time (sec)	N/A	0.016	0.009	0.081	0.540	2.220	0.000	0.421	0.682

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	19	29	24	27	0	25	17
N.S.	1	1.00	0.68	1.04	0.86	0.96	0.00	0.89	0.61
time (sec)	N/A	0.063	0.028	0.171	0.281	2.720	0.000	0.421	0.586

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	32	38	27	77	0	49	-1
N.S.	1	1.00	1.03	1.23	0.87	2.48	0.00	1.58	-0.03
time (sec)	N/A	0.067	0.037	0.198	0.579	2.992	0.000	0.410	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	8	27	12	11	10
N.S.	1	1.00	1.00	1.10	0.80	2.70	1.20	1.10	1.00
time (sec)	N/A	0.033	0.013	0.111	0.291	2.906	0.647	0.440	0.476

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	49	56	52	78	0	12	20
N.S.	1	1.00	1.36	1.56	1.44	2.17	0.00	0.33	0.56
time (sec)	N/A	0.057	0.039	0.572	0.498	2.947	0.000	0.457	0.423

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	19	33	18	35	0	25	34
N.S.	1	1.00	0.66	1.14	0.62	1.21	0.00	0.86	1.17
time (sec)	N/A	0.072	0.036	0.421	0.527	2.395	0.000	0.423	0.727

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	65	90	0	330	0	0	66
N.S.	1	1.00	0.98	1.36	0.00	5.00	0.00	0.00	1.00
time (sec)	N/A	0.082	0.199	0.222	0.000	3.398	0.000	0.000	3.157

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	77	0	248	0	95	53
N.S.	1	1.00	1.00	1.60	0.00	5.17	0.00	1.98	1.10
time (sec)	N/A	0.049	0.030	0.135	0.000	3.186	0.000	0.478	1.170

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	591	0	351	0	187	69
N.S.	1	1.00	1.00	9.85	0.00	5.85	0.00	3.12	1.15
time (sec)	N/A	0.068	0.031	0.829	0.000	2.722	0.000	0.497	0.477

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	2105	177	0	768	0	0	-1
N.S.	1	1.00	23.65	1.99	0.00	8.63	0.00	0.00	-0.01
time (sec)	N/A	0.084	22.859	0.151	0.000	2.491	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	78	141	0	515	0	210	-1
N.S.	1	1.00	1.20	2.17	0.00	7.92	0.00	3.23	-0.02
time (sec)	N/A	0.031	0.169	0.177	0.000	2.594	0.000	1.724	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	44	752	0	193	0	239	-1
N.S.	1	1.00	0.86	14.75	0.00	3.78	0.00	4.69	-0.02
time (sec)	N/A	0.059	0.109	0.889	0.000	3.503	0.000	0.467	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	174	951	0	239	0	476	-1
N.S.	1	1.00	2.05	11.19	0.00	2.81	0.00	5.60	-0.01
time (sec)	N/A	0.096	1.677	0.643	0.000	2.787	0.000	0.489	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	91	176	0	486	0	0	120
N.S.	1	1.00	1.03	2.00	0.00	5.52	0.00	0.00	1.36
time (sec)	N/A	0.091	0.554	0.161	0.000	3.497	0.000	0.000	11.132

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	253	323	0	1134	0	0	-1
N.S.	1	1.00	1.99	2.54	0.00	8.93	0.00	0.00	-0.01
time (sec)	N/A	0.152	1.286	0.125	0.000	3.435	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	63	163	0	330	0	0	70
N.S.	1	1.00	0.91	2.36	0.00	4.78	0.00	0.00	1.01
time (sec)	N/A	0.062	0.184	0.125	0.000	4.167	0.000	0.000	3.535

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	2628	0	565	0	0	506
N.S.	1	1.00	1.00	35.04	0.00	7.53	0.00	0.00	6.75
time (sec)	N/A	0.083	0.082	0.730	0.000	5.083	0.000	0.000	0.540

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	222	1276	0	543	0	0	-1
N.S.	1	1.00	2.78	15.95	0.00	6.79	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.795	0.642	0.000	6.165	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	169	580	0	1520	0	0	-1
N.S.	1	1.00	0.99	3.39	0.00	8.89	0.00	0.00	-0.01
time (sec)	N/A	0.130	0.881	0.329	0.000	3.819	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	143	329	0	1071	0	0	-1
N.S.	1	1.00	1.13	2.61	0.00	8.50	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.481	0.233	0.000	3.490	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	107	169	0	703	0	0	-1
N.S.	1	1.00	1.23	1.94	0.00	8.08	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.061	0.238	0.000	3.118	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	111	68	0	239	0	97	41
N.S.	1	1.00	2.36	1.45	0.00	5.09	0.00	2.06	0.87
time (sec)	N/A	0.024	0.482	0.322	0.000	3.938	0.000	1.380	0.854

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	231	102	0	526	0	300	-1
N.S.	1	1.00	2.72	1.20	0.00	6.19	0.00	3.53	-0.01
time (sec)	N/A	0.051	3.761	0.197	0.000	3.317	0.000	0.964	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	367	162	0	898	0	1160	-1
N.S.	1	1.00	2.72	1.20	0.00	6.65	0.00	8.59	-0.01
time (sec)	N/A	0.082	7.996	0.224	0.000	3.338	0.000	1.485	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	2553	253	0	1452	0	3249	-1
N.S.	1	1.00	13.44	1.33	0.00	7.64	0.00	17.10	-0.01
time (sec)	N/A	0.146	14.780	0.226	0.000	3.188	0.000	2.694	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	123	51	0	110	0	257	104
N.S.	1	1.00	2.28	0.94	0.00	2.04	0.00	4.76	1.93
time (sec)	N/A	0.032	0.438	0.197	0.000	2.404	0.000	0.483	0.837

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	62	34	0	68	0	170	88
N.S.	1	1.00	1.94	1.06	0.00	2.12	0.00	5.31	2.75
time (sec)	N/A	0.018	0.075	0.208	0.000	2.585	0.000	0.502	0.963

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	42	31	90	56	0	34	85
N.S.	1	1.00	1.50	1.11	3.21	2.00	0.00	1.21	3.04
time (sec)	N/A	0.013	0.072	0.255	0.549	3.131	0.000	0.452	0.626

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	121	48	0	170	0	179	-1
N.S.	1	1.00	1.98	0.79	0.00	2.79	0.00	2.93	-0.02
time (sec)	N/A	0.028	0.133	0.178	0.000	2.642	0.000	0.744	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	60	35	0	123	0	0	34
N.S.	1	1.00	1.43	0.83	0.00	2.93	0.00	0.00	0.81
time (sec)	N/A	0.017	0.049	0.167	0.000	2.606	0.000	0.000	0.432

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	45	21	143	60	0	45	20
N.S.	1	1.00	1.73	0.81	5.50	2.31	0.00	1.73	0.77
time (sec)	N/A	0.012	0.034	0.184	0.562	3.394	0.000	0.433	0.502

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	44	0	284	0	96	44
N.S.	1	1.00	1.00	0.85	0.00	5.46	0.00	1.85	0.85
time (sec)	N/A	0.066	0.160	0.133	0.000	3.659	0.000	0.478	1.207

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	158	80	0	588	0	229	-1
N.S.	1	1.00	2.47	1.25	0.00	9.19	0.00	3.58	-0.02
time (sec)	N/A	0.064	0.294	0.174	0.000	2.979	0.000	1.923	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	29	0	127	0	61	27
N.S.	1	1.00	1.00	0.88	0.00	3.85	0.00	1.85	0.82
time (sec)	N/A	0.041	0.018	0.206	0.000	3.158	0.000	0.459	0.963

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	376	0	419	0	236	93
N.S.	1	1.00	1.00	6.27	0.00	6.98	0.00	3.93	1.55
time (sec)	N/A	0.065	0.049	0.627	0.000	2.489	0.000	0.487	0.513

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	134	328	0	229	0	216	-1
N.S.	1	1.00	2.48	6.07	0.00	4.24	0.00	4.00	-0.02
time (sec)	N/A	0.064	1.670	0.678	0.000	3.504	0.000	0.472	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	68	0	385	0	109	52
N.S.	1	1.00	1.00	1.15	0.00	6.53	0.00	1.85	0.88
time (sec)	N/A	0.076	0.239	0.129	0.000	2.487	0.000	0.470	1.917

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	137	99	0	388	0	161	-1
N.S.	1	1.00	2.32	1.68	0.00	6.58	0.00	2.73	-0.02
time (sec)	N/A	0.064	0.753	0.147	0.000	4.059	0.000	0.498	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	44	56	0	344	48	105	47
N.S.	1	1.00	0.80	1.02	0.00	6.25	0.87	1.91	0.85
time (sec)	N/A	0.056	0.044	0.132	0.000	3.000	4.743	0.471	1.779

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	75	962	0	863	0	340	1451
N.S.	1	1.00	0.89	11.45	0.00	10.27	0.00	4.05	17.27
time (sec)	N/A	0.089	0.056	0.661	0.000	2.890	0.000	0.491	0.482

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	674	421	0	393	0	359	-1
N.S.	1	1.00	7.33	4.58	0.00	4.27	0.00	3.90	-0.01
time (sec)	N/A	0.106	6.891	1.728	0.000	2.851	0.000	0.459	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	69	88	0	698	0	219	88
N.S.	1	1.00	0.84	1.07	0.00	8.51	0.00	2.67	1.07
time (sec)	N/A	0.090	0.131	0.165	0.000	2.881	0.000	0.495	4.236

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	200	161	0	720	0	278	-1
N.S.	1	1.00	2.13	1.71	0.00	7.66	0.00	2.96	-0.01
time (sec)	N/A	0.090	6.864	0.145	0.000	3.253	0.000	0.517	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	47	75	0	627	70	215	82
N.S.	1	1.00	0.60	0.96	0.00	8.04	0.90	2.76	1.05
time (sec)	N/A	0.066	0.049	0.116	0.000	3.278	8.210	0.461	4.462

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	78	0	0	1531	0	527	2500
N.S.	1	1.00	0.66	0.00	0.00	12.97	0.00	4.47	21.19
time (sec)	N/A	0.130	0.072	0.370	0.000	3.338	0.000	0.498	1.051

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	1450	1040	0	647	0	537	-1
N.S.	1	1.00	10.28	7.38	0.00	4.59	0.00	3.81	-0.01
time (sec)	N/A	0.162	8.078	2.145	0.000	2.694	0.000	0.528	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	43	39	33	24	34	34	37
N.S.	1	1.00	1.16	1.05	0.89	0.65	0.92	0.92	1.00
time (sec)	N/A	0.042	0.039	0.143	0.521	3.225	0.118	0.447	0.720

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	86	139	0	1063	0	204	-1
N.S.	1	1.00	0.96	1.54	0.00	11.81	0.00	2.27	-0.01
time (sec)	N/A	0.093	0.171	0.323	0.000	3.959	0.000	0.474	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	167	312	0	1486	0	445	-1
N.S.	1	1.00	1.33	2.48	0.00	11.79	0.00	3.53	-0.01
time (sec)	N/A	0.143	4.627	0.244	0.000	3.300	0.000	0.651	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	65	0	264	0	58	-1
N.S.	1	1.00	1.00	1.59	0.00	6.44	0.00	1.41	-0.02
time (sec)	N/A	0.044	0.020	0.366	0.000	3.800	0.000	0.470	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	248	0	670	0	111	-1
N.S.	1	1.00	0.99	3.35	0.00	9.05	0.00	1.50	-0.01
time (sec)	N/A	0.077	0.304	3.330	0.000	3.427	0.000	0.467	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	114	602	0	1365	0	276	-1
N.S.	1	1.00	0.97	5.15	0.00	11.67	0.00	2.36	-0.01
time (sec)	N/A	0.131	0.781	3.327	0.000	3.872	0.000	0.457	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [1] had the largest ratio of [25]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	15	12	1.00	25	0.480
2	A	3	2	1.00	12	0.167
3	A	4	3	1.00	14	0.214
4	A	4	3	1.00	14	0.214
5	A	3	3	1.00	14	0.214
6	A	5	5	1.00	14	0.357
7	A	6	6	1.00	14	0.429
8	A	4	4	1.00	10	0.400
9	A	3	3	1.00	10	0.300
10	A	3	3	1.00	10	0.300
11	A	5	5	1.00	12	0.417
12	A	4	4	1.00	12	0.333
13	A	3	3	1.00	12	0.250
14	A	4	3	1.00	17	0.176
15	A	5	5	1.00	17	0.294
16	A	3	3	1.00	15	0.200
17	A	5	5	1.00	15	0.333
18	A	5	4	1.00	17	0.235
19	A	6	6	1.00	17	0.353
20	A	5	5	1.00	15	0.333
21	A	7	5	1.00	15	0.333
22	A	7	7	1.00	17	0.412
23	A	6	6	1.00	12	0.500
24	A	5	5	1.00	17	0.294
25	A	6	6	1.00	17	0.353

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	7	6	1.00	17	0.353
27	A	8	8	1.00	17	0.471
28	A	6	5	1.00	15	0.333
29	A	8	6	1.00	15	0.400
30	A	7	7	1.00	17	0.412
31	A	8	8	1.00	16	0.500
32	A	7	7	1.00	16	0.438
33	A	6	6	1.00	16	0.375
34	A	3	3	1.00	16	0.188
35	A	4	4	1.00	16	0.250
36	A	6	6	1.00	16	0.375
37	A	7	6	1.00	16	0.375
38	A	6	6	1.00	12	0.500
39	A	5	5	1.00	12	0.417
40	A	3	3	1.00	12	0.250
41	A	7	6	1.00	10	0.600
42	A	6	5	1.00	10	0.500
43	A	3	3	1.00	10	0.300
44	A	5	5	1.00	17	0.294
45	A	6	6	1.00	17	0.353
46	A	4	4	1.00	15	0.267
47	A	7	5	1.00	15	0.333
48	A	5	5	1.00	17	0.294
49	A	5	5	1.00	17	0.294
50	A	4	4	1.00	17	0.235
51	A	5	5	1.00	15	0.333
52	A	8	6	1.00	15	0.400
53	A	6	6	1.00	17	0.353
54	A	6	6	1.00	17	0.353
55	A	6	6	1.00	17	0.353
56	A	6	5	1.00	15	0.333
57	A	9	7	1.00	15	0.467
58	A	7	7	1.00	17	0.412
59	A	7	6	1.00	8	0.750
60	A	8	7	1.00	15	0.467

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	9	8	1.00	15	0.533
62	A	4	4	1.00	15	0.267
63	A	6	6	1.00	15	0.400
64	A	7	7	1.00	15	0.467

Chapter 3

Listing of integrals

Local contents

3.1 $\int \frac{A+C\cot^2(c+dx)}{\sqrt{b\tan(c+dx)}}\,dx$

Optimal. Leaf size=233

$$-\frac{(A-C)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\sqrt{b}\,d}+\frac{(A-C)\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\sqrt{b}\,d}-\frac{(A-C)\log$$

```
[Out] -1/2*(A-C)*arctan(1-2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/d*2^(1/2)/b^(1/2)
+1/2*(A-C)*arctan(1+2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/d*2^(1/2)/b^(1/2)
-1/4*(A-C)*ln(b^(1/2)-2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/d*2^
(1/2)/b^(1/2)+1/4*(A-C)*ln(b^(1/2)+2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan
(d*x+c))/d*2^(1/2)/b^(1/2)-2/3*b*C/d/(b*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.21, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3754, 3710, 12, 16, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{(A-C)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\sqrt{b}\,d}+\frac{(A-C)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}+1\right)}{\sqrt{2}\sqrt{b}\,d}-\frac{(A-C)\log\left(\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}\sqrt{b}\,d}+\frac{(A-C)\log\left(\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}\sqrt{b}\,d}-\frac{2bC}{3d(b\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cot[c + d*x]^2)/Sqrt[b*Tan[c + d*x]],x]
```

```
[Out] -(((A - C)*ArcTan[1 - (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]])/(Sqrt[2]*Sqr
t[b]*d)) + ((A - C)*ArcTan[1 + (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]])/(Sq
rt[2]*Sqrt[b]*d) - ((A - C)*Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] - Sqrt[2]*Sq
rt[b*Tan[c + d*x]]])/(2*Sqrt[2]*Sqrt[b]*d) + ((A - C)*Log[Sqrt[b] + Sqrt[b]
*Tan[c + d*x] + Sqrt[2]*Sqrt[b*Tan[c + d*x]]])/(2*Sqrt[2]*Sqrt[b]*d) - (2*b
*C)/(3*d*(b*Tan[c + d*x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3710

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 + a^2*C)*((a + b*Tan[e + f*x])^(m
 + 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e
 + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /; F
reeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[a
^2 + b^2, 0]
```

Rule 3754

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Cot[e + f*x])^(m - n*p)
*(b + a*Cot[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+C\cot^2(c+dx)}{\sqrt{b\tan(c+dx)}}\,dx &= b^2\int \frac{C+A\tan^2(c+dx)}{(b\tan(c+dx))^{5/2}}\,dx \\
&= -\frac{2bC}{3d(b\tan(c+dx))^{3/2}}+\int \frac{b(A-C)\tan(c+dx)}{(b\tan(c+dx))^{3/2}}\,dx \\
&= -\frac{2bC}{3d(b\tan(c+dx))^{3/2}}+(b(A-C))\int \frac{\tan(c+dx)}{(b\tan(c+dx))^{3/2}}\,dx \\
&= -\frac{2bC}{3d(b\tan(c+dx))^{3/2}}+(A-C)\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx \\
&= -\frac{2bC}{3d(b\tan(c+dx))^{3/2}}+\frac{(b(A-C))\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}\,(b^2+x^2)}\,dx,x,b\tan(c+dx)\right)}{d} \\
&= -\frac{2bC}{3d(b\tan(c+dx))^{3/2}}+\frac{(2b(A-C))\mathrm{Subst}\left(\int \frac{1}{b^2+x^4}\,dx,x,\sqrt{b\tan(c+dx)}\right)}{d} \\
&= -\frac{2bC}{3d(b\tan(c+dx))^{3/2}}+\frac{(A-C)\mathrm{Subst}\left(\int \frac{b-x^2}{b^2+x^4}\,dx,x,\sqrt{b\tan(c+dx)}\right)}{d}+\frac{(A \\
&= -\frac{2bC}{3d(b\tan(c+dx))^{3/2}}+\frac{(A-C)\mathrm{Subst}\left(\int \frac{1}{b-\sqrt{2}\sqrt{b}x+x^2}\,dx,x,\sqrt{b\tan(c+dx}\right.}{2d} \\
&= -\frac{(A-C)\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}\sqrt{b}\,d}+\frac{(A-C)\log}{} \\
&= -\frac{(A-C)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\sqrt{b}\,d}+\frac{(A-C)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{bt}}{}\right.}{\sqrt{2}\sqrt{b}\,d}
\end{aligned}
$$

Mathematica [A]
time = 0.90, size = 148, normalized size = 0.64

$$
\frac{-8C\cot(c+dx)-3\sqrt{2}(A-C)\left(2\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-2\mathrm{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)+\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)-\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\right)\sqrt{\tan(c+dx)}}{12d\sqrt{b\tan(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cot[c + d*x]^2)/Sqrt[b*Tan[c + d*x]],x]
```

```
[Out] (-8*C*Cot[c + d*x] - 3*Sqrt[2]*(A - C)*(2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d
*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + Log[1 - Sqrt[2]*Sqrt[Tan
[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d
*x]])*Sqrt[Tan[c + d*x]])/(12*d*Sqrt[b*Tan[c + d*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 12.09, size = 2454, normalized size = 10.53

method	result	size
default	Expression too large to display	2454

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cot(d*x+c)^2)/(b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)^2*(3*I*A*((cos(d*x+c)-1)/sin(d*x+c))
^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x
+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin
(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-3*I*A*((cos(d*x+c)-1)/sin(d*x+c))^(1/
2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*I*C*((cos(d*x+c)-1)/sin(d*x+
c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(
d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-3*I*A*((cos(d*x+c)-1)/sin(d*x+c))^
(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*I*C*((cos(d*x+c)-1)/sin(d*x+c))^(1/2
)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/
sin(d*x+c))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c
))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*I*C*((cos(d*x+c)-1)/sin(d*x+c
))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/s
in(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*A*((cos(d*x+c)-1)/sin(d*x+c))^(1/
2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))
/sin(d*x+c))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+
c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-3*A*((cos(d*x+c)-1)/sin(d*x+c)
)^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*
x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+6*A*((cos(d*x+c)-1)/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticF(((1-cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+3*I*C*((cos(d*x+c)-1)/sin(d*x+c
))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticPi(((1-cos(d*x+c)+si
n(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+3*I*A*((cos(d*x+c)-1)/si
n(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c
)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticPi(((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*C*((cos(d*x+c)-
1)/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(
```

```
d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticPi(((1-c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*C*((cos(d*
x+c)-1)/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1
-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticPi(
((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-6*C*((c
os(d*x+c)-1)/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2
)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*cos(d*x+c)*Ellipt
icF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-3*A*((cos(d*x
+c)-1)/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-
cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi(((1-cos(d*x+
c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-3*A*((cos(d*x+c)-1)
/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*
x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+6*A*((cos(d*x+c)-1)/sin(d
*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+s
in(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticF(((1-cos(d*x+c)+sin(d*x+c)
)/sin(d*x+c))^(1/2),1/2*2^(1/2))+3*C*((cos(d*x+c)-1)/sin(d*x+c))^(1/2)*((-1
+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*
x+c))^(1/2)*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1
/2),1/2+1/2*I,1/2*2^(1/2))+3*C*((cos(d*x+c)-1)/sin(d*x+c))^(1/2)*((-1+cos(d
*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^
(1/2)*sin(d*x+c)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/
2-1/2*I,1/2*2^(1/2))-6*C*((cos(d*x+c)-1)/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+
sin(d*x+c))/sin(d*x+c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*
sin(d*x+c)*EllipticF(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/
2))+2*C*2^(1/2)*cos(d*x+c)^2)/sin(d*x+c)^5/cos(d*x+c)/(b*sin(d*x+c)/cos(d*x
+c))^(1/2)*2^(1/2)
```

<u>Maxima</u> [**A**]
time = 0.53, size = 179, normalized size = 0.77

$$\frac{3\left(2\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}+2\sqrt{b\tan\left(dx+c\right)}\right)}{2\sqrt{b}}\right)+2\sqrt{2}\sqrt{b}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}-2\sqrt{b\tan\left(dx+c\right)}\right)}{2\sqrt{b}}\right)+\sqrt{2}\sqrt{b}\log\left(b\tan\left(dx+c\right)+\sqrt{2}\sqrt{b\tan\left(dx+c\right)}\sqrt{b}+b\right)-\sqrt{2}\sqrt{b}\log\left(b\tan\left(dx+c\right)-\sqrt{2}\sqrt{b\tan\left(dx+c\right)}\sqrt{b}+b\right)\right)\left(A-C\right)-\frac{8Cb^2}{\left(b\tan(dx+c)\right)^{\frac{3}{2}}}}{12\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cot(d*x+c)^2)/(b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(3*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(b) + 2*sqrt(b*t
an(d*x + c)))/sqrt(b)) + 2*sqrt(2)*sqrt(b)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqr
t(b) - 2*sqrt(b*tan(d*x + c)))/sqrt(b)) + sqrt(2)*sqrt(b)*log(b*tan(d*x + c
) + sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b) - sqrt(2)*sqrt(b)*log(b*tan(d
*x + c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b))*(A - C) - 8*C*b^2/(b*t
an(d*x + c))^(3/2))/(b*d)
```

<u>Fricas</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1236 vs. $2(182) = 364$.

time = 2.75, size = 1236, normalized size = 5.30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cot(d*x+c)^2)/(b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(8*C*sqrt(b*sin(d*x + c)/cos(d*x + c))*cos(d*x + c)^2 + 12*(sqrt(2)*b*
d*cos(d*x + c)^2 - sqrt(2)*b*d)*((A^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^4
)/(b^2*d^4))^(1/4)*arctan((sqrt(2)*(A - C)*b*d^3*sqrt(b*sin(d*x + c)/cos(d*
x + c))*((A^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^4)/(b^2*d^4))^(3/4) + sqr
t(2)*b*d^3*sqrt((b^2*d^2*sqrt((A^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^4)/(
b^2*d^4))*cos(d*x + c) + sqrt(2)*(A - C)*b*d*sqrt(b*sin(d*x + c)/cos(d*x +
c))*((A^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^4)/(b^2*d^4))^(1/4)*cos(d*x +
 c) + (A^2 - 2*A*C + C^2)*b*sin(d*x + c))/cos(d*x + c))*((A^4 - 4*A^3*C + 6
*A^2*C^2 - 4*A*C^3 + C^4)/(b^2*d^4))^(3/4) + A^4 - 4*A^3*C + 6*A^2*C^2 - 4*
A*C^3 + C^4)/(A^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^4)) + 12*(sqrt(2)*b*d
*cos(d*x + c)^2 - sqrt(2)*b*d)*((A^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^4)
/(b^2*d^4))^(1/4)*arctan((sqrt(2)*(A - C)*b*d^3*sqrt(b*sin(d*x + c)/cos(d*x
 + c))*((A^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^4)/(b^2*d^4))^(3/4) + sqrt
(2)*b*d^3*sqrt((b^2*d^2*sqrt((A^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^4)/(b
^2*d^4))*cos(d*x + c) - sqrt(2)*(A - C)*b*d*sqrt(b*sin(d*x + c)/cos(d*x + c
))*((A^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^4)/(b^2*d^4))^(1/4)*cos(d*x +
c) + (A^2 - 2*A*C + C^2)*b*sin(d*x + c))/cos(d*x + c))*((A^4 - 4*A^3*C + 6*
A^2*C^2 - 4*A*C^3 + C^4)/(b^2*d^4))^(3/4) - A^4 + 4*A^3*C - 6*A^2*C^2 + 4*A
*C^3 - C^4)/(A^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^4)) + 3*(sqrt(2)*b*d*c
os(d*x + c)^2 - sqrt(2)*b*d)*((A^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^4)/(
b^2*d^4))^(1/4)*log((b^2*d^2*sqrt((A^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^
4)/(b^2*d^4))*cos(d*x + c) + sqrt(2)*(A - C)*b*d*sqrt(b*sin(d*x + c)/cos(d*
x + c))*((A^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^4)/(b^2*d^4))^(1/4)*cos(d
*x + c) + (A^2 - 2*A*C + C^2)*b*sin(d*x + c))/cos(d*x + c)) - 3*(sqrt(2)*b*
d*cos(d*x + c)^2 - sqrt(2)*b*d)*((A^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^4
)/(b^2*d^4))^(1/4)*log((b^2*d^2*sqrt((A^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 +
 C^4)/(b^2*d^4))*cos(d*x + c) - sqrt(2)*(A - C)*b*d*sqrt(b*sin(d*x + c)/cos
(d*x + c))*((A^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^4)/(b^2*d^4))^(1/4)*co
s(d*x + c) + (A^2 - 2*A*C + C^2)*b*sin(d*x + c))/cos(d*x + c)))/(b*d*cos(d*
x + c)^2 - b*d)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + C\cot^2\left(c + dx\right)}{\sqrt{b\tan\left(c + dx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cot(d*x+c)**2)/(b*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + C*cot(c + d*x)**2)/sqrt(b*tan(c + d*x)), x)
```

Giac [A]
time = 0.65, size = 248, normalized size = 1.06

$$\frac{\sqrt{2}\left(A\sqrt{|b|}-C\sqrt{|b|}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{2bd}+\frac{\sqrt{2}\left(A\sqrt{|b|}-C\sqrt{|b|}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{2bd}+\frac{\sqrt{2}\left(A\sqrt{|b|}-C\sqrt{|b|}\right)\log\left(b\tan(dx+c)+\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{|b|}+|b|\right)}{4bd}-\frac{\sqrt{2}\left(A\sqrt{|b|}-C\sqrt{|b|}\right)\log\left(b\tan(dx+c)-\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{|b|}+|b|\right)}{4bd}-\frac{2C}{3\sqrt{b\tan(dx+c)}\,d\tan(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cot(d*x+c)^2)/(b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(A*sqrt(abs(b)) - C*sqrt(abs(b)))*arctan(1/2*sqrt(2)*(sqrt(2)*s
qrt(abs(b)) + 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/(b*d) + 1/2*sqrt(2)*(A*
sqrt(abs(b)) - C*sqrt(abs(b)))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) -
2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/(b*d) + 1/4*sqrt(2)*(A*sqrt(abs(b)) -
 C*sqrt(abs(b)))*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs
(b)) + abs(b))/(b*d) - 1/4*sqrt(2)*(A*sqrt(abs(b)) - C*sqrt(abs(b)))*log(b*
tan(d*x + c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/(b*d) -
2/3*C/(sqrt(b*tan(d*x + c))*d*tan(d*x + c))
```

Mupad [B]
time = 0.92, size = 828, normalized size = 3.55

$$-\frac{2Cb}{3d\,(b\tan(c+dx))^{3/2}}+\frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{\frac{(-1)^{1/4}(A-C)\left(\sqrt{b\tan(c+dx)}\,(16A^2b^2d^3-32ACb^2d^3+16C^2b^2d^3)-\frac{(-1)^{1/4}(A-C)(32Ab^3d^4-32Cb^3d^4)}{2\sqrt{b}\,d}\right)1i}{2\sqrt{b}\,d}+\frac{(-1)^{1/4}(A-C)\left(\sqrt{b\tan(c+dx)}\,(16A^2b^2d^3-32ACb^2d^3+16C^2b^2d^3)+\frac{(-1)^{1/4}(A-C)(32Ab^3d^4-32Cb^3d^4)}{2\sqrt{b}\,d}\right)1i}{2\sqrt{b}\,d}}{\frac{(-1)^{1/4}(A-C)\left(\sqrt{b\tan(c+dx)}\,(16A^2b^2d^3-32ACb^2d^3+16C^2b^2d^3)-\frac{(-1)^{1/4}(A-C)(32Ab^3d^4-32Cb^3d^4)}{2\sqrt{b}\,d}\right)}{2\sqrt{b}\,d}-\frac{(-1)^{1/4}(A-C)\left(\sqrt{b\tan(c+dx)}\,(16A^2b^2d^3-32ACb^2d^3+16C^2b^2d^3)+\frac{(-1)^{1/4}(A-C)(32Ab^3d^4-32Cb^3d^4)}{2\sqrt{b}\,d}\right)}{2\sqrt{b}\,d}}\right)(A-C)\,1i}{\sqrt{b}\,d}+\frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{\frac{(-1)^{1/4}(A-C)\left(\sqrt{b\tan(c+dx)}\,(16A^2b^2d^3-32ACb^2d^3+16C^2b^2d^3)-\frac{(-1)^{1/4}(A-C)(32Ab^3d^4-32Cb^3d^4)\,1i}{2\sqrt{b}\,d}\right)}{2\sqrt{b}\,d}+\frac{(-1)^{1/4}(A-C)\left(\sqrt{b\tan(c+dx)}\,(16A^2b^2d^3-32ACb^2d^3+16C^2b^2d^3)+\frac{(-1)^{1/4}(A-C)(32Ab^3d^4-32Cb^3d^4)\,1i}{2\sqrt{b}\,d}\right)}{2\sqrt{b}\,d}}{\frac{(-1)^{1/4}(A-C)\left(\sqrt{b\tan(c+dx)}\,(16A^2b^2d^3-32ACb^2d^3+16C^2b^2d^3)-\frac{(-1)^{1/4}(A-C)(32Ab^3d^4-32Cb^3d^4)\,1i}{2\sqrt{b}\,d}\right)1i}{2\sqrt{b}\,d}-\frac{(-1)^{1/4}(A-C)\left(\sqrt{b\tan(c+dx)}\,(16A^2b^2d^3-32ACb^2d^3+16C^2b^2d^3)+\frac{(-1)^{1/4}(A-C)(32Ab^3d^4-32Cb^3d^4)\,1i}{2\sqrt{b}\,d}\right)1i}{2\sqrt{b}\,d}}\right)(A-C)}{\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cot(c + d*x)^2)/(b*tan(c + d*x))^(1/2),x)
```

```
[Out] ((-1)^(1/4)*atan((((-1)^(1/4)*(A - C)*((b*tan(c + d*x))^(1/2)*(16*A^2*b^2*d
^3 + 16*C^2*b^2*d^3 - 32*A*C*b^2*d^3) - ((-1)^(1/4)*(A - C)*(32*A*b^3*d^4 -
 32*C*b^3*d^4))/(2*b^(1/2)*d))*1i)/(2*b^(1/2)*d) + ((-1)^(1/4)*(A - C)*((b*
tan(c + d*x))^(1/2)*(16*A^2*b^2*d^3 + 16*C^2*b^2*d^3 - 32*A*C*b^2*d^3) + ((
-1)^(1/4)*(A - C)*(32*A*b^3*d^4 - 32*C*b^3*d^4))/(2*b^(1/2)*d))*1i)/(2*b^(1
/2)*d))/(((-1)^(1/4)*(A - C)*((b*tan(c + d*x))^(1/2)*(16*A^2*b^2*d^3 + 16*C
^2*b^2*d^3 - 32*A*C*b^2*d^3) - ((-1)^(1/4)*(A - C)*(32*A*b^3*d^4 - 32*C*b^3
*d^4))/(2*b^(1/2)*d)))/(2*b^(1/2)*d) - ((-1)^(1/4)*(A - C)*((b*tan(c + d*x)
)^(1/2)*(16*A^2*b^2*d^3 + 16*C^2*b^2*d^3 - 32*A*C*b^2*d^3) + ((-1)^(1/4)*(A
 - C)*(32*A*b^3*d^4 - 32*C*b^3*d^4))/(2*b^(1/2)*d)))/(2*b^(1/2)*d)))*(A - C
)*1i)/(b^(1/2)*d) - (2*C*b)/(3*d*(b*tan(c + d*x))^(3/2)) + ((-1)^(1/4)*atan
((((-1)^(1/4)*(A - C)*((b*tan(c + d*x))^(1/2)*(16*A^2*b^2*d^3 + 16*C^2*b^2*
d^3 - 32*A*C*b^2*d^3) - ((-1)^(1/4)*(A - C)*(32*A*b^3*d^4 - 32*C*b^3*d^4)*1
i)/(2*b^(1/2)*d)))/(2*b^(1/2)*d) + ((-1)^(1/4)*(A - C)*((b*tan(c + d*x))^(1
/2)*(16*A^2*b^2*d^3 + 16*C^2*b^2*d^3 - 32*A*C*b^2*d^3) + ((-1)^(1/4)*(A - C
)*(32*A*b^3*d^4 - 32*C*b^3*d^4)*1i)/(2*b^(1/2)*d)))/(2*b^(1/2)*d))/(((-1)^(
1/4)*(A - C)*((b*tan(c + d*x))^(1/2)*(16*A^2*b^2*d^3 + 16*C^2*b^2*d^3 - 32*
```

```
A*C*b^2*d^3) - ((-1)^(1/4)*(A - C)*(32*A*b^3*d^4 - 32*C*b^3*d^4)*1i)/(2*b^(
1/2)*d))*1i)/(2*b^(1/2)*d) - ((-1)^(1/4)*(A - C)*((b*tan(c + d*x))^(1/2)*(1
6*A^2*b^2*d^3 + 16*C^2*b^2*d^3 - 32*A*C*b^2*d^3) + ((-1)^(1/4)*(A - C)*(32*
A*b^3*d^4 - 32*C*b^3*d^4)*1i)/(2*b^(1/2)*d))*1i)/(2*b^(1/2)*d)))*(A - C))/(
b^(1/2)*d)
```

3.2 $\int (a+b\cot^2(c+dx))\,dx$

Optimal. Leaf size=20

$$ax-bx-\frac{b\cot(c+dx)}{d}$$

```
[Out] a*x-b*x-b*cot(d*x+c)/d
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3554, 8}

$$ax-\frac{b\cot(c+dx)}{d}-bx$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Cot[c + d*x]^2,x]
```

```
[Out] a*x - b*x - (b*Cot[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int (a+b\cot^2(c+dx))\,dx &= ax+b\int \cot^2(c+dx)\,dx\\
&= ax-\frac{b\cot(c+dx)}{d}-b\int 1\,dx\\
&= ax-bx-\frac{b\cot(c+dx)}{d}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 34, normalized size = 1.70

$$ax-\frac{b\cot(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Cot[c + d*x]^2,x]
[Out] a*x - (b*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d*x]^2])/d
```

Maple [A]
time = 0.08, size = 31, normalized size = 1.55

method	result	size
risch	$ax-bx-\frac{2ib}{d\left(e^{2i(dx+c)}-1\right)}$	29
norman	$\frac{(a-b)x\tan(dx+c)-\frac{b}{d}}{\tan(dx+c)}$	30
default	$ax+\frac{b\left(-\cot(dx+c)+\frac{\pi}{2}-\mathrm{arccot}(\cot(dx+c))\right)}{d}$	31
derivativedivides	$\frac{-b\cot(dx+c)+(-a+b)\left(\frac{\pi}{2}-\mathrm{arccot}(\cot(dx+c))\right)}{d}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*cot(d*x+c)^2,x,method=_RETURNVERBOSE)
[Out] a*x+b/d*(-cot(d*x+c)+1/2*Pi-arccot(cot(d*x+c)))
```

Maxima [A]
time = 0.52, size = 23, normalized size = 1.15

$$ax-\frac{\left(dx+c+\frac{1}{\tan(dx+c)}\right)b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cot(d*x+c)^2,x, algorithm="maxima")
[Out] a*x - (d*x + c + 1/tan(d*x + c))*b/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(20) = 40.
time = 2.72, size = 48, normalized size = 2.40

$$\frac{(a-b)dx\sin(2\,dx+2\,c)-b\cos(2\,dx+2\,c)-b}{d\sin(2\,dx+2\,c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cot(d*x+c)^2,x, algorithm="fricas")
[Out] ((a - b)*d*x*sin(2*d*x + 2*c) - b*cos(2*d*x + 2*c) - b)/(d*sin(2*d*x + 2*c)
)
```

Sympy [A]
time = 0.05, size = 22, normalized size = 1.10

$$ax + b \left(\begin{cases} -x - \frac{\cot(c+dx)}{d} & \text{for } d \neq 0 \\ x\cot^2(c) & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cot(d*x+c)**2,x)
```

```
[Out] a*x + b*Piecewise((-x - cot(c + d*x)/d, Ne(d, 0)), (x*cot(c)**2, True))
```

Giac [A]
time = 0.46, size = 40, normalized size = 2.00

$$ax - \frac{\left(2\,dx + 2\,c + \frac{1}{\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)} - \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)b}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cot(d*x+c)^2,x, algorithm="giac")
```

```
[Out] a*x - 1/2*(2*d*x + 2*c + 1/tan(1/2*d*x + 1/2*c) - tan(1/2*d*x + 1/2*c))*b/d
```

Mupad [B]
time = 0.34, size = 20, normalized size = 1.00

$$x\,(a-b) - \frac{b\cot(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*cot(c + d*x)^2,x)
```

```
[Out] x*(a - b) - (b*cot(c + d*x))/d
```

3.3 $\int (a+b\cot^2(c+dx))^2\,dx$

Optimal. Leaf size=47

$$(a-b)^2x-\frac{(2a-b)b\cot(c+dx)}{d}-\frac{b^2\cot^3(c+dx)}{3d}$$

```
[Out] (a-b)^2*x-(2*a-b)*b*cot(d*x+c)/d-1/3*b^2*cot(d*x+c)^3/d
```

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.214$, Rules used = {3742, 398, 209}

$$-\frac{b(2a-b)\cot(c+dx)}{d}+x(a-b)^2-\frac{b^2\cot^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[c + d*x]^2)^2,x]
```

```
[Out] (a - b)^2*x - ((2*a - b)*b*Cot[c + d*x])/d - (b^2*Cot[c + d*x]^3)/(3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \left(a+b\cot^2(c+dx)\right)^2\,dx=-\frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{1+x^2}\,dx,x,\cot(c+dx)\right)}{d}$$
$$=-\frac{\text{Subst}\left(\int \left((2a-b)b+b^2x^2+\frac{(a-b)^2}{1+x^2}\right)\,dx,x,\cot(c+dx)\right)}{d}$$
$$=-\frac{(2a-b)b\cot(c+dx)}{d}-\frac{b^2\cot^3(c+dx)}{3d}-\frac{(a-b)^2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\cot(c+dx)\right)}{d}$$
$$=(a-b)^2x-\frac{(2a-b)b\cot(c+dx)}{d}-\frac{b^2\cot^3(c+dx)}{3d}$$

Mathematica [A]
time = 1.25, size = 71, normalized size = 1.51

$$-\frac{\cot(c+dx)\left(b(6a-3b+b\cot^2(c+dx))+3(a-b)^2\tanh^{-1}\left(\sqrt{-\tan^2(c+dx)}\right)\sqrt{-\tan^2(c+dx)}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[c + d*x]^2)^2,x]
```

```
[Out] -1/3*(Cot[c + d*x]*(b*(6*a - 3*b + b*Cot[c + d*x]^2) + 3*(a - b)^2*ArcTanh[
Sqrt[-Tan[c + d*x]^2]]*Sqrt[-Tan[c + d*x]^2]))/d
```

Maple [A]
time = 0.09, size = 68, normalized size = 1.45

method	result	size
norman	$\frac{(a^2-2ab+b^2)x(\tan^3(dx+c))-\frac{b^2}{3d}-\frac{b(2a-b)\left(\tan^2(dx+c)\right)}{d}}{\tan(dx+c)^3}$	61
derivativedivides	$\frac{-\frac{b^2\left(\cot^3(dx+c)\right)}{3}-2ab\cot(dx+c)+b^2\cot(dx+c)+(-a^2+2ab-b^2)\left(\frac{\pi}{2}-\text{arccot}(\cot(dx+c))\right)}{d}$	68
default	$\frac{-\frac{b^2\left(\cot^3(dx+c)\right)}{3}-2ab\cot(dx+c)+b^2\cot(dx+c)+(-a^2+2ab-b^2)\left(\frac{\pi}{2}-\text{arccot}(\cot(dx+c))\right)}{d}$	68
risch	$x\,a^2-2xab+x\,b^2+\frac{4ib\left(-3a\,e^{4i(dx+c)}+3b\,e^{4i(dx+c)}+6a\,e^{2i(dx+c)}-3b\,e^{2i(dx+c)}-3a+2b\right)}{3d\left(e^{2i(dx+c)}-1\right)^3}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*b^2*cot(d*x+c)^3-2*a*b*cot(d*x+c)+b^2*cot(d*x+c)+(-a^2+2*a*b-b^2)
*(1/2*Pi-arccot(cot(d*x+c))))
```

Maxima [A]
time = 0.53, size = 63, normalized size = 1.34

$$a^2x - \frac{2\left(dx + c + \frac{1}{\tan(dx+c)}\right)ab}{d} + \frac{\left(3\,dx + 3\,c + \frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)b^2}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*x - 2*(d*x + c + 1/tan(d*x + c))*a*b/d + 1/3*(3*d*x + 3*c + (3*tan(d*x
+ c)^2 - 1)/tan(d*x + c)^3)*b^2/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(45) = 90$.
time = 2.94, size = 127, normalized size = 2.70

$$\frac{2\,b^2\cos\left(2\,dx + 2\,c\right) - 2\left(3\,ab - 2\,b^2\right)\cos\left(2\,dx + 2\,c\right)^2 + 6\,ab - 2\,b^2 + 3\left((a^2 - 2\,ab + b^2)dx\cos\left(2\,dx + 2\,c\right) - (a^2 - 2\,ab + b^2)dx\right)\sin\left(2\,dx + 2\,c\right)}{3\left(d\cos\left(2\,dx + 2\,c\right) - d\right)\sin\left(2\,dx + 2\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*b^2*cos(2*d*x + 2*c) - 2*(3*a*b - 2*b^2)*cos(2*d*x + 2*c)^2 + 6*a*b
- 2*b^2 + 3*((a^2 - 2*a*b + b^2)*d*x*cos(2*d*x + 2*c) - (a^2 - 2*a*b + b^2)
*d*x)*sin(2*d*x + 2*c))/((d*cos(2*d*x + 2*c) - d)*sin(2*d*x + 2*c))
```

Sympy [A]
time = 0.09, size = 68, normalized size = 1.45

$$\begin{cases} a^2x - 2abx - \frac{2ab\cot\left(c+dx\right)}{d} + b^2x - \frac{b^2\cot^3\left(c+dx\right)}{3d} + \frac{b^2\cot\left(c+dx\right)}{d} & \text{for}\ d \neq 0 \\ x\left(a + b\cot^2\left(c\right)\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*x - 2*a*b*x - 2*a*b*cot(c + d*x)/d + b**2*x - b**2*cot(c +
d*x)**3/(3*d) + b**2*cot(c + d*x)/d, Ne(d, 0)), (x*(a + b*cot(c)**2)**2, Tr
ue))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(45) = 90$.
time = 0.44, size = 114, normalized size = 2.43

$$\frac{b^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 + 24\,ab\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 15\,b^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 24\left(a^2 - 2\,ab + b^2\right)(dx + c) - \frac{24\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 15\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + b^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(b^2*tan(1/2*d*x + 1/2*c)^3 + 24*a*b*tan(1/2*d*x + 1/2*c) - 15*b^2*tan
(1/2*d*x + 1/2*c) + 24*(a^2 - 2*a*b + b^2)*(d*x + c) - (24*a*b*tan(1/2*d*x
+ 1/2*c)^2 - 15*b^2*tan(1/2*d*x + 1/2*c)^2 + b^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B]
time = 0.12, size = 45, normalized size = 0.96

$$x\,{\left(a-b\right)}^2-\frac{b^2\,{\mathrm{cot}\left(c+d\,x\right)}^3}{3\,d}-\frac{b\,\mathrm{cot}\left(c+d\,x\right)\,\left(2\,a-b\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(c + d*x)^2)^2,x)
```

```
[Out] x*(a - b)^2 - (b^2*cot(c + d*x)^3)/(3*d) - (b*cot(c + d*x)*(2*a - b))/d
```

3.4 $\int (a+b\cot^2(c+dx))^3 \, dx$

Optimal. Leaf size=78

$$(a-b)^3x-\frac{b(3a^2-3ab+b^2)\cot(c+dx)}{d}-\frac{(3a-b)b^2\cot^3(c+dx)}{3d}-\frac{b^3\cot^5(c+dx)}{5d}$$

```
[Out] (a-b)^3*x-b*(3*a^2-3*a*b+b^2)*cot(d*x+c)/d-1/3*(3*a-b)*b^2*cot(d*x+c)^3/d-1
/5*b^3*cot(d*x+c)^5/d
```

Rubi [A]

time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3742, 398, 209}

$$-\frac{b(3a^2-3ab+b^2)\cot(c+dx)}{d}-\frac{b^2(3a-b)\cot^3(c+dx)}{3d}+x(a-b)^3-\frac{b^3\cot^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[c + d*x]^2)^3,x]
```

```
[Out] (a - b)^3*x - (b*(3*a^2 - 3*a*b + b^2)*Cot[c + d*x])/d - ((3*a - b)*b^2*Cot
[c + d*x]^3)/(3*d) - (b^3*Cot[c + d*x]^5)/(5*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int (a+b\cot^2(c+dx))^3\,dx = -\frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{1+x^2}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(b(3a^2-3ab+b^2)+(3a-b)b^2x^2+b^3x^4+\frac{(a-b)^3}{1+x^2}\right)dx, x, \cot(c+$$
$$= -\frac{b(3a^2-3ab+b^2)\cot(c+dx)}{d} - \frac{(3a-b)b^2\cot^3(c+dx)}{3d} - \frac{b^3\cot^5(c+dx)}{5d}$$
$$= (a-b)^3x - \frac{b(3a^2-3ab+b^2)\cot(c+dx)}{d} - \frac{(3a-b)b^2\cot^3(c+dx)}{3d} - \frac{b^3\cot$$

Mathematica [A]
time = 2.90, size = 111, normalized size = 1.42

$$-\frac{\cot^5(c+dx)\left(\frac{15(a-b)^3\tanh^{-1}\left(\sqrt{-\tan^2(c+dx)}\right)\tan^8(c+dx)}{(-\tan^2(c+dx))^{3/2}}+b(3b^2+5(3a-b)b\tan^2(c+dx)+15(3a^2-3ab+b^2)\tan^4(c+dx))\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[c + d*x]^2)^3,x]
```

```
[Out] -1/15*(Cot[c + d*x]^5*((15*(a - b)^3*ArcTanh[Sqrt[-Tan[c + d*x]^2]]*Tan[c +
 d*x]^8)/(-Tan[c + d*x]^2)^(3/2) + b*(3*b^2 + 5*(3*a - b)*b*Tan[c + d*x]^2
+ 15*(3*a^2 - 3*a*b + b^2)*Tan[c + d*x]^4)))/d
```

Maple [A]
time = 0.10, size = 116, normalized size = 1.49

method	result
norman	$\frac{(a^3-3a^2b+3ab^2-b^3)x\left(\tan^5(dx+c)\right)-\frac{b^3}{5d}-\frac{b\left(3a^2-3ab+b^2\right)\left(\tan^4(dx+c)\right)}{d}-\frac{b^2(3a-b)\left(\tan^2(dx+c)\right)}{3d}}{\tan(dx+c)^5}$
derivativedivides	$\frac{-\frac{b^3\left(\cot^5(dx+c)\right)}{5}-ab^2\left(\cot^3(dx+c)\right)+\frac{b^3\left(\cot^3(dx+c)\right)}{3}-3a^2b\cot(dx+c)+3ab^2\cot(dx+c)-b^3\cot(dx+c)+\left(-a^3+3a^2b-3\right.}{d}$
default	$\frac{-\frac{b^3\left(\cot^5(dx+c)\right)}{5}-ab^2\left(\cot^3(dx+c)\right)+\frac{b^3\left(\cot^3(dx+c)\right)}{3}-3a^2b\cot(dx+c)+3ab^2\cot(dx+c)-b^3\cot(dx+c)+\left(-a^3+3a^2b-3\right.}{d}$
risch	$a^3x-3a^2bx+3ab^2x-b^3x-\frac{2ib\left(45a^2e^{8i(dx+c)}-90ab\,e^{8i(dx+c)}+45b^2e^{8i(dx+c)}-180a^2e^{6i(dx+c)}+270ab\,e^{6\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5*b^3*cot(d*x+c)^5-a*b^2*cot(d*x+c)^3+1/3*b^3*cot(d*x+c)^3-3*a^2*b*
cot(d*x+c)+3*a*b^2*cot(d*x+c)-b^3*cot(d*x+c)+(-a^3+3*a^2*b-3*a*b^2+b^3)*(1/
2*Pi-arccot(cot(d*x+c))))
```

Maxima [A]
time = 0.49, size = 112, normalized size = 1.44

$$a^3x - \frac{3\left(dx + c + \frac{1}{\tan(dx+c)}\right)a^2b}{d} + \frac{\left(3\,dx + 3\,c + \frac{3\,\tan(dx+c)^2 - 1}{\tan(dx+c)^3}\right)ab^2}{d} - \frac{\left(15\,dx + 15\,c + \frac{15\,\tan(dx+c)^4 - 5\,\tan(dx+c)^2 + 3}{\tan(dx+c)^5}\right)b^3}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a^3*x - 3*(d*x + c + 1/tan(d*x + c))*a^2*b/d + (3*d*x + 3*c + (3*tan(d*x +
c)^2 - 1)/tan(d*x + c)^3)*a*b^2/d - 1/15*(15*d*x + 15*c + (15*tan(d*x + c)^
4 - 5*tan(d*x + c)^2 + 3)/tan(d*x + c)^5)*b^3/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(74) = 148$.
time = 3.26, size = 253, normalized size = 3.24

$$-\frac{(45\,a^2b - 60\,ab^2 + 23\,b^3)\cos(2\,dx + 2\,c)^3 + 45\,a^2b - 30\,ab^2 + 13\,b^3 - (45\,a^2b - 30\,ab^2 + b^3)\cos(2\,dx + 2\,c)^2 - (45\,a^2b - 60\,ab^2 + 11\,b^3)\cos(2\,dx + 2\,c) - 15\left((a^3 - 3\,a^2b + 3\,ab^2 - b^3)dx\cos(2\,dx + 2\,c)^2 - 2\,(a^3 - 3\,a^2b + 3\,ab^2 - b^3)dx\cos(2\,dx + 2\,c) + (a^3 - 3\,a^2b + 3\,ab^2 - b^3)dx\right)\sin(2\,dx + 2\,c)}{15\left(d\cos(2\,dx + 2\,c)^2 - 2\,d\cos(2\,dx + 2\,c) + d\right)\sin(2\,dx + 2\,c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/15*((45*a^2*b - 60*a*b^2 + 23*b^3)*cos(2*d*x + 2*c)^3 + 45*a^2*b - 30*a*
b^2 + 13*b^3 - (45*a^2*b - 30*a*b^2 + b^3)*cos(2*d*x + 2*c)^2 - (45*a^2*b -
 60*a*b^2 + 11*b^3)*cos(2*d*x + 2*c) - 15*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*
d*x*cos(2*d*x + 2*c)^2 - 2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x*cos(2*d*x +
2*c) + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x)*sin(2*d*x + 2*c))/((d*cos(2*d*x
 + 2*c)^2 - 2*d*cos(2*d*x + 2*c) + d)*sin(2*d*x + 2*c))
```

Sympy [A]
time = 0.13, size = 126, normalized size = 1.62

$$\begin{cases} a^3x - 3a^2bx - \frac{3a^2b\cot(c+dx)}{d} + 3ab^2x - \frac{ab^2\cot^3(c+dx)}{d} + \frac{3ab^2\cot(c+dx)}{d} - b^3x - \frac{b^3\cot^5(c+dx)}{5d} + \frac{b^3\cot^3(c+dx)}{3d} - \frac{b^3\cot(c+dx)}{d} & \text{for } d \neq 0 \\ x\left(a + b\cot^2(c)\right)^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*x - 3*a**2*b*x - 3*a**2*b*cot(c + d*x)/d + 3*a*b**2*x - a*b
**2*cot(c + d*x)**3/d + 3*a*b**2*cot(c + d*x)/d - b**3*x - b**3*cot(c + d*x
)**5/(5*d) + b**3*cot(c + d*x)**3/(3*d) - b**3*cot(c + d*x)/d, Ne(d, 0)), (
x*(a + b*cot(c)**2)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(74) = 148$.

time = 0.53, size = 229, normalized size = 2.94

$$\frac{3\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+60\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-35\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+720\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-900\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+330\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+480\,(a^3-3\,a^2b+3\,ab^2-b^3)(dx+c)-\frac{720\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-900\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+330\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+60\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-35\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,b^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/480*(3*b^3*tan(1/2*d*x + 1/2*c)^5 + 60*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 35*
b^3*tan(1/2*d*x + 1/2*c)^3 + 720*a^2*b*tan(1/2*d*x + 1/2*c) - 900*a*b^2*tan
(1/2*d*x + 1/2*c) + 330*b^3*tan(1/2*d*x + 1/2*c) + 480*(a^3 - 3*a^2*b + 3*a
*b^2 - b^3)*(d*x + c) - (720*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 900*a*b^2*tan(1
/2*d*x + 1/2*c)^4 + 330*b^3*tan(1/2*d*x + 1/2*c)^4 + 60*a*b^2*tan(1/2*d*x +
 1/2*c)^2 - 35*b^3*tan(1/2*d*x + 1/2*c)^2 + 3*b^3)/tan(1/2*d*x + 1/2*c)^5)/
d
```

Mupad [B]

time = 0.45, size = 76, normalized size = 0.97

$$x\,{\left(a-b\right)}^3-\frac{b^3\,{\mathrm{cot}\left(c+d\,x\right)}^5}{5\,d}-\frac{{\mathrm{cot}\left(c+d\,x\right)}^3\,\left(3\,a\,b^2-b^3\right)}{3\,d}-\frac{b\,\mathrm{cot}\left(c+d\,x\right)\,\left(3\,a^2-3\,a\,b+b^2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(c + d*x)^2)^3,x)
```

```
[Out] x*(a - b)^3 - (b^3*cot(c + d*x)^5)/(5*d) - (cot(c + d*x)^3*(3*a*b^2 - b^3))
/(3*d) - (b*cot(c + d*x)*(3*a^2 - 3*a*b + b^2))/d
```

3.5 $\int \frac{1}{a+b\cot^2(c+dx)}\,dx$

Optimal. Leaf size=49

$$\frac{x}{a-b}+\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\cot(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)d}$$

```
[Out] x/(a-b)+arctan(cot(d*x+c)*b^(1/2)/a^(1/2))*b^(1/2)/(a-b)/d/a^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3741, 3756, 211}

$$\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\cot(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)}+\frac{x}{a-b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[c + d*x]^2)^(-1),x]
```

```
[Out] x/(a - b) + (Sqrt[b]*ArcTan[(Sqrt[b]*Cot[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a -
b)*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3741

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/(a -
 b), x] - Dist[b/(a - b), Int[Sec[e + f*x]^2/(a + b*Tan[e + f*x]^2), x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a, b]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{a+b\cot^2(c+dx)}\,dx &= \frac{x}{a-b}-\frac{b\int \frac{\csc^2(c+dx)}{a+b\cot^2(c+dx)}\,dx}{a-b}\\
&= \frac{x}{a-b}+\frac{b\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\cot(c+dx)\right)}{(a-b)d}\\
&= \frac{x}{a-b}+\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\cot(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)d}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 49, normalized size = 1.00

$$
\frac{\mathrm{ArcTan}(\tan(c+dx))-\frac{\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}}}{ad-bd}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[c + d*x]^2)^(-1),x]
[Out] (ArcTan[Tan[c + d*x]] - (Sqrt[b]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[b]])/Sq
rt[a])/(a*d - b*d)
```

Maple [A]
time = 0.26, size = 56, normalized size = 1.14

method	result	size
derivativedivides	$\frac{-\frac{\frac{\pi}{2}-\mathrm{arccot}(\cot(dx+c))}{a-b}+\frac{b\arctan\left(\frac{b\cot(dx+c)}{\sqrt{ab}}\right)}{(a-b)\sqrt{ab}}}{d}$	56
default	$\frac{-\frac{\frac{\pi}{2}-\mathrm{arccot}(\cot(dx+c))}{a-b}+\frac{b\arctan\left(\frac{b\cot(dx+c)}{\sqrt{ab}}\right)}{(a-b)\sqrt{ab}}}{d}$	56
risch	$\frac{x}{a-b}+\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2i(dx+c)}-\frac{2i\sqrt{-ab}+a+b}{a-b}\right)}{2a(a-b)d}-\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2i(dx+c)}+\frac{2i\sqrt{-ab}-a-b}{a-b}\right)}{2a(a-b)d}$	120

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cot(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/(a-b)*(1/2*Pi-arccot(cot(d*x+c)))+1/(a-b)*b/(a*b)^(1/2)*arctan(b*co
t(d*x+c)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.49, size = 48, normalized size = 0.98

$$-\frac{\frac{b\arctan\left(\frac{a\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}\,(a-b)}-\frac{dx+c}{a-b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -(b*arctan(a*tan(d*x + c)/sqrt(a*b))/(sqrt(a*b)*(a - b)) - (d*x + c)/(a - b
))/d
```

Fricas [A]
time = 3.49, size = 252, normalized size = 5.14

$$\left[\frac{4\,dx-\sqrt{-\frac{b}{a}}\log\left(\frac{\left(a^2+6\,ab+b^2\right)\cos\left(2\,dx+2\,c\right)^2+4\left(a^2-ab-\left(a^2+ab\right)\cos\left(2\,dx+2\,c\right)\right)\sqrt{-\frac{b}{a}}\sin\left(2\,dx+2\,c\right)+a^2-6\,ab+b^2-2\left(a^2-b^2\right)\cos\left(2\,dx+2\,c\right)}{\left(a^2-2\,ab+b^2\right)\cos\left(2\,dx+2\,c\right)^2+a^2+2\,ab+b^2-2\left(a^2-b^2\right)\cos\left(2\,dx+2\,c\right)}\right)}{4\left(a-b\right)d},\ \frac{2\,dx+\sqrt{\frac{b}{a}}\arctan\left(\frac{\left(\left(a+b\right)\cos\left(2\,dx+2\,c\right)-a+b\right)\sqrt{\frac{b}{a}}}{2\,b\sin\left(2\,dx+2\,c\right)}\right)}{2\left(a-b\right)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*d*x - sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(2*d*x + 2*c)^2 + 4*(a
^2 - a*b - (a^2 + a*b)*cos(2*d*x + 2*c))*sqrt(-b/a)*sin(2*d*x + 2*c) + a^2
- 6*a*b + b^2 - 2*(a^2 - b^2)*cos(2*d*x + 2*c))/((a^2 - 2*a*b + b^2)*cos(2*
d*x + 2*c)^2 + a^2 + 2*a*b + b^2 - 2*(a^2 - b^2)*cos(2*d*x + 2*c))))/((a -
b)*d), 1/2*(2*d*x + sqrt(b/a)*arctan(1/2*((a + b)*cos(2*d*x + 2*c) - a + b)
*sqrt(b/a)/(b*sin(2*d*x + 2*c))))/((a - b)*d)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(37) = 74$.
time = 0.70, size = 238, normalized size = 4.86

$$\begin{cases} \frac{\tilde{\infty}x}{\cot^2(c)} & \text{for } a=0\wedge b=0\wedge d=0 \\ \frac{x}{a} & \text{for } b=0 \\ \frac{-x+\frac{1}{d\cot(c+dx)}}{b} & \text{for } a=0 \\ \frac{dx\cot^2(c+dx)}{2bd\cot^2(c+dx)+2bd}+\frac{dx}{2bd\cot^2(c+dx)+2bd}-\frac{\cot(c+dx)}{2bd\cot^2(c+dx)+2bd} & \text{for } a=b \\ \frac{x}{a+b\cot^2(c)} & \text{for } d=0 \\ \frac{2dx\sqrt{-\frac{a}{b}}}{2ad\sqrt{-\frac{a}{b}}-2bd\sqrt{-\frac{a}{b}}}+\frac{\log\left(-\sqrt{-\frac{a}{b}}+\cot(c+dx)\right)}{2ad\sqrt{-\frac{a}{b}}-2bd\sqrt{-\frac{a}{b}}}-\frac{\log\left(\sqrt{-\frac{a}{b}}+\cot(c+dx)\right)}{2ad\sqrt{-\frac{a}{b}}-2bd\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x/cot(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/a, Eq(b, 0)
), ((-x + 1/(d*cot(c + d*x)))/b, Eq(a, 0)), (d*x*cot(c + d*x)**2/(2*b*d*cot
(c + d*x)**2 + 2*b*d) + d*x/(2*b*d*cot(c + d*x)**2 + 2*b*d) - cot(c + d*x)/
(2*b*d*cot(c + d*x)**2 + 2*b*d), Eq(a, b)), (x/(a + b*cot(c)**2), Eq(d, 0))
, (2*d*x*sqrt(-a/b)/(2*a*d*sqrt(-a/b) - 2*b*d*sqrt(-a/b)) + log(-sqrt(-a/b)
 + cot(c + d*x))/(2*a*d*sqrt(-a/b) - 2*b*d*sqrt(-a/b)) - log(sqrt(-a/b) + c
ot(c + d*x))/(2*a*d*sqrt(-a/b) - 2*b*d*sqrt(-a/b)), True))
```

Giac [A]
time = 0.49, size = 65, normalized size = 1.33

$$-\frac{\frac{\left(\pi \left\lfloor \frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(a)+\arctan\left(\frac{a\tan(dx+c)}{\sqrt{ab}}\right)\right)b}{\sqrt{ab}\,(a-b)}-\frac{dx+c}{a-b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((d*x + c)/pi + 1/2)*sgn(a) + arctan(a*tan(d*x + c)/sqrt(a*b)))*
b/(sqrt(a*b)*(a - b)) - (d*x + c)/(a - b))/d
```

Mupad [B]
time = 0.12, size = 41, normalized size = 0.84

$$\frac{x}{a-b}+\frac{b\,\mathrm{atan}\left(\frac{b\cot(c+d\,x)}{\sqrt{a\,b}}\right)}{d\,\sqrt{a\,b}\,\left(a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cot(c + d*x)^2),x)
```

```
[Out] x/(a - b) + (b*atan((b*cot(c + d*x))/(a*b)^(1/2)))/(d*(a*b)^(1/2)*(a - b))
```

3.6 $\int \frac{1}{(a+b\cot^2(c+dx))^2}\,dx$

Optimal. Leaf size=97

$$\frac{x}{(a-b)^2}+\frac{(3a-b)\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\cot(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^2d}+\frac{b\cot(c+dx)}{2a(a-b)d\,(a+b\cot^2(c+dx))}$$

```
[Out] x/(a-b)^2+1/2*b*cot(d*x+c)/a/(a-b)/d/(a+b*cot(d*x+c)^2)+1/2*(3*a-b)*arctan(
cot(d*x+c)*b^(1/2)/a^(1/2))*b^(1/2)/a^(3/2)/(a-b)^2/d
```

Rubi [A]
time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3742, 425, 536, 209, 211}

$$\frac{\sqrt{b}\,(3a-b)\text{ArcTan}\left(\frac{\sqrt{b}\,\cot(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^2}+\frac{b\cot(c+dx)}{2ad(a-b)\,(a+b\cot^2(c+dx))}+\frac{x}{(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[c + d*x]^2)^(-2),x]
```

```
[Out] x/(a - b)^2 + ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Cot[c + d*x])/Sqrt[a]])/(2
*a^(3/2)*(a - b)^2*d) + (b*Cot[c + d*x])/(2*a*(a - b)*d*(a + b*Cot[c + d*x]
^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^2}\,dx, x, \cot(c+dx)\right)}{d} \\
&= \frac{b\cot(c+dx)}{2a(a-b)d\left(a+b\cot^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{2a-b-bx^2}{(1+x^2)(a+bx^2)}\,dx, x, \cot(c+dx)\right)}{2a(a-b)d} \\
&= \frac{b\cot(c+dx)}{2a(a-b)d\left(a+b\cot^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \cot(c+dx)\right)}{(a-b)^2d} + \frac{((3a-
\\
&= \frac{x}{(a-b)^2} + \frac{(3a-b)\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\cot(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^2d} + \frac{b\cot(c+dx)}{2a(a-b)d\left(a+b\cot^2(c+d
\end{aligned}
$$

Mathematica [A]
time = 0.96, size = 90, normalized size = 0.93

$$
\frac{-2\text{ArcTan}(\cot(c+dx)) + \frac{(3a-b)\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\cot(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} + \frac{(a-b)b\cot(c+dx)}{a\left(a+b\cot^2(c+dx)\right)}}{2(a-b)^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[c + d*x]^2)^(-2),x]
```

```
[Out] (-2*ArcTan[Cot[c + d*x]] + ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Cot[c + d*x])
/Sqrt[a]])/a^(3/2) + ((a - b)*b*Cot[c + d*x])/(a*(a + b*Cot[c + d*x]^2)))/(
2*(a - b)^2*d)
```

Maple [A]
time = 0.33, size = 99, normalized size = 1.02

method	result
derivativedivides	$\dfrac{\dfrac{b\left(\dfrac{(a-b)\cot(dx+c)}{2a\left(a+b\left(\cot^2(dx+c)\right)\right)}+\dfrac{(3a-b)\arctan\left(\dfrac{b\cot(dx+c)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a-b)^2}-\dfrac{\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))}{(a-b)^2}}{d}$
default	$\dfrac{\dfrac{b\left(\dfrac{(a-b)\cot(dx+c)}{2a\left(a+b\left(\cot^2(dx+c)\right)\right)}+\dfrac{(3a-b)\arctan\left(\dfrac{b\cot(dx+c)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a-b)^2}-\dfrac{\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))}{(a-b)^2}}{d}$
risch	$\dfrac{x}{a^2-2ab+b^2}-\dfrac{ib\left(a\,e^{2i(dx+c)}+b\,e^{2i(dx+c)}-a+b\right)}{da(-a+b)^2\left(-a\,e^{4i(dx+c)}+b\,e^{4i(dx+c)}+2a\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}-a+b\right)}+\dfrac{3\sqrt{-ab}\,\ln\left(e^{2i(dx+c)}-\right.}{4a(a-b)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cot(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a-b)^2*b*(1/2/a*(a-b)*cot(d*x+c)/(a+b*cot(d*x+c)^2)+1/2*(3*a-b)/a/(
a*b)^(1/2)*arctan(b*cot(d*x+c)/(a*b)^(1/2)))-1/(a-b)^2*(1/2*Pi-arccot(cot(d
*x+c))))
```

Maxima [A]
time = 0.51, size = 115, normalized size = 1.19

$$\frac{\frac{b\tan(dx+c)}{a^2b-ab^2+(a^3-a^2b)\tan(dx+c)^2}-\frac{(3\,ab-b^2)\arctan\left(\frac{a\tan(dx+c)}{\sqrt{ab}}\right)}{(a^3-2\,a^2b+ab^2)\sqrt{ab}}+\frac{2\,(dx+c)}{a^2-2\,ab+b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*tan(d*x + c)/(a^2*b - a*b^2 + (a^3 - a^2*b)*tan(d*x + c)^2) - (3*a*b
 - b^2)*arctan(a*tan(d*x + c)/sqrt(a*b))/((a^3 - 2*a^2*b + a*b^2)*sqrt(a*b)
) + 2*(d*x + c)/(a^2 - 2*a*b + b^2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(85) = 170$.

time = 3.06, size = 534, normalized size = 5.51

$$\left[\frac{8\,(a^2-ab)dx\cos\left(2\,dx+2\,c\right)-8\,(a^2+ab)dx+(3\,a^2+2\,ab-b^2-(3\,a^2-4\,ab+b^2)\cos\left(2\,dx+2\,c\right))\sqrt{-\frac{b}{a}}\log\left(\frac{(a^2+6\,ab+b^2)\cos(2\,dx+2\,c)^2+4\,(a^2-ab-(a^2+ab)\cos(2\,dx+2\,c))\sqrt{-\frac{b}{a}}\sin(2\,dx+2\,c)+a^2-6\,ab+b^2-2\,(a^2-b^2)\cos(2\,dx+2\,c)}{(a^2-2\,ab+b^2)\cos(2\,dx+2\,c)^2+a^2+2\,ab+b^2-2\,(a^2-b^2)\cos(2\,dx+2\,c)}\right)-4\,(ab-b^2)\sin\left(2\,dx+2\,c\right)}{8\,((a^4-3\,a^3b+3\,a^2b^2-ab^3)d\cos\left(2\,dx+2\,c\right)-(a^4-a^3b-a^2b^2+ab^3)d)},\ \frac{4\,(a^2-ab)dx\cos\left(2\,dx+2\,c\right)-4\,(a^2+ab)dx-(3\,a^2+2\,ab-b^2-(3\,a^2-4\,ab+b^2)\cos\left(2\,dx+2\,c\right))\sqrt{\frac{b}{a}}\arctan\left(\frac{((a+b)\cos(2\,dx+2\,c)-a+b)\sqrt{\frac{b}{a}}}{2\,b\sin(2\,dx+2\,c)}\right)-2\,(ab-b^2)\sin\left(2\,dx+2\,c\right)}{4\,((a^4-3\,a^3b+3\,a^2b^2-ab^3)d\cos\left(2\,dx+2\,c\right)-(a^4-a^3b-a^2b^2+ab^3)d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(8*(a^2 - a*b)*d*x*cos(2*d*x + 2*c) - 8*(a^2 + a*b)*d*x + (3*a^2 + 2*a
*b - b^2 - (3*a^2 - 4*a*b + b^2)*cos(2*d*x + 2*c))*sqrt(-b/a)*log(((a^2 + 6
*a*b + b^2)*cos(2*d*x + 2*c)^2 + 4*(a^2 - a*b - (a^2 + a*b)*cos(2*d*x + 2*c
))*sqrt(-b/a)*sin(2*d*x + 2*c) + a^2 - 6*a*b + b^2 - 2*(a^2 - b^2)*cos(2*d*
x + 2*c))/((a^2 - 2*a*b + b^2)*cos(2*d*x + 2*c)^2 + a^2 + 2*a*b + b^2 - 2*(
a^2 - b^2)*cos(2*d*x + 2*c))) - 4*(a*b - b^2)*sin(2*d*x + 2*c))/((a^4 - 3*a
^3*b + 3*a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) - (a^4 - a^3*b - a^2*b^2 + a*b
^3)*d), 1/4*(4*(a^2 - a*b)*d*x*cos(2*d*x + 2*c) - 4*(a^2 + a*b)*d*x - (3*a^
2 + 2*a*b - b^2 - (3*a^2 - 4*a*b + b^2)*cos(2*d*x + 2*c))*sqrt(b/a)*arctan(
1/2*((a + b)*cos(2*d*x + 2*c) - a + b)*sqrt(b/a)/(b*sin(2*d*x + 2*c))) - 2*
(a*b - b^2)*sin(2*d*x + 2*c))/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cos(2*
d*x + 2*c) - (a^4 - a^3*b - a^2*b^2 + a*b^3)*d)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2125 vs. $2(78) = 156$.
time = 9.36, size = 2125, normalized size = 21.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((zoo*x/cot(c)**4, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/a**2, Eq(b,
 0)), ((x - 1/(d*cot(c + d*x)) + 1/(3*d*cot(c + d*x)**3))/b**2, Eq(a, 0)),
(3*d*x*cot(c + d*x)**4/(8*b**2*d*cot(c + d*x)**4 + 16*b**2*d*cot(c + d*x)**
2 + 8*b**2*d) + 6*d*x*cot(c + d*x)**2/(8*b**2*d*cot(c + d*x)**4 + 16*b**2*d
*cot(c + d*x)**2 + 8*b**2*d) + 3*d*x/(8*b**2*d*cot(c + d*x)**4 + 16*b**2*d*
cot(c + d*x)**2 + 8*b**2*d) - 3*cot(c + d*x)**3/(8*b**2*d*cot(c + d*x)**4 +
 16*b**2*d*cot(c + d*x)**2 + 8*b**2*d) - 5*cot(c + d*x)/(8*b**2*d*cot(c + d
*x)**4 + 16*b**2*d*cot(c + d*x)**2 + 8*b**2*d), Eq(a, b)), (x/(a + b*cot(c)
**2)**2, Eq(d, 0)), (4*a**2*d*x*sqrt(-a/b)/(4*a**4*d*sqrt(-a/b) + 4*a**3*b*
d*sqrt(-a/b)*cot(c + d*x)**2 - 8*a**3*b*d*sqrt(-a/b) - 8*a**2*b**2*d*sqrt(-
a/b)*cot(c + d*x)**2 + 4*a**2*b**2*d*sqrt(-a/b) + 4*a*b**3*d*sqrt(-a/b)*cot
(c + d*x)**2) + 3*a**2*log(-sqrt(-a/b) + cot(c + d*x))/(4*a**4*d*sqrt(-a/b)
 + 4*a**3*b*d*sqrt(-a/b)*cot(c + d*x)**2 - 8*a**3*b*d*sqrt(-a/b) - 8*a**2*b
**2*d*sqrt(-a/b)*cot(c + d*x)**2 + 4*a**2*b**2*d*sqrt(-a/b) + 4*a*b**3*d*sq
rt(-a/b)*cot(c + d*x)**2) - 3*a**2*log(sqrt(-a/b) + cot(c + d*x))/(4*a**4*d
```

```
*sqrt(-a/b) + 4*a**3*b*d*sqrt(-a/b)*cot(c + d*x)**2 - 8*a**3*b*d*sqrt(-a/b)
 - 8*a**2*b**2*d*sqrt(-a/b)*cot(c + d*x)**2 + 4*a**2*b**2*d*sqrt(-a/b) + 4*
a*b**3*d*sqrt(-a/b)*cot(c + d*x)**2) + 4*a*b*d*x*sqrt(-a/b)*cot(c + d*x)**2
/(4*a**4*d*sqrt(-a/b) + 4*a**3*b*d*sqrt(-a/b)*cot(c + d*x)**2 - 8*a**3*b*d*
sqrt(-a/b) - 8*a**2*b**2*d*sqrt(-a/b)*cot(c + d*x)**2 + 4*a**2*b**2*d*sqrt(
-a/b) + 4*a*b**3*d*sqrt(-a/b)*cot(c + d*x)**2) + 2*a*b*sqrt(-a/b)*cot(c + d
*x)/(4*a**4*d*sqrt(-a/b) + 4*a**3*b*d*sqrt(-a/b)*cot(c + d*x)**2 - 8*a**3*b
*d*sqrt(-a/b) - 8*a**2*b**2*d*sqrt(-a/b)*cot(c + d*x)**2 + 4*a**2*b**2*d*sq
rt(-a/b) + 4*a*b**3*d*sqrt(-a/b)*cot(c + d*x)**2) + 3*a*b*log(-sqrt(-a/b) +
 cot(c + d*x))*cot(c + d*x)**2/(4*a**4*d*sqrt(-a/b) + 4*a**3*b*d*sqrt(-a/b)
*cot(c + d*x)**2 - 8*a**3*b*d*sqrt(-a/b) - 8*a**2*b**2*d*sqrt(-a/b)*cot(c +
 d*x)**2 + 4*a**2*b**2*d*sqrt(-a/b) + 4*a*b**3*d*sqrt(-a/b)*cot(c + d*x)**2
) - a*b*log(-sqrt(-a/b) + cot(c + d*x))/(4*a**4*d*sqrt(-a/b) + 4*a**3*b*d*s
qrt(-a/b)*cot(c + d*x)**2 - 8*a**3*b*d*sqrt(-a/b) - 8*a**2*b**2*d*sqrt(-a/b
)*cot(c + d*x)**2 + 4*a**2*b**2*d*sqrt(-a/b) + 4*a*b**3*d*sqrt(-a/b)*cot(c
+ d*x)**2) - 3*a*b*log(sqrt(-a/b) + cot(c + d*x))*cot(c + d*x)**2/(4*a**4*d
*sqrt(-a/b) + 4*a**3*b*d*sqrt(-a/b)*cot(c + d*x)**2 - 8*a**3*b*d*sqrt(-a/b)
 - 8*a**2*b**2*d*sqrt(-a/b)*cot(c + d*x)**2 + 4*a**2*b**2*d*sqrt(-a/b) + 4*
a*b**3*d*sqrt(-a/b)*cot(c + d*x)**2) + a*b*log(sqrt(-a/b) + cot(c + d*x))/(
4*a**4*d*sqrt(-a/b) + 4*a**3*b*d*sqrt(-a/b)*cot(c + d*x)**2 - 8*a**3*b*d*sq
rt(-a/b) - 8*a**2*b**2*d*sqrt(-a/b)*cot(c + d*x)**2 + 4*a**2*b**2*d*sqrt(-a
/b) + 4*a*b**3*d*sqrt(-a/b)*cot(c + d*x)**2) - 2*b**2*sqrt(-a/b)*cot(c + d*
x)/(4*a**4*d*sqrt(-a/b) + 4*a**3*b*d*sqrt(-a/b)*cot(c + d*x)**2 - 8*a**3*b*
d*sqrt(-a/b) - 8*a**2*b**2*d*sqrt(-a/b)*cot(c + d*x)**2 + 4*a**2*b**2*d*sqr
t(-a/b) + 4*a*b**3*d*sqrt(-a/b)*cot(c + d*x)**2) - b**2*log(-sqrt(-a/b) + c
ot(c + d*x))*cot(c + d*x)**2/(4*a**4*d*sqrt(-a/b) + 4*a**3*b*d*sqrt(-a/b)*c
ot(c + d*x)**2 - 8*a**3*b*d*sqrt(-a/b) - 8*a**2*b**2*d*sqrt(-a/b)*cot(c + d
*x)**2 + 4*a**2*b**2*d*sqrt(-a/b) + 4*a*b**3*d*sqrt(-a/b)*cot(c + d*x)**2)
+ b**2*log(sqrt(-a/b) + cot(c + d*x))*cot(c + d*x)**2/(4*a**4*d*sqrt(-a/b)
+ 4*a**3*b*d*sqrt(-a/b)*cot(c + d*x)**2 - 8*a**3*b*d*sqrt(-a/b) - 8*a**2*b*
*2*d*sqrt(-a/b)*cot(c + d*x)**2 + 4*a**2*b**2*d*sqrt(-a/b) + 4*a*b**3*d*sqr
t(-a/b)*cot(c + d*x)**2), True))
```

Giac [**A**]
time = 0.47, size = 123, normalized size = 1.27

$$-\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan(dx+c)}{\sqrt{ab}}\right)\right)(3\,ab-b^2)}{(a^3-2\,a^2b+ab^2)\sqrt{ab}}-\frac{2\,(dx+c)}{a^2-2\,ab+b^2}-\frac{b\tan(dx+c)}{\left(a\tan(dx+c)^2+b\right)(a^2-ab)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((pi*floor((d*x + c)/pi + 1/2)*sgn(a) + arctan(a*tan(d*x + c)/sqrt(a*b
)))*(3*a*b - b^2)/((a^3 - 2*a^2*b + a*b^2)*sqrt(a*b)) - 2*(d*x + c)/(a^2 -
2*a*b + b^2) - b*tan(d*x + c)/((a*tan(d*x + c)^2 + b)*(a^2 - a*b)))/d
```

Mupad [B]
time = 0.79, size = 119, normalized size = 1.23

$$\frac{\frac{a\,x}{(a-b)^2}+\frac{b\,x\cot(c+d\,x)^2}{(a-b)^2}+\frac{b\cot(c+d\,x)}{2\,a\,d\,(a-b)}}{b\cot\left(c+d\,x\right)^2+a}+\frac{\mathrm{atan}\left(\frac{b\cot(c+d\,x)}{\sqrt{a\,b}}\right)\,\left(3\,a\,b-b^2\right)}{\sqrt{a\,b}\,\left(2\,a^3\,d-a\,b\,\left(4\,a\,d-2\,b\,d\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cot(c + d*x)^2)^2,x)
```

```
[Out] ((a*x)/(a - b)^2 + (b*x*cot(c + d*x)^2)/(a - b)^2 + (b*cot(c + d*x))/(2*a*d
*(a - b)))/(a + b*cot(c + d*x)^2) + (atan((b*cot(c + d*x))/(a*b)^(1/2))*(3*
a*b - b^2))/((a*b)^(1/2)*(2*a^3*d - a*b*(4*a*d - 2*b*d)))
```

3.7 $\int \frac{1}{(a+b\cot^2(c+dx))^3}\,dx$

Optimal. Leaf size=150

$$\frac{x}{(a-b)^3}+\frac{\sqrt{b}\,(15a^2-10ab+3b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\cot(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^3d}+\frac{b\cot(c+dx)}{4a(a-b)d\,(a+b\cot^2(c+dx))^2}+\frac{(7a-3b)}{8a^2(a-b)^2d\,(}$$

```
[Out] x/(a-b)^3+1/4*b*cot(d*x+c)/a/(a-b)/d/(a+b*cot(d*x+c)^2)^2+1/8*(7*a-3*b)*b*c
ot(d*x+c)/a^2/(a-b)^2/d/(a+b*cot(d*x+c)^2)+1/8*(15*a^2-10*a*b+3*b^2)*arctan
(cot(d*x+c)*b^(1/2)/a^(1/2))*b^(1/2)/a^(5/2)/(a-b)^3/d
```

Rubi [A]
time = 0.12, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3742, 425, 541, 536, 209, 211}

$$\frac{b(7a-3b)\cot(c+dx)}{8a^2d(a-b)^2\,(a+b\cot^2(c+dx))}+\frac{\sqrt{b}\,(15a^2-10ab+3b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\cot(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^3}+\frac{b\cot(c+dx)}{4ad(a-b)\,(a+b\cot^2(c+dx))^2}+\frac{x}{(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[c + d*x]^2)^(-3),x]
```

```
[Out] x/(a - b)^3 + (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Cot[c + d*
x])/Sqrt[a]])/(8*a^(5/2)*(a - b)^3*d) + (b*Cot[c + d*x])/(4*a*(a - b)*d*(a
+ b*Cot[c + d*x]^2)^2) + ((7*a - 3*b)*b*Cot[c + d*x])/(8*a^2*(a - b)^2*d*(a
 + b*Cot[c + d*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
```

```
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^3}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^3}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= \frac{b\cot(c+dx)}{4a(a-b)d\left(a+b\cot^2(c+dx)\right)^2} - \frac{\operatorname{Subst}\left(\int \frac{4a-3b-3bx^2}{(1+x^2)(a+bx^2)^2}\,dx, x, \cot(c+dx)\right)}{4a(a-b)d}$$

$$= \frac{b\cot(c+dx)}{4a(a-b)d\left(a+b\cot^2(c+dx)\right)^2} + \frac{(7a-3b)b\cot(c+dx)}{8a^2(a-b)^2d\left(a+b\cot^2(c+dx)\right)} - \frac{\operatorname{Subst}\Big(}{}$$

$$= \frac{b\cot(c+dx)}{4a(a-b)d\left(a+b\cot^2(c+dx)\right)^2} + \frac{(7a-3b)b\cot(c+dx)}{8a^2(a-b)^2d\left(a+b\cot^2(c+dx)\right)} - \frac{\operatorname{Subst}\Big(}{}$$

$$= \frac{x}{(a-b)^3} + \frac{\sqrt{b}\,(15a^2-10ab+3b^2)\tan^{-1}\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^3d} + \frac{b\cot(c+}{4a(a-b)d\,(a+b}$$

Mathematica [A]
time = 0.33, size = 138, normalized size = 0.92

$$\frac{-8\operatorname{ArcTan}(\cot(c+dx)) + \frac{\sqrt{b}\,(15a^2-10ab+3b^2)\operatorname{ArcTan}\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a}}\right)}{a^{5/2}} + \frac{2(a-b)^2b\cot(c+dx)}{a\left(a+b\cot^2(c+dx)\right)^2} + \frac{(7a-3b)(a-b)b\cot(c+dx)}{a^2\left(a+b\cot^2(c+dx)\right)}}{8(a-b)^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[c + d*x]^2)^(-3),x]
```

```
[Out] (-8*ArcTan[Cot[c + d*x]] + (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(Sqrt[
b]*Cot[c + d*x])/Sqrt[a]])/a^(5/2) + (2*(a - b)^2*b*Cot[c + d*x])/(a*(a + b
*Cot[c + d*x]^2)^2) + ((7*a - 3*b)*(a - b)*b*Cot[c + d*x])/(a^2*(a + b*Cot[
c + d*x]^2)))/(8*(a - b)^3*d)
```

Maple [A]
time = 0.45, size = 148, normalized size = 0.99

method	result
derivativedivides	$\frac{-\frac{\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))}{(a-b)^3} + \frac{b\left(\frac{\frac{b\left(7a^2-10ab+3b^2\right)\left(\cot^3(dx+c)\right)}{8a^2} + \frac{\left(9a^2-14ab+5b^2\right)\cot(dx+c)}{8a}}{\left(a+b\left(\cot^2(dx+c)\right)\right)^2} + \frac{\left(15a^2-10ab+3b^2\right)\arctan\left(\frac{b\cot(dx+c}{\sqrt{ab}}\right.}{8a^2\sqrt{ab}}\right.}{(a-b)^3}}{d}$

default	$\dfrac{-\frac{\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))}{(a-b)^3}+\frac{b\left(\frac{\frac{b(7a^2-10ab+3b^2)(\cot^3(dx+c))}{8a^2}+\frac{(9a^2-14ab+5b^2)\cot(dx+c)}{8a}}{(a+b(\cot^2(dx+c)))^2}+\frac{(15a^2-10ab+3b^2)\arctan\left(\frac{b\cot(dx}{\sqrt{ab}}\right.}{8a^2\sqrt{ab}}\right.}{(a-b)^3}}{d}$
risch	$\dfrac{x}{a^3-3a^2b+3ab^2-b^3}-\dfrac{ib(9a^3e^{6i(dx+c)}+a^2b\,e^{6i(dx+c)}-13a\,b^2e^{6i(dx+c)}+3b^3e^{6i(dx+c)}-27a^3e^{4i(dx+c)}-9a^2b\,e^{4i(dx+c)}}{4(-a\,e^{4i(dx+c)}+b\,e^{4i(dx+c)}+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cot(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/(a-b)^3*(1/2*Pi-arccot(cot(d*x+c)))+b/(a-b)^3*((1/8*b*(7*a^2-10*a*b
+3*b^2)/a^2*cot(d*x+c)^3+1/8*(9*a^2-14*a*b+5*b^2)/a*cot(d*x+c))/(a+b*cot(d*
x+c)^2)^2+1/8*(15*a^2-10*a*b+3*b^2)/a^2/(a*b)^(1/2)*arctan(b*cot(d*x+c)/(a*
b)^(1/2))))
```

Maxima [A]
time = 0.52, size = 228, normalized size = 1.52

$$-\frac{\frac{(15a^2b-10ab^2+3b^3)\arctan\left(\frac{a\tan(dx+c)}{\sqrt{ab}}\right)}{(a^5-3a^4b+3a^3b^2-a^2b^3)\sqrt{ab}}-\frac{(9a^2b-5ab^2)\tan(dx+c)^3+(7ab^2-3b^3)\tan(dx+c)}{a^4b^2-2a^3b^3+a^2b^4+(a^6-2a^5b+a^4b^2)\tan(dx+c)^4+2(a^5b-2a^4b^2+a^3b^3)\tan(dx+c)^2}-\frac{8(dx+c)}{a^3-3a^2b+3ab^2-b^3}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((15*a^2*b - 10*a*b^2 + 3*b^3)*arctan(a*tan(d*x + c)/sqrt(a*b))/((a^5
- 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*sqrt(a*b)) - ((9*a^2*b - 5*a*b^2)*tan(d*x
+ c)^3 + (7*a*b^2 - 3*b^3)*tan(d*x + c))/(a^4*b^2 - 2*a^3*b^3 + a^2*b^4 + (
a^6 - 2*a^5*b + a^4*b^2)*tan(d*x + c)^4 + 2*(a^5*b - 2*a^4*b^2 + a^3*b^3)*t
an(d*x + c)^2) - 8*(d*x + c)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(136) = 272$.
time = 3.18, size = 1068, normalized size = 7.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(32*(a^4 - 2*a^3*b + a^2*b^2)*d*x*cos(2*d*x + 2*c)^2 - 64*(a^4 - a^2*
b^2)*d*x*cos(2*d*x + 2*c) + 32*(a^4 + 2*a^3*b + a^2*b^2)*d*x - (15*a^4 + 20
*a^3*b - 2*a^2*b^2 - 4*a*b^3 + 3*b^4 + (15*a^4 - 40*a^3*b + 38*a^2*b^2 - 16
```

```
*a*b^3 + 3*b^4)*cos(2*d*x + 2*c)^2 - 2*(15*a^4 - 10*a^3*b - 12*a^2*b^2 + 10
*a*b^3 - 3*b^4)*cos(2*d*x + 2*c))*sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(2
*d*x + 2*c)^2 + 4*(a^2 - a*b - (a^2 + a*b)*cos(2*d*x + 2*c))*sqrt(-b/a)*sin
(2*d*x + 2*c) + a^2 - 6*a*b + b^2 - 2*(a^2 - b^2)*cos(2*d*x + 2*c))/((a^2 -
 2*a*b + b^2)*cos(2*d*x + 2*c)^2 + a^2 + 2*a*b + b^2 - 2*(a^2 - b^2)*cos(2*
d*x + 2*c))) + 4*(9*a^3*b - 7*a^2*b^2 - 5*a*b^3 + 3*b^4 - 3*(3*a^3*b - 7*a^
2*b^2 + 5*a*b^3 - b^4)*cos(2*d*x + 2*c))*sin(2*d*x + 2*c))/((a^7 - 5*a^6*b
+ 10*a^5*b^2 - 10*a^4*b^3 + 5*a^3*b^4 - a^2*b^5)*d*cos(2*d*x + 2*c)^2 - 2*(
a^7 - 3*a^6*b + 2*a^5*b^2 + 2*a^4*b^3 - 3*a^3*b^4 + a^2*b^5)*d*cos(2*d*x +
2*c) + (a^7 - a^6*b - 2*a^5*b^2 + 2*a^4*b^3 + a^3*b^4 - a^2*b^5)*d), 1/16*(
16*(a^4 - 2*a^3*b + a^2*b^2)*d*x*cos(2*d*x + 2*c)^2 - 32*(a^4 - a^2*b^2)*d*
x*cos(2*d*x + 2*c) + 16*(a^4 + 2*a^3*b + a^2*b^2)*d*x + (15*a^4 + 20*a^3*b
- 2*a^2*b^2 - 4*a*b^3 + 3*b^4 + (15*a^4 - 40*a^3*b + 38*a^2*b^2 - 16*a*b^3
+ 3*b^4)*cos(2*d*x + 2*c)^2 - 2*(15*a^4 - 10*a^3*b - 12*a^2*b^2 + 10*a*b^3
- 3*b^4)*cos(2*d*x + 2*c))*sqrt(b/a)*arctan(1/2*((a + b)*cos(2*d*x + 2*c) -
 a + b)*sqrt(b/a)/(b*sin(2*d*x + 2*c))) + 2*(9*a^3*b - 7*a^2*b^2 - 5*a*b^3
+ 3*b^4 - 3*(3*a^3*b - 7*a^2*b^2 + 5*a*b^3 - b^4)*cos(2*d*x + 2*c))*sin(2*d
*x + 2*c))/((a^7 - 5*a^6*b + 10*a^5*b^2 - 10*a^4*b^3 + 5*a^3*b^4 - a^2*b^5)
*d*cos(2*d*x + 2*c)^2 - 2*(a^7 - 3*a^6*b + 2*a^5*b^2 + 2*a^4*b^3 - 3*a^3*b^
4 + a^2*b^5)*d*cos(2*d*x + 2*c) + (a^7 - a^6*b - 2*a^5*b^2 + 2*a^4*b^3 + a^
3*b^4 - a^2*b^5)*d)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 8964 vs. $2(133) = 266$.
time = 49.59, size = 8964, normalized size = 59.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((zoo*x/cot(c)**6, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((-x + 1/(d*co
t(c + d*x)) - 1/(3*d*cot(c + d*x)**3) + 1/(5*d*cot(c + d*x)**5))/b**3, Eq(a
, 0)), (15*d*x*cot(c + d*x)**6/(48*b**3*d*cot(c + d*x)**6 + 144*b**3*d*cot(
c + d*x)**4 + 144*b**3*d*cot(c + d*x)**2 + 48*b**3*d) + 45*d*x*cot(c + d*x)
**4/(48*b**3*d*cot(c + d*x)**6 + 144*b**3*d*cot(c + d*x)**4 + 144*b**3*d*co
t(c + d*x)**2 + 48*b**3*d) + 45*d*x*cot(c + d*x)**2/(48*b**3*d*cot(c + d*x)
**6 + 144*b**3*d*cot(c + d*x)**4 + 144*b**3*d*cot(c + d*x)**2 + 48*b**3*d)
+ 15*d*x/(48*b**3*d*cot(c + d*x)**6 + 144*b**3*d*cot(c + d*x)**4 + 144*b**3
*d*cot(c + d*x)**2 + 48*b**3*d) - 15*cot(c + d*x)**5/(48*b**3*d*cot(c + d*x
)**6 + 144*b**3*d*cot(c + d*x)**4 + 144*b**3*d*cot(c + d*x)**2 + 48*b**3*d)
 - 40*cot(c + d*x)**3/(48*b**3*d*cot(c + d*x)**6 + 144*b**3*d*cot(c + d*x)*
*4 + 144*b**3*d*cot(c + d*x)**2 + 48*b**3*d) - 33*cot(c + d*x)/(48*b**3*d*c
ot(c + d*x)**6 + 144*b**3*d*cot(c + d*x)**4 + 144*b**3*d*cot(c + d*x)**2 +
48*b**3*d), Eq(a, b)), (x/(a + b*cot(c)**2)**3, Eq(d, 0)), (x/a**3, Eq(b, 0
```

```
)), (16*a**4*d*x*sqrt(-a/b)/(16*a**7*d*sqrt(-a/b) + 32*a**6*b*d*sqrt(-a/b)*
cot(c + d*x)**2 - 48*a**6*b*d*sqrt(-a/b) + 16*a**5*b**2*d*sqrt(-a/b)*cot(c
+ d*x)**4 - 96*a**5*b**2*d*sqrt(-a/b)*cot(c + d*x)**2 + 48*a**5*b**2*d*sqrt
(-a/b) - 48*a**4*b**3*d*sqrt(-a/b)*cot(c + d*x)**4 + 96*a**4*b**3*d*sqrt(-a
/b)*cot(c + d*x)**2 - 16*a**4*b**3*d*sqrt(-a/b) + 48*a**3*b**4*d*sqrt(-a/b)
*cot(c + d*x)**4 - 32*a**3*b**4*d*sqrt(-a/b)*cot(c + d*x)**2 - 16*a**2*b**5
*d*sqrt(-a/b)*cot(c + d*x)**4) + 15*a**4*log(-sqrt(-a/b) + cot(c + d*x))/(1
6*a**7*d*sqrt(-a/b) + 32*a**6*b*d*sqrt(-a/b)*cot(c + d*x)**2 - 48*a**6*b*d*
sqrt(-a/b) + 16*a**5*b**2*d*sqrt(-a/b)*cot(c + d*x)**4 - 96*a**5*b**2*d*sqr
t(-a/b)*cot(c + d*x)**2 + 48*a**5*b**2*d*sqrt(-a/b) - 48*a**4*b**3*d*sqrt(-
a/b)*cot(c + d*x)**4 + 96*a**4*b**3*d*sqrt(-a/b)*cot(c + d*x)**2 - 16*a**4*
b**3*d*sqrt(-a/b) + 48*a**3*b**4*d*sqrt(-a/b)*cot(c + d*x)**4 - 32*a**3*b**
4*d*sqrt(-a/b)*cot(c + d*x)**2 - 16*a**2*b**5*d*sqrt(-a/b)*cot(c + d*x)**4)
 - 15*a**4*log(sqrt(-a/b) + cot(c + d*x))/(16*a**7*d*sqrt(-a/b) + 32*a**6*b
*d*sqrt(-a/b)*cot(c + d*x)**2 - 48*a**6*b*d*sqrt(-a/b) + 16*a**5*b**2*d*sqr
t(-a/b)*cot(c + d*x)**4 - 96*a**5*b**2*d*sqrt(-a/b)*cot(c + d*x)**2 + 48*a*
*5*b**2*d*sqrt(-a/b) - 48*a**4*b**3*d*sqrt(-a/b)*cot(c + d*x)**4 + 96*a**4*
b**3*d*sqrt(-a/b)*cot(c + d*x)**2 - 16*a**4*b**3*d*sqrt(-a/b) + 48*a**3*b**
4*d*sqrt(-a/b)*cot(c + d*x)**4 - 32*a**3*b**4*d*sqrt(-a/b)*cot(c + d*x)**2
- 16*a**2*b**5*d*sqrt(-a/b)*cot(c + d*x)**4) + 32*a**3*b*d*x*sqrt(-a/b)*cot
(c + d*x)**2/(16*a**7*d*sqrt(-a/b) + 32*a**6*b*d*sqrt(-a/b)*cot(c + d*x)**2
 - 48*a**6*b*d*sqrt(-a/b) + 16*a**5*b**2*d*sqrt(-a/b)*cot(c + d*x)**4 - 96*
a**5*b**2*d*sqrt(-a/b)*cot(c + d*x)**2 + 48*a**5*b**2*d*sqrt(-a/b) - 48*a**
4*b**3*d*sqrt(-a/b)*cot(c + d*x)**4 + 96*a**4*b**3*d*sqrt(-a/b)*cot(c + d*x
)**2 - 16*a**4*b**3*d*sqrt(-a/b) + 48*a**3*b**4*d*sqrt(-a/b)*cot(c + d*x)**
4 - 32*a**3*b**4*d*sqrt(-a/b)*cot(c + d*x)**2 - 16*a**2*b**5*d*sqrt(-a/b)*c
ot(c + d*x)**4) + 18*a**3*b*sqrt(-a/b)*cot(c + d*x)/(16*a**7*d*sqrt(-a/b) +
 32*a**6*b*d*sqrt(-a/b)*cot(c + d*x)**2 - 48*a**6*b*d*sqrt(-a/b) + 16*a**5*
b**2*d*sqrt(-a/b)*cot(c + d*x)**4 - 96*a**5*b**2*d*sqrt(-a/b)*cot(c + d*x)*
*2 + 48*a**5*b**2*d*sqrt(-a/b) - 48*a**4*b**3*d*sqrt(-a/b)*cot(c + d*x)**4
+ 96*a**4*b**3*d*sqrt(-a/b)*cot(c + d*x)**2 - 16*a**4*b**3*d*sqrt(-a/b) + 4
8*a**3*b**4*d*sqrt(-a/b)*cot(c + d*x)**4 - 32*a**3*b**4*d*sqrt(-a/b)*cot(c
+ d*x)**2 - 16*a**2*b**5*d*sqrt(-a/b)*cot(c + d*x)**4) + 30*a**3*b*log(-sqr
t(-a/b) + cot(c + d*x))*cot(c + d*x)**2/(16*a**7*d*sqrt(-a/b) + 32*a**6*b*d
*sqrt(-a/b)*cot(c + d*x)**2 - 48*a**6*b*d*sqrt(-a/b) + 16*a**5*b**2*d*sqrt(
-a/b)*cot(c + d*x)**4 - 96*a**5*b**2*d*sqrt(-a/b)*cot(c + d*x)**2 + 48*a**5
*b**2*d*sqrt(-a/b) - 48*a**4*b**3*d*sqrt(-a/b)*cot(c + d*x)**4 + 96*a**4*b*
*3*d*sqrt(-a/b)*cot(c + d*x)**2 - 16*a**4*b**3*d*sqrt(-a/b) + 48*a**3*b**4*
d*sqrt(-a/b)*cot(c + d*x)**4 - 32*a**3*b**4*d*sqrt(-a/b)*cot(c + d*x)**2 -
16*a**2*b**5*d*sqrt(-a/b)*cot(c + d*x)**4) - 10*a**3*b*log(-sqrt(-a/b) + co
t(c + d*x))/(16*a**7*d*sqrt(-a/b) + 32*a**6*b*d*sqrt(-a/b)*cot(c + d*x)**2
- 48*a**6*b*d*sqrt(-a/b) + 16*a**5*b**2*d*sqrt(-a/b)*cot(c + d*x)**4 - 96*a
**5*b**2*d*sqrt(-a/b)*cot(c + d*x)**2 + 48*a**5*b**2*d*sqrt(-a/b) - 48*a**4
*b**3*d*sqrt(-a/b)*cot(c + d*x)**4 + 96*a**4*b**3*d*sqrt(-a/b)*cot(c + d*x)
**2 - 16*a**4*b**3*d*sqrt(-a/b) + 48*a**3*b**4*d*sqrt(-a/b)*cot(c + d*x)**4
```

```
 - 32*a**3*b**4*d*sqrt(-a/b)*cot(c + d*x)**2 - 16*a**2*b**5*d*sqrt(-a/b)*co
t(c + d*x)**4) - 30*a**3*b*log(sqrt(-a/b) + cot(c + d*x))*cot(c + d*x)**2/(
16*a**7*d*sqrt(-a/b) + 32*a**6*b*d*sqrt(-a/b)*cot(c + d*x)**2 - 48*a**6*b*d
*sqrt(-a/b) + 16*a**5*b**2*d*sqrt(-a/b)*cot(c + d*x)**4 - 96*a**5*b**2*d*sq
rt(-a/b)*cot(c + d*x)**2 + 48*a**5*b**2*d*sqrt(-a/b) - 48*a**4*b**3*d*sqrt(
-a/b)*cot(c + d*x)**4 + 96*a**4*b**3*d*sqrt(-a/b)*cot(c + d*x)**2 - 16*a**4
*b**3*d*sqrt(-a/b) + 48*a**3*b**4*d*sqrt(-a/b)*cot(c + d*x)**4 - 32*a**3*b*
*4*d*sqrt(-a/b)*cot(c + d*x)**2 - 16*a**2*b**5*...
```

Giac [A]
time = 0.55, size = 206, normalized size = 1.37

$$-\frac{\frac{\left(15\,a^2b-10\,ab^2+3\,b^3\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan(dx+c)}{\sqrt{ab}}\right)\right)}{\left(a^5-3\,a^4b+3\,a^3b^2-a^2b^3\right)\sqrt{ab}}-\frac{8\,(dx+c)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{9\,a^2b\tan(dx+c)^3-5\,ab^2\tan(dx+c)^3+7\,ab^2\tan(dx+c)-3\,b^3\tan(dx+c)}{\left(a^4-2\,a^3b+a^2b^2\right)\left(a\tan(dx+c)^2+b\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((15*a^2*b - 10*a*b^2 + 3*b^3)*(pi*floor((d*x + c)/pi + 1/2)*sgn(a) +
arctan(a*tan(d*x + c)/sqrt(a*b)))/((a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*sq
rt(a*b)) - 8*(d*x + c)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (9*a^2*b*tan(d*x +
 c)^3 - 5*a*b^2*tan(d*x + c)^3 + 7*a*b^2*tan(d*x + c) - 3*b^3*tan(d*x + c))
/((a^4 - 2*a^3*b + a^2*b^2)*(a*tan(d*x + c)^2 + b)^2))/d
```

Mupad [B]
time = 3.29, size = 2500, normalized size = 16.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cot(c + d*x)^2)^3,x)
```

```
[Out] ((cot(c + d*x)^3*(7*a*b^2 - 3*b^3))/(8*a^2*(a^2 - 2*a*b + b^2)) + (cot(c +
d*x)*(9*a*b - 5*b^2))/(8*a*(a^2 - 2*a*b + b^2)))/(a^2*d + b^2*d*cot(c + d*x
)^4 + 2*a*b*d*cot(c + d*x)^2) + (2*atan((((((96*a^2*b^10*d^2 - 800*a^3*b^9*
d^2 + 3040*a^4*b^8*d^2 - 6816*a^5*b^7*d^2 + 9760*a^6*b^6*d^2 - 9056*a^7*b^5
*d^2 + 5280*a^8*b^4*d^2 - 1760*a^9*b^3*d^2 + 256*a^10*b^2*d^2)/(64*(a^10*d^
3 - 6*a^9*b*d^3 + a^4*b^6*d^3 - 6*a^5*b^5*d^3 + 15*a^6*b^4*d^3 - 20*a^7*b^3
*d^3 + 15*a^8*b^2*d^3)) - (cot(c + d*x)*(256*a^4*b^9*d^2 - 1280*a^5*b^8*d^2
 + 2304*a^6*b^7*d^2 - 1280*a^7*b^6*d^2 - 1280*a^8*b^5*d^2 + 2304*a^9*b^4*d^
2 - 1280*a^10*b^3*d^2 + 256*a^11*b^2*d^2)*1i)/(32*(2*a^3*d - 2*b^3*d + 6*a*
b^2*d - 6*a^2*b*d)*(a^8*d^2 - 4*a^7*b*d^2 + a^4*b^4*d^2 - 4*a^5*b^3*d^2 + 6
*a^6*b^2*d^2)))*1i)/(2*a^3*d - 2*b^3*d + 6*a*b^2*d - 6*a^2*b*d) - (cot(c +
d*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a^8
*d^2 - 4*a^7*b*d^2 + a^4*b^4*d^2 - 4*a^5*b^3*d^2 + 6*a^6*b^2*d^2)))/(2*a^3*
```

d - 2*b^3*d + 6*a*b^2*d - 6*a^2*b*d) - ((((96*a^2*b^10*d^2 - 800*a^3*b^9*d^
2 + 3040*a^4*b^8*d^2 - 6816*a^5*b^7*d^2 + 9760*a^6*b^6*d^2 - 9056*a^7*b^5*d
^2 + 5280*a^8*b^4*d^2 - 1760*a^9*b^3*d^2 + 256*a^10*b^2*d^2)/(64*(a^10*d^3
- 6*a^9*b*d^3 + a^4*b^6*d^3 - 6*a^5*b^5*d^3 + 15*a^6*b^4*d^3 - 20*a^7*b^3*d
^3 + 15*a^8*b^2*d^3)) + (cot(c + d*x)*(256*a^4*b^9*d^2 - 1280*a^5*b^8*d^2 +
2304*a^6*b^7*d^2 - 1280*a^7*b^6*d^2 - 1280*a^8*b^5*d^2 + 2304*a^9*b^4*d^2
- 1280*a^10*b^3*d^2 + 256*a^11*b^2*d^2)*1i)/(32*(2*a^3*d - 2*b^3*d + 6*a*b^
2*d - 6*a^2*b*d)*(a^8*d^2 - 4*a^7*b*d^2 + a^4*b^4*d^2 - 4*a^5*b^3*d^2 + 6*a
^6*b^2*d^2)))*1i)/(2*a^3*d - 2*b^3*d + 6*a*b^2*d - 6*a^2*b*d) + (cot(c + d*
x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a^8*d
^2 - 4*a^7*b*d^2 + a^4*b^4*d^2 - 4*a^5*b^3*d^2 + 6*a^6*b^2*d^2)))/(2*a^3*d
- 2*b^3*d + 6*a*b^2*d - 6*a^2*b*d))/((((((96*a^2*b^10*d^2 - 800*a^3*b^9*d^2
+ 3040*a^4*b^8*d^2 - 6816*a^5*b^7*d^2 + 9760*a^6*b^6*d^2 - 9056*a^7*b^5*d^
2 + 5280*a^8*b^4*d^2 - 1760*a^9*b^3*d^2 + 256*a^10*b^2*d^2)/(64*(a^10*d^3 -
6*a^9*b*d^3 + a^4*b^6*d^3 - 6*a^5*b^5*d^3 + 15*a^6*b^4*d^3 - 20*a^7*b^3*d^
3 + 15*a^8*b^2*d^3)) - (cot(c + d*x)*(256*a^4*b^9*d^2 - 1280*a^5*b^8*d^2 +
2304*a^6*b^7*d^2 - 1280*a^7*b^6*d^2 - 1280*a^8*b^5*d^2 + 2304*a^9*b^4*d^2 -
1280*a^10*b^3*d^2 + 256*a^11*b^2*d^2)*1i)/(32*(2*a^3*d - 2*b^3*d + 6*a*b^2
*d - 6*a^2*b*d)*(a^8*d^2 - 4*a^7*b*d^2 + a^4*b^4*d^2 - 4*a^5*b^3*d^2 + 6*a^
6*b^2*d^2)))*1i)/(2*a^3*d - 2*b^3*d + 6*a*b^2*d - 6*a^2*b*d) - (cot(c + d*x
)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a^8*d^
2 - 4*a^7*b*d^2 + a^4*b^4*d^2 - 4*a^5*b^3*d^2 + 6*a^6*b^2*d^2)))*1i)/(2*a^3
*d - 2*b^3*d + 6*a*b^2*d - 6*a^2*b*d) + (((((96*a^2*b^10*d^2 - 800*a^3*b^9*
d^2 + 3040*a^4*b^8*d^2 - 6816*a^5*b^7*d^2 + 9760*a^6*b^6*d^2 - 9056*a^7*b^5
*d^2 + 5280*a^8*b^4*d^2 - 1760*a^9*b^3*d^2 + 256*a^10*b^2*d^2)/(64*(a^10*d^
3 - 6*a^9*b*d^3 + a^4*b^6*d^3 - 6*a^5*b^5*d^3 + 15*a^6*b^4*d^3 - 20*a^7*b^3
*d^3 + 15*a^8*b^2*d^3)) + (cot(c + d*x)*(256*a^4*b^9*d^2 - 1280*a^5*b^8*d^2
+ 2304*a^6*b^7*d^2 - 1280*a^7*b^6*d^2 - 1280*a^8*b^5*d^2 + 2304*a^9*b^4*d^
2 - 1280*a^10*b^3*d^2 + 256*a^11*b^2*d^2)*1i)/(32*(2*a^3*d - 2*b^3*d + 6*a*
b^2*d - 6*a^2*b*d)*(a^8*d^2 - 4*a^7*b*d^2 + a^4*b^4*d^2 - 4*a^5*b^3*d^2 + 6
*a^6*b^2*d^2)))*1i)/(2*a^3*d - 2*b^3*d + 6*a*b^2*d - 6*a^2*b*d) + (cot(c +
d*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a^8
*d^2 - 4*a^7*b*d^2 + a^4*b^4*d^2 - 4*a^5*b^3*d^2 + 6*a^6*b^2*d^2)))*1i)/(2*
a^3*d - 2*b^3*d + 6*a*b^2*d - 6*a^2*b*d) + (51*a*b^5 - 9*b^6 - 115*a^2*b^4
+ 105*a^3*b^3)/(32*(a^10*d^3 - 6*a^9*b*d^3 + a^4*b^6*d^3 - 6*a^5*b^5*d^3 +
15*a^6*b^4*d^3 - 20*a^7*b^3*d^3 + 15*a^8*b^2*d^3)))))/(2*a^3*d - 2*b^3*d +
6*a*b^2*d - 6*a^2*b*d) - (atan((((-a^5*b)^(1/2)*((cot(c + d*x)*(9*b^7 - 60*
a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a^8*d^2 - 4*a^7*b*d^
2 + a^4*b^4*d^2 - 4*a^5*b^3*d^2 + 6*a^6*b^2*d^2)) - (((96*a^2*b^10*d^2 - 80
0*a^3*b^9*d^2 + 3040*a^4*b^8*d^2 - 6816*a^5*b^7*d^2 + 9760*a^6*b^6*d^2 - 90
56*a^7*b^5*d^2 + 5280*a^8*b^4*d^2 - 1760*a^9*b^3*d^2 + 256*a^10*b^2*d^2)/(6
4*(a^10*d^3 - 6*a^9*b*d^3 + a^4*b^6*d^3 - 6*a^5*b^5*d^3 + 15*a^6*b^4*d^3 -
20*a^7*b^3*d^3 + 15*a^8*b^2*d^3)) - (cot(c + d*x)*(-a^5*b)^(1/2)*(15*a^2 -
10*a*b + 3*b^2)*(256*a^4*b^9*d^2 - 1280*a^5*b^8*d^2 + 2304*a^6*b^7*d^2 - 12
80*a^7*b^6*d^2 - 1280*a^8*b^5*d^2 + 2304*a^9*b^4*d^2 - 1280*a^10*b^3*d^2 +

```
256*a^11*b^2*d^2))/(512*(a^8*d - a^5*b^3*d + 3*a^6*b^2*d - 3*a^7*b*d)*(a^8*
d^2 - 4*a^7*b*d^2 + a^4*b^4*d^2 - 4*a^5*b^3*d^2 + 6*a^6*b^2*d^2)))*(-a^5*b)
^(1/2)*(15*a^2 - 10*a*b + 3*b^2))/(16*(a^8*d - a^5*b^3*d + 3*a^6*b^2*d - 3*
a^7*b*d)))*(15*a^2 - 10*a*b + 3*b^2)*1i)/(16*(a^8*d - a^5*b^3*d + 3*a^6*b^2
*d - 3*a^7*b*d)) + ((-a^5*b)^(1/2)*((cot(c + d*x)*(9*b^7 - 60*a*b^6 + 190*a
^2*b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a^8*d^2 - 4*a^7*b*d^2 + a^4*b^4*d
^2 - 4*a^5*b^3*d^2 + 6*a^6*b^2*d^2)) + (((96*a^2*b^10*d^2 - 800*a^3*b^9*d^2
 + 3040*a^4*b^8*d^2 - 6816*a^5*b^7*d^2 + 9760*a^6*b^6*d^2 - 9056*a^7*b^5*d^
2 + 5280*a^8*b^4*d^2 - 1760*a^9*b^3*d^2 + 256*a...
```

3.8 $\int \left(1+\cot^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=22

$$-\frac{1}{2}\sinh^{-1}(\cot(x))-\frac{1}{2}\cot(x)\sqrt{\csc^2(x)}$$

```
[Out] -1/2*arcsinh(cot(x))-1/2*cot(x)*(csc(x)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3738, 4207, 201, 221}

$$-\frac{1}{2}\cot(x)\sqrt{\csc^2(x)}-\frac{1}{2}\sinh^{-1}(\cot(x))$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cot[x]^2)^(3/2),x]
```

```
[Out] -1/2*ArcSinh[Cot[x]] - (Cot[x]*Sqrt[Csc[x]^2])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \left(1+\cot^2(x)\right)^{3/2}\,dx &= \int \csc^2(x)^{3/2}\,dx \\
&= -\text{Subst}\left(\int \sqrt{1+x^2}\,dx, x, \cot(x)\right) \\
&= -\frac{1}{2}\cot(x)\sqrt{\csc^2(x)} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \cot(x)\right) \\
&= -\frac{1}{2}\sinh^{-1}(\cot(x)) - \frac{1}{2}\cot(x)\sqrt{\csc^2(x)}
\end{aligned}
$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 51 vs. $2(22) = 44$.
time = 0.11, size = 51, normalized size = 2.32

$$\frac{1}{8}\sqrt{\csc^2(x)}\left(-\csc^2\left(\frac{x}{2}\right) - 4\log\left(\cos\left(\frac{x}{2}\right)\right) + 4\log\left(\sin\left(\frac{x}{2}\right)\right) + \sec^2\left(\frac{x}{2}\right)\right)\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cot[x]^2)^(3/2),x]
[Out] (Sqrt[Csc[x]^2]*(-Csc[x/2]^2 - 4*Log[Cos[x/2]] + 4*Log[Sin[x/2]] + Sec[x/2]
^2)*Sin[x])/8
```

Maple [**A**]
time = 0.18, size = 19, normalized size = 0.86

method	result
derivativedivides	$-\frac{\cot(x)\sqrt{1+\cot^2(x)}}{2} - \frac{\operatorname{arcsinh}(\cot(x))}{2}$
default	$-\frac{\cot(x)\sqrt{1+\cot^2(x)}}{2} - \frac{\operatorname{arcsinh}(\cot(x))}{2}$
risch	$-\frac{i\sqrt{-\frac{e^{2ix}}{(e^{2ix}-1)^2}}\,(e^{2ix}+1)}{e^{2ix}-1} - \sqrt{-\frac{e^{2ix}}{(e^{2ix}-1)^2}}\ \ln(e^{ix}+1)\sin(x) + \sqrt{-\frac{e^{2ix}}{(e^{2ix}-1)^2}}\ \ln(e^{ix}-1)\sin$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cot(x)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/2*cot(x)*(1+cot(x)^2)^(1/2)-1/2*arcsinh(cot(x))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(16) = 32$.
time = 0.55, size = 300, normalized size = 13.64

$$-\frac{4\left(\cos(3x)+\cos(x)\right)\cos(4x) - 4\left(2\cos(2x)-1\right)\cos(3x) - 8\cos(2x)\cos(x) + \left(2\left(2\cos(2x)-1\right)\cos(4x) - \cos(4x)^2 - 4\cos(2x)^2 - \sin(4x)^2 + 4\sin(4x)\sin(2x) - 4\sin(2x)^2 + 4\cos(2x) - 1\right)\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right) - \left(2\left(2\cos(2x)-1\right)\cos(4x) - \cos(4x)^2 - 4\cos(2x)^2 - \sin(4x)^2 + 4\sin(4x)\sin(2x) - 4\sin(2x)^2 + 4\cos(2x) - 1\right)\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right) + 4\left(\sin(3x)+\sin(x)\right)\sin(4x) - 8\sin(3x)\sin(2x) - 8\sin(2x)\sin(x) + 4\cos(x)}{4\left(2\left(2\cos(2x)-1\right)\cos(4x) - \cos(4x)^2 - 4\cos(2x)^2 - \sin(4x)^2 + 4\sin(4x)\sin(2x) - 4\sin(2x)^2 + 4\cos(2x) - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(cos(3*x) + cos(x))*cos(4*x) - 4*(2*cos(2*x) - 1)*cos(3*x) - 8*cos(
2*x)*cos(x) + (2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - si
n(4*x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)*log(cos(x)^
2 + sin(x)^2 + 2*cos(x) + 1) - (2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 -
4*cos(2*x)^2 - sin(4*x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x)
 - 1)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) + 4*(sin(3*x) + sin(x))*sin(4
*x) - 8*sin(3*x)*sin(2*x) - 8*sin(2*x)*sin(x) + 4*cos(x))/(2*(2*cos(2*x) -
1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - sin(4*x)^2 + 4*sin(4*x)*sin(2*x)
- 4*sin(2*x)^2 + 4*cos(2*x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(16) = 32$.
time = 3.02, size = 91, normalized size = 4.14

$$-\frac{2\sqrt{2}\sqrt{-\frac{1}{\cos(2x)-1}}(\cos(2x)+1)+\log\left(\frac{1}{2}\sqrt{2}\sqrt{-\frac{1}{\cos(2x)-1}}\sin(2x)+1\right)\sin(2x)-\log\left(-\frac{1}{2}\sqrt{2}\sqrt{-\frac{1}{\cos(2x)-1}}\sin(2x)+1\right)\sin(2x)}{4\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(2)*sqrt(-1/(cos(2*x) - 1))*(cos(2*x) + 1) + log(1/2*sqrt(2)*sq
rt(-1/(cos(2*x) - 1))*sin(2*x) + 1)*sin(2*x) - log(-1/2*sqrt(2)*sqrt(-1/(co
s(2*x) - 1))*sin(2*x) + 1)*sin(2*x))/sin(2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\cot^2(x)+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(x)**2)**(3/2),x)
```

```
[Out] Integral((cot(x)**2 + 1)**(3/2), x)
```

Giac [A]
time = 0.43, size = 32, normalized size = 1.45

$$\frac{1}{4}\left(\frac{2\cos(x)}{\cos(x)^2-1}-\log(\cos(x)+1)+\log(-\cos(x)+1)\right)\mathrm{sgn}(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*cos(x)/(cos(x)^2 - 1) - log(cos(x) + 1) + log(-cos(x) + 1))*sgn(sin(
x))
```

Mupad [B]
time = 0.37, size = 18, normalized size = 0.82

$$-\frac{\operatorname{asinh}(\cot(x))}{2}-\frac{\cot(x)\,\sqrt{\cot\left(x\right)^2+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(x)^2 + 1)^(3/2),x)
[Out] - asinh(cot(x))/2 - (cot(x)*(cot(x)^2 + 1)^(1/2))/2
```

3.9 $\int \sqrt{1+\cot^2(x)}\,dx$

Optimal. Leaf size=5

$$-\sinh^{-1}(\cot(x))$$

```
[Out] -arcsinh(cot(x))
```

Rubi [A]
time = 0.01, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3738, 4207, 221}

$$-\sinh^{-1}(\cot(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Cot[x]^2],x]
```

```
[Out] -ArcSinh[Cot[x]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1+\cot^2(x)}\,dx &= \int \sqrt{\csc^2(x)}\,dx\\
&= -\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \cot(x)\right)\\
&= -\sinh^{-1}(\cot(x))
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 28 vs. $2(5) = 10$.
time = 0.02, size = 28, normalized size = 5.60

$$\sqrt{\csc^2(x)}\left(-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)\right)\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Cot[x]^2],x]
```

```
[Out] Sqrt[Csc[x]^2]*(-Log[Cos[x/2]] + Log[Sin[x/2]])*Sin[x]
```

Maple [A]
time = 0.20, size = 6, normalized size = 1.20

method	result	size
derivativedivides	$-\operatorname{arcsinh}(\cot(x))$	6
default	$-\operatorname{arcsinh}(\cot(x))$	6
risch	$2\sqrt{-\frac{e^{2ix}}{(e^{2ix}-1)^2}}\ \ln(e^{ix}-1)\sin(x)-2\sqrt{-\frac{e^{2ix}}{(e^{2ix}-1)^2}}\ \ln(e^{ix}+1)\sin(x)$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -arcsinh(cot(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(5) = 10$.
time = 0.60, size = 35, normalized size = 7.00

$$-\frac{1}{2}\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)+\frac{1}{2}\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/2*log(cos(x)^2 + sin(x)^2
- 2*cos(x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(5) = 10$.
time = 2.26, size = 53, normalized size = 10.60

$$-\frac{1}{2}\log\left(\frac{1}{2}\sqrt{2}\sqrt{-\frac{1}{\cos(2x)-1}}\sin(2x)+1\right)+\frac{1}{2}\log\left(-\frac{1}{2}\sqrt{2}\sqrt{-\frac{1}{\cos(2x)-1}}\sin(2x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*log(1/2*sqrt(2)*sqrt(-1/(cos(2*x) - 1))*sin(2*x) + 1) + 1/2*log(-1/2*s
qrt(2)*sqrt(-1/(cos(2*x) - 1))*sin(2*x) + 1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\cot^2(x) + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(cot(x)**2 + 1), x)
```

Giac [**A**]
time = 0.45, size = 10, normalized size = 2.00

$$\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)\operatorname{sgn}(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] log(abs(tan(1/2*x)))*sgn(sin(x))
```

Mupad [**B**]
time = 0.32, size = 5, normalized size = 1.00

$$-\operatorname{asinh}(\cot(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(x)^2 + 1)^(1/2),x)
```

```
[Out] -asinh(cot(x))
```

3.10 $\int \frac{1}{\sqrt{1+\cot^2(x)}}\,dx$

Optimal. Leaf size=12

$$-\frac{\cot(x)}{\sqrt{\csc^2(x)}}$$

```
[Out] -cot(x)/(csc(x)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3738, 4207, 197}

$$-\frac{\cot(x)}{\sqrt{\csc^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + Cot[x]^2],x]
```

```
[Out] -(Cot[x]/Sqrt[Csc[x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{1+\cot^2(x)}}\,dx &= \int \frac{1}{\sqrt{\csc^2(x)}}\,dx \\
&= -\mathrm{Subst}\left(\int \frac{1}{(1+x^2)^{3/2}}\,dx, x, \cot(x)\right) \\
&= -\frac{\cot(x)}{\sqrt{\csc^2(x)}}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.00

$$-\frac{\cot(x)}{\sqrt{\csc^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + Cot[x]^2],x]
[Out] -(Cot[x]/Sqrt[Csc[x]^2])
```

Maple [A]
time = 0.13, size = 13, normalized size = 1.08

method	result	size
derivativedivides	$-\frac{\cot(x)}{\sqrt{1+\cot^2(x)}}$	13
default	$-\frac{\cot(x)}{\sqrt{1+\cot^2(x)}}$	13
risch	$-\frac{i e^{2ix}}{2\sqrt{-\frac{e^{2ix}}{(e^{2ix}-1)^2}}\,(e^{2ix}-1)}-\frac{i}{2(e^{2ix}-1)\sqrt{-\frac{e^{2ix}}{(e^{2ix}-1)^2}}}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -cot(x)/(1+cot(x)^2)^(1/2)
```

Maxima [A]
time = 0.50, size = 10, normalized size = 0.83

$$-\frac{1}{\sqrt{\tan(x)^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/sqrt(tan(x)^2 + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(10) = 20$.
time = 2.66, size = 21, normalized size = 1.75

$$-\frac{1}{2}\sqrt{2}\sqrt{-\frac{1}{\cos(2x)-1}}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*sqrt(-1/(cos(2*x) - 1))*sin(2*x)
```

Sympy [A]
time = 0.16, size = 14, normalized size = 1.17

$$-\frac{\cot(x)}{\sqrt{\cot^2(x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cot(x)**2)**(1/2),x)
```

```
[Out] -cot(x)/sqrt(cot(x)**2 + 1)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(10) = 20$.
time = 0.42, size = 28, normalized size = 2.33

$$\frac{2}{\left(\frac{\cos(x)-1}{\cos(x)+1}-1\right)\operatorname{sgn}(\sin(x))}+2\operatorname{sgn}(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/(((cos(x) - 1)/(cos(x) + 1) - 1)*sgn(sin(x))) + 2*sgn(sin(x))
```

Mupad [B]
time = 0.39, size = 12, normalized size = 1.00

$$-\frac{\sin(2x)}{2\sqrt{\sin(x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)^2 + 1)^(1/2),x)
```

```
[Out] -sin(2*x)/(2*(sin(x)^2)^(1/2))
```

3.11 $\int \left(-1-\cot^2(x)\right)^{3/2} \, dx$

Optimal. Leaf size=35

$$-\frac{1}{2}\text{ArcTan}\left(\frac{\cot(x)}{\sqrt{-\csc^2(x)}}\right)+\frac{1}{2}\cot(x)\sqrt{-\csc^2(x)}$$

```
[Out] -1/2*arctan(cot(x)/(-csc(x)^2)^(1/2))+1/2*cot(x)*(-csc(x)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3738, 4207, 201, 223, 209}

$$\frac{1}{2}\cot(x)\sqrt{-\csc^2(x)}-\frac{1}{2}\text{ArcTan}\left(\frac{\cot(x)}{\sqrt{-\csc^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - Cot[x]^2)^(3/2),x]
```

```
[Out] -1/2*ArcTan[Cot[x]/Sqrt[-Csc[x]^2]] + (Cot[x]*Sqrt[-Csc[x]^2])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
```

```
[a, b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \left(-1-\cot^2(x)\right)^{3/2}\,dx &= \int \left(-\csc^2(x)\right)^{3/2}\,dx\\
&= \text{Subst}\left(\int \sqrt{-1-x^2}\,dx, x, \cot(x)\right)\\
&= \frac{1}{2}\cot(x)\sqrt{-\csc^2(x)} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1-x^2}}\,dx, x, \cot(x)\right)\\
&= \frac{1}{2}\cot(x)\sqrt{-\csc^2(x)} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\cot(x)}{\sqrt{-\csc^2(x)}}\right)\\
&= -\frac{1}{2}\tan^{-1}\left(\frac{\cot(x)}{\sqrt{-\csc^2(x)}}\right) + \frac{1}{2}\cot(x)\sqrt{-\csc^2(x)}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 48, normalized size = 1.37

$$
-\frac{\csc\left(\frac{x}{2}\right)\left(\cot(x)\csc(x)+\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)\sec\left(\frac{x}{2}\right)}{4\sqrt{-\csc^2(x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - Cot[x]^2)^(3/2),x]
[Out] -1/4*(Csc[x/2]*(Cot[x]*Csc[x] + Log[Cos[x/2]] - Log[Sin[x/2]])*Sec[x/2])/Sq
rt[-Csc[x]^2]
```

Maple [A]
time = 0.12, size = 32, normalized size = 0.91

method	result	s
derivativedivides	$\frac{\cot(x)\sqrt{-1-\left(\cot^2(x)\right)}}{2} - \frac{\arctan\left(\frac{\cot(x)}{\sqrt{-1-\left(\cot^2(x)\right)}}\right)}{2}$	3

default	$\frac{\cot(x)\sqrt{-1-(\cot^2(x))}}{2}-\frac{\arctan\left(\frac{\cot(x)}{\sqrt{-1-(\cot^2(x))}}\right)}{2}$
risch	$\frac{i\sqrt{\frac{e^{2ix}}{(e^{2ix}-1)^2}}\,(e^{2ix}+1)}{e^{2ix}-1}+\sqrt{\frac{e^{2ix}}{(e^{2ix}-1)^2}}\,\ln(e^{ix}+1)\sin(x)-\sqrt{\frac{e^{2ix}}{(e^{2ix}-1)^2}}\,\ln(e^{ix}-1)\sin(x)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1-cot(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*cot(x)*(-1-cot(x)^2)^(1/2)-1/2*arctan(cot(x)/(-1-cot(x)^2)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(27) = 54$.
time = 0.62, size = 284, normalized size = 8.11

$$\frac{(2(2\cos(2x)-1)\cos(4x)-\cos(4x)^2-4\cos(2x)^2-\sin(4x)^2+4\sin(4x)\sin(2x)-4\sin(2x)^2+4\cos(2x)-1)\arctan(\sin(x),\cos(x)+1)-(2(2\cos(2x)-1)\cos(4x)-\cos(4x)^2-4\cos(2x)^2-\sin(4x)^2+4\sin(4x)\sin(2x)-4\sin(2x)^2+4\cos(2x)-1)\arctan(\sin(x),\cos(x)-1)+2(\sin(3x)+\sin(x))\cos(4x)-2(\cos(3x)+\cos(x))\sin(4x)-2(2\cos(2x)-1)\sin(3x)+4\cos(3x)\sin(2x)+4\cos(x)\sin(2x)-4\cos(2x)\sin(x)+2\sin(x)}{2(2(2\cos(2x)-1)\cos(4x)-\cos(4x)^2-4\cos(2x)^2-\sin(4x)^2+4\sin(4x)\sin(2x)-4\sin(2x)^2+4\cos(2x)-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cot(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*((2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - sin(4*x)^2
+ 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)*arctan2(sin(x), cos(
x) + 1) - (2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - sin(4*
x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)*arctan2(sin(x),
 cos(x) - 1) + 2*(sin(3*x) + sin(x))*cos(4*x) - 2*(cos(3*x) + cos(x))*sin(4
*x) - 2*(2*cos(2*x) - 1)*sin(3*x) + 4*cos(3*x)*sin(2*x) + 4*cos(x)*sin(2*x)
 - 4*cos(2*x)*sin(x) + 2*sin(x))/(2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2
- 4*cos(2*x)^2 - sin(4*x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*
x) - 1)
```

Fricas [C] Result contains complex when optimal does not.
time = 2.28, size = 73, normalized size = 2.09

$$\frac{\left(-i\,e^{(4ix)}+2i\,e^{(2ix)}-i\right)\log\left(e^{(ix)}+1\right)+\left(i\,e^{(4ix)}-2i\,e^{(2ix)}+i\right)\log\left(e^{(ix)}-1\right)+2i\,e^{(3ix)}+2i\,e^{(ix)}}{2\left(e^{(4ix)}-2\,e^{(2ix)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cot(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((-I*e^(4*I*x) + 2*I*e^(2*I*x) - I)*log(e^(I*x) + 1) + (I*e^(4*I*x) - 2
*I*e^(2*I*x) + I)*log(e^(I*x) - 1) + 2*I*e^(3*I*x) + 2*I*e^(I*x))/(e^(4*I*x
) - 2*e^(2*I*x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-\cot^2(x)-1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cot(x)**2)**(3/2),x)
```

```
[Out] Integral((-cot(x)**2 - 1)**(3/2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.43, size = 34, normalized size = 0.97

$$-\frac{1}{4}\left(\frac{2i\cos\left(x\right)}{\cos\left(x\right)^{2}-1}-i\log\left(\cos\left(x\right)+1\right)+i\log\left(-\cos\left(x\right)+1\right)\right)\operatorname{sgn}(\sin\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cot(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*(2*I*cos(x)/(cos(x)^2 - 1) - I*log(cos(x) + 1) + I*log(-cos(x) + 1))*s
gn(sin(x))
```

Mupad [B]
time = 0.37, size = 31, normalized size = 0.89

$$\frac{\cot(x)\sqrt{-\cot\left(x\right)^{2}-1}}{2}-\frac{\operatorname{atan}\left(\frac{\cot(x)}{\sqrt{-\cot\left(x\right)^{2}-1}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- cot(x)^2 - 1)^(3/2),x)
```

```
[Out] (cot(x)*(- cot(x)^2 - 1)^(1/2))/2 - atan(cot(x)/(- cot(x)^2 - 1)^(1/2))/2
```

3.12 $\int \sqrt{-1-\cot^2(x)}\,dx$

Optimal. Leaf size=14

$$\text{ArcTan}\left(\frac{\cot(x)}{\sqrt{-\csc^2(x)}}\right)$$

```
[Out] arctan(cot(x)/(-csc(x)^2)^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3738, 4207, 223, 209}

$$\text{ArcTan}\left(\frac{\cot(x)}{\sqrt{-\csc^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 - Cot[x]^2],x]
```

```
[Out] ArcTan[Cot[x]/Sqrt[-Csc[x]^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{-1-\cot^2(x)}\,dx &= \int \sqrt{-\csc^2(x)}\,dx \\
&= \text{Subst}\left(\int \frac{1}{\sqrt{-1-x^2}}\,dx, x, \cot(x)\right) \\
&= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\cot(x)}{\sqrt{-\csc^2(x)}}\right) \\
&= \tan^{-1}\left(\frac{\cot(x)}{\sqrt{-\csc^2(x)}}\right)
\end{aligned}
$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 30 vs. $2(14) = 28$.
time = 0.02, size = 30, normalized size = 2.14

$$
\frac{\csc(x)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)}{\sqrt{-\csc^2(x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 - Cot[x]^2],x]
[Out] (Csc[x]*(Log[Cos[x/2]] - Log[Sin[x/2]]))/Sqrt[-Csc[x]^2]
```

Maple [**A**]
time = 0.13, size = 15, normalized size = 1.07

method	result	size
derivativedivides	$\arctan\left(\frac{\cot(x)}{\sqrt{-1-\left(\cot^2(x)\right)}}\right)$	15
default	$\arctan\left(\frac{\cot(x)}{\sqrt{-1-\left(\cot^2(x)\right)}}\right)$	15
risch	$-2\sqrt{\frac{e^{2ix}}{\left(e^{2ix}-1\right)^2}}\,\ln\left(e^{ix}+1\right)\sin(x)+2\sqrt{\frac{e^{2ix}}{\left(e^{2ix}-1\right)^2}}\,\ln\left(e^{ix}-1\right)\sin(x)$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1-cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] arctan(cot(x)/(-1-cot(x)^2)^(1/2))
```

Maxima [**A**]
time = 0.53, size = 17, normalized size = 1.21

$$
-\arctan\left(\sin(x), \cos(x)+1\right)+\arctan\left(\sin(x), \cos(x)-1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arctan2(sin(x), cos(x) + 1) + arctan2(sin(x), cos(x) - 1)
```

Fricas [C] Result contains complex when optimal does not.
time = 2.70, size = 19, normalized size = 1.36

$$i \log \left(e^{(i x)} + 1\right) - i \log \left(e^{(i x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] I*log(e^(I*x) + 1) - I*log(e^(I*x) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\cot^{2}(x) - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cot(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-cot(x)**2 - 1), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.43, size = 11, normalized size = 0.79

$$i \log \left(\left| \tan \left(\frac{1}{2} x \right) \right| \right) \operatorname{sgn}(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] I*log(abs(tan(1/2*x)))*sgn(sin(x))
```

Mupad [B]
time = 0.39, size = 14, normalized size = 1.00

$$\operatorname{atan}\left(\frac{\cot(x)}{\sqrt{-\cot(x)^2 - 1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- cot(x)^2 - 1)^(1/2),x)
```

```
[Out] atan(cot(x)/(- cot(x)^2 - 1)^(1/2))
```

3.13 $\int \frac{1}{\sqrt{-1-\cot^2(x)}}\,dx$

Optimal. Leaf size=14

$$-\frac{\cot(x)}{\sqrt{-\csc^2(x)}}$$

```
[Out] -cot(x)/(-csc(x)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3738, 4207, 197}

$$-\frac{\cot(x)}{\sqrt{-\csc^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-1 - Cot[x]^2],x]
```

```
[Out] -(Cot[x]/Sqrt[-Csc[x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1-\cot^2(x)}}\,dx = \int \frac{1}{\sqrt{-\csc^2(x)}}\,dx$$
$$= \text{Subst}\left(\int \frac{1}{(-1-x^2)^{3/2}}\,dx, x, \cot(x)\right)$$
$$= -\frac{\cot(x)}{\sqrt{-\csc^2(x)}}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$-\frac{\cot(x)}{\sqrt{-\csc^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 - Cot[x]^2],x]
[Out] -(Cot[x]/Sqrt[-Csc[x]^2])
```

Maple [A]
time = 0.08, size = 15, normalized size = 1.07

method	result	size
derivativedivides	$-\frac{\cot(x)}{\sqrt{-1-\left(\cot^2(x)\right)}}$	15
default	$-\frac{\cot(x)}{\sqrt{-1-\left(\cot^2(x)\right)}}$	15
risch	$-\frac{i e^{2ix}}{2\sqrt{\frac{e^{2ix}}{(e^{2ix}-1)^2}}\,(e^{2ix}-1)}-\frac{i}{2(e^{2ix}-1)\sqrt{\frac{e^{2ix}}{(e^{2ix}-1)^2}}}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1-cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -cot(x)/(-1-cot(x)^2)^(1/2)
```

Maxima [A]
time = 0.54, size = 12, normalized size = 0.86

$$-\frac{1}{\sqrt{-\tan(x)^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/sqrt(-tan(x)^2 - 1)
```

Fricas [C] Result contains complex when optimal does not.
time = 2.22, size = 14, normalized size = 1.00

$$\frac{1}{2}\left(-i\,e^{(2\,i\,x)}-i\right)e^{(-i\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*e^(2*I*x) - I)*e^(-I*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-\cot^{2}(x)-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-cot(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-cot(x)**2 - 1), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.42, size = 28, normalized size = 2.00

$$-\frac{2i}{\left(\frac{\cos(x)-1}{\cos(x)+1}-1\right)\operatorname{sgn}(\sin(x))}-2i\operatorname{sgn}(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*I/(((cos(x) - 1)/(cos(x) + 1) - 1)*sgn(sin(x))) - 2*I*sgn(sin(x))
```

Mupad [B]
time = 0.68, size = 13, normalized size = 0.93

$$\frac{\sin(2\,x)\;1\mathrm{i}}{2\,\sqrt{\sin(x)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- cot(x)^2 - 1)^(1/2),x)
```

```
[Out] (sin(2*x)*1i)/(2*(sin(x)^2)^(1/2))
```

3.14 $\int \frac{\cot^3(x)}{\sqrt{a+a\cot^2(x)}}\,dx$

Optimal. Leaf size=28

$$-\frac{1}{\sqrt{a\csc^2(x)}}-\frac{\sqrt{a\csc^2(x)}}{a}$$

[Out] -1/(a*csc(x)^2)^(1/2)-(a*csc(x)^2)^(1/2)/a

Rubi [A]

time = 0.06, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3738, 4209, 45}

$$-\frac{\sqrt{a\csc^2(x)}}{a}-\frac{1}{\sqrt{a\csc^2(x)}}$$

Antiderivative was successfully verified.

[In] Int[Cot[x]^3/Sqrt[a + a*Cot[x]^2],x]

[Out] -(1/Sqrt[a*Csc[x]^2]) - Sqrt[a*Csc[x]^2]/a

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4209

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^3(x)}{\sqrt{a+a\cot^2(x)}}\,dx &= \int \frac{\cot^3(x)}{\sqrt{a\csc^2(x)}}\,dx \\
&= -\left(\frac{1}{2}a\text{Subst}\left(\int \frac{-1+x}{(ax)^{3/2}}\,dx, x, \csc^2(x)\right)\right) \\
&= -\left(\frac{1}{2}a\text{Subst}\left(\int \left(-\frac{1}{(ax)^{3/2}}+\frac{1}{a\sqrt{ax}}\right)dx, x, \csc^2(x)\right)\right) \\
&= -\frac{1}{\sqrt{a\csc^2(x)}}-\frac{\sqrt{a\csc^2(x)}}{a}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 19, normalized size = 0.68

$$\frac{-1-\csc^2(x)}{\sqrt{a\csc^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3/Sqrt[a + a*Cot[x]^2],x]
```

```
[Out] (-1 - Csc[x]^2)/Sqrt[a*Csc[x]^2]
```

Maple [A]
time = 0.17, size = 29, normalized size = 1.04

method	result	size
derivativedivides	$-\frac{\sqrt{a+a\left(\cot^2\left(x\right)\right)}}{a}-\frac{1}{\sqrt{a+a\left(\cot^2\left(x\right)\right)}}$	29
default	$-\frac{\sqrt{a+a\left(\cot^2\left(x\right)\right)}}{a}-\frac{1}{\sqrt{a+a\left(\cot^2\left(x\right)\right)}}$	29
risch	$-\frac{e^{2ix}}{2\sqrt{-\frac{a\,e^{2ix}}{\left(e^{2ix}-1\right)^2}}\,\left(e^{2ix}-1\right)}+\frac{1}{2\left(e^{2ix}-1\right)\sqrt{-\frac{a\,e^{2ix}}{\left(e^{2ix}-1\right)^2}}}+\frac{2\,e^{2ix}}{\sqrt{-\frac{a\,e^{2ix}}{\left(e^{2ix}-1\right)^2}}\,\left(e^{2ix}-1\right)^2}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3/(a+a*cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a*(a+a*cot(x)^2)^(1/2)-1/(a+a*cot(x)^2)^(1/2)
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.86

$$-\frac{1}{\sqrt{\frac{a}{\sin(x)^2}}}-\frac{\sqrt{\frac{a}{\sin(x)^2}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+a*cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/sqrt(a/sin(x)^2) - sqrt(a/sin(x)^2)/a
```

Fricas [A]
time = 2.72, size = 27, normalized size = 0.96

$$\frac{\sqrt{2}\sqrt{-\frac{a}{\cos(2x)-1}}\left(\cos(2x)-3\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+a*cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*sqrt(-a/(cos(2*x) - 1))*(cos(2*x) - 3)/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot^3(x)}{\sqrt{a\left(\cot^2(x)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**3/(a+a*cot(x)**2)**(1/2),x)
```

```
[Out] Integral(cot(x)**3/sqrt(a*(cot(x)**2 + 1)), x)
```

Giac [A]
time = 0.42, size = 25, normalized size = 0.89

$$-\frac{\sqrt{a}\sin(x)+\frac{\sqrt{a}}{\sin(x)}}{a\mathrm{sgn}\left(\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+a*cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(sqrt(a)*sin(x) + sqrt(a)/sin(x))/(a*sgn(sin(x)))
```

Mupad [B]
time = 0.59, size = 17, normalized size = 0.61

$$-\frac{\sin(x)^2+1}{\sqrt{a}\,\sqrt{\sin(x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3/(a + a*cot(x)^2)^(1/2),x)
[Out] -(sin(x)^2 + 1)/(a^(1/2)*(sin(x)^2)^(1/2))
```

3.15 $\int \frac{\cot^2(x)}{\sqrt{a+a\cot^2(x)}}\,dx$

Optimal. Leaf size=31

$$\frac{\cot(x)}{\sqrt{a\csc^2(x)}}-\frac{\tanh^{-1}(\cos(x))\csc(x)}{\sqrt{a\csc^2(x)}}$$

```
[Out] cot(x)/(a*csc(x)^2)^(1/2)-arctanh(cos(x))*csc(x)/(a*csc(x)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3738, 4210, 2672, 327, 212}

$$\frac{\cot(x)}{\sqrt{a\csc^2(x)}}-\frac{\csc(x)\tanh^{-1}(\cos(x))}{\sqrt{a\csc^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2/Sqrt[a + a*Cot[x]^2],x]
```

```
[Out] Cot[x]/Sqrt[a*Csc[x]^2] - (ArcTanh[Cos[x]]*Csc[x])/Sqrt[a*Csc[x]^2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4210

```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sec[e + f*x]^
n)^FracPart[p]/(Sec[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sec
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^2(x)}{\sqrt{a+a\cot^2(x)}}\,dx &= \int \frac{\cot^2(x)}{\sqrt{a\csc^2(x)}}\,dx \\
&= \frac{\csc(x)\int \cos(x)\cot(x)\,dx}{\sqrt{a\csc^2(x)}} \\
&= -\frac{\csc(x)\mathrm{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cos(x)\right)}{\sqrt{a\csc^2(x)}} \\
&= \frac{\cot(x)}{\sqrt{a\csc^2(x)}} - \frac{\csc(x)\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(x)\right)}{\sqrt{a\csc^2(x)}} \\
&= \frac{\cot(x)}{\sqrt{a\csc^2(x)}} - \frac{\tanh^{-1}(\cos(x))\csc(x)}{\sqrt{a\csc^2(x)}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.04, size = 32, normalized size = 1.03

$$\frac{\csc(x)\left(\cos(x)-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)\right)}{\sqrt{a\csc^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2/Sqrt[a + a*Cot[x]^2],x]
[Out] (Csc[x]*(Cos[x] - Log[Cos[x/2]] + Log[Sin[x/2]]))/Sqrt[a*Csc[x]^2]
```

Maple [**A**]
time = 0.20, size = 38, normalized size = 1.23

method	result
derivativedivides	$-\frac{\ln\left(\sqrt{a}\,\cot(x)+\sqrt{a+a\left(\cot^2(x)\right)}\right)}{\sqrt{a}}+\frac{\cot(x)}{\sqrt{a+a\left(\cot^2(x)\right)}}$
default	$-\frac{\ln\left(\sqrt{a}\,\cot(x)+\sqrt{a+a\left(\cot^2(x)\right)}\right)}{\sqrt{a}}+\frac{\cot(x)}{\sqrt{a+a\left(\cot^2(x)\right)}}$
risch	$\frac{i e^{2ix}}{2\sqrt{-\frac{a\,e^{2ix}}{(e^{2ix}-1)^2}}\,(e^{2ix}-1)}+\frac{i}{2(e^{2ix}-1)\sqrt{-\frac{a\,e^{2ix}}{(e^{2ix}-1)^2}}}+\frac{i e^{ix}\ln(e^{ix}-1)}{\sqrt{-\frac{a\,e^{2ix}}{(e^{2ix}-1)^2}}\,(e^{2ix}-1)}-\frac{i e^{ix}\ln(e^{ix}+1)}{\sqrt{-\frac{a\,e^{2ix}}{(e^{2ix}-1)^2}}\,(e^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/(a+a*cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -ln(a^(1/2)*cot(x)+(a+a*cot(x)^2)^(1/2))/a^(1/2)+cot(x)/(a+a*cot(x)^2)^(1/2
)
```

Maxima [A]
time = 0.58, size = 27, normalized size = 0.87

$$-\frac{\sqrt{-a}\,(\arctan(\sin(x),\cos(x)+1)-\arctan(\sin(x),\cos(x)-1))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+a*cot(x)^2)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-a)*(arctan2(sin(x), cos(x) + 1) - arctan2(sin(x), cos(x) - 1))/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(27) = 54$.
time = 2.99, size = 77, normalized size = 2.48

$$\frac{\sqrt{2}\,\sqrt{-\frac{a}{\cos(2x)-1}}\,\sin(2x)+\sqrt{a}\,\log\left(\frac{2\sqrt{2}\,\sqrt{a}\,\sqrt{-\frac{a}{\cos(2x)-1}}\,\sin(2x)-a\cos(2x)-3a}{\cos(2x)-1}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+a*cot(x)^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*(sqrt(2)*sqrt(-a/(cos(2*x) - 1))*sin(2*x) + sqrt(a)*log((2*sqrt(2)*sqrt
(a)*sqrt(-a/(cos(2*x) - 1))*sin(2*x) - a*cos(2*x) - 3*a)/(cos(2*x) - 1)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(x)}{\sqrt{a(\cot^2(x)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2/(a+a*cot(x)**2)**(1/2),x)
```

```
[Out] Integral(cot(x)**2/sqrt(a*(cot(x)**2 + 1)), x)
```

Giac [A]
time = 0.41, size = 49, normalized size = 1.58

$$\frac{1}{2}\sqrt{a}\left(\frac{2\cos(x)}{a\operatorname{sgn}(\sin(x))}-\frac{\log(\cos(x)+1)}{a\operatorname{sgn}(\sin(x))}+\frac{\log(-\cos(x)+1)}{a\operatorname{sgn}(\sin(x))}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+a*cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a)*(2*cos(x)/(a*sgn(sin(x))) - log(cos(x) + 1)/(a*sgn(sin(x))) + l
og(-cos(x) + 1)/(a*sgn(sin(x))))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\cot(x)}^2}{\sqrt{a\,{\cot(x)}^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/(a + a*cot(x)^2)^(1/2),x)
```

```
[Out] int(cot(x)^2/(a + a*cot(x)^2)^(1/2), x)
```

3.16 $\int \frac{\cot(x)}{\sqrt{a+a\cot^2(x)}}\,dx$

Optimal. Leaf size=10

$$\frac{1}{\sqrt{a\csc^2(x)}}$$

[Out] 1/(a*csc(x)^2)^(1/2)

Rubi [A]
time = 0.03, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3738, 4209, 32}

$$\frac{1}{\sqrt{a\csc^2(x)}}$$

Antiderivative was successfully verified.

[In] Int[Cot[x]/Sqrt[a + a*Cot[x]^2],x]

[Out] 1/Sqrt[a*Csc[x]^2]

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4209

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cot(x)}{\sqrt{a+a\cot^2(x)}}\,dx = \int \frac{\cot(x)}{\sqrt{a\csc^2(x)}}\,dx$$
$$= -\left(\frac{1}{2}a\text{Subst}\left(\int \frac{1}{(ax)^{3/2}}\,dx, x, \csc^2(x)\right)\right)$$
$$= \frac{1}{\sqrt{a\csc^2(x)}}$$

Mathematica [A]
time = 0.01, size = 10, normalized size = 1.00

$$\frac{1}{\sqrt{a\csc^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/Sqrt[a + a*Cot[x]^2],x]
```

```
[Out] 1/Sqrt[a*Csc[x]^2]
```

Maple [A]
time = 0.11, size = 11, normalized size = 1.10

method	result	size
derivativedivides	$\frac{1}{\sqrt{a+a\left(\cot^2(x)\right)}}$	11
default	$\frac{1}{\sqrt{a+a\left(\cot^2(x)\right)}}$	11
risch	$\frac{e^{2ix}}{2\sqrt{-\frac{a\,e^{2ix}}{(e^{2ix}-1)^2}}\,(e^{2ix}-1)}-\frac{1}{2(e^{2ix}-1)\sqrt{-\frac{a\,e^{2ix}}{(e^{2ix}-1)^2}}}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a+a*cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a+a*cot(x)^2)^(1/2)
```

Maxima [A]
time = 0.29, size = 8, normalized size = 0.80

$$\frac{1}{\sqrt{\frac{a}{\sin(x)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+a*cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/sqrt(a/sin(x)^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(8) = 16$.
time = 2.91, size = 27, normalized size = 2.70

$$-\frac{\sqrt{2}\sqrt{-\frac{a}{\cos(2x)-1}}(\cos(2x)-1)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+a*cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*sqrt(-a/(cos(2*x) - 1))*(cos(2*x) - 1)/a
```

Sympy [A]
time = 0.65, size = 12, normalized size = 1.20

$$\frac{1}{\sqrt{a\cot^2(x)+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+a*cot(x)**2)**(1/2),x)
```

```
[Out] 1/sqrt(a*cot(x)**2 + a)
```

Giac [A]
time = 0.44, size = 11, normalized size = 1.10

$$\frac{\sin(x)}{\sqrt{a}\,\mathrm{sgn}(\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+a*cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] sin(x)/(sqrt(a)*sgn(sin(x)))
```

Mupad [B]
time = 0.48, size = 10, normalized size = 1.00

$$\frac{\sqrt{\sin(x)^2}}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a + a*cot(x)^2)^(1/2),x)
```

```
[Out] (sin(x)^2)^(1/2)/a^(1/2)
```

3.17 $\int \frac{\tan(x)}{\sqrt{a+a\cot^2(x)}}\,dx$

Optimal. Leaf size=36

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a\csc^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{1}{\sqrt{a\csc^2(x)}}$$

[Out] arctanh((a*csc(x)^2)^(1/2)/a^(1/2))/a^(1/2)-1/(a*csc(x)^2)^(1/2)

Rubi [A]
time = 0.06, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3738, 4209, 53, 65, 213}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a\csc^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{1}{\sqrt{a\csc^2(x)}}$$

Antiderivative was successfully verified.

[In] Int[Tan[x]/Sqrt[a + a*Cot[x]^2],x]

[Out] ArcTanh[Sqrt[a*Csc[x]^2]/Sqrt[a]]/Sqrt[a] - 1/Sqrt[a*Csc[x]^2]

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4209

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan(x)}{\sqrt{a+a\cot^2(x)}}\,dx &= \int \frac{\tan(x)}{\sqrt{a\csc^2(x)}}\,dx\\
&= -\left(\frac{1}{2}a\text{Subst}\left(\int \frac{1}{(-1+x)(ax)^{3/2}}\,dx, x, \csc^2(x)\right)\right)\\
&= -\frac{1}{\sqrt{a\csc^2(x)}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{ax}}\,dx, x, \csc^2(x)\right)\\
&= -\frac{1}{\sqrt{a\csc^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a\csc^2(x)}\right)}{a}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a\csc^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{1}{\sqrt{a\csc^2(x)}}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 49, normalized size = 1.36

$$-\frac{\csc(x)\left(\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+\sin(x)\right)}{\sqrt{a\csc^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/Sqrt[a + a*Cot[x]^2],x]
[Out] -((Csc[x]*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]] + Sin[x]))/S
qrt[a*Csc[x]^2])
```

Maple [A]
time = 0.57, size = 56, normalized size = 1.56

method	result	si
default	$-\frac{\left(\sin(x)-\ln\left(-\frac{\cos(x)-1-\sin(x)}{\sin(x)}\right)+\ln\left(-\frac{\cos(x)-1+\sin(x)}{\sin(x)}\right)\right)\sqrt{4}}{2\sin(x)\sqrt{-\frac{a}{\cos^2(x)-1}}}$	56
risch	$-\frac{e^{2ix}}{2\sqrt{-\frac{a\,e^{2ix}}{(e^{2ix}-1)^2}}\,(e^{2ix}-1)}+\frac{1}{2(e^{2ix}-1)\sqrt{-\frac{a\,e^{2ix}}{(e^{2ix}-1)^2}}}-\frac{i e^{ix}\ln(e^{ix}-i)}{\sqrt{-\frac{a\,e^{2ix}}{(e^{2ix}-1)^2}}\,(e^{2ix}-1)}+\frac{i e^{ix}\ln(e^{ix}+i)}{\sqrt{-\frac{a\,e^{2ix}}{(e^{2ix}-1)^2}}\,(e^{2ix}-1)}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a+a*cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(sin(x)-ln(-(cos(x)-1-sin(x))/sin(x))+ln(-(cos(x)-1+sin(x))/sin(x)))/s
in(x)/(-1/(cos(x)^2-1)*a)^(1/2)*4^(1/2)
```

Maxima [A]
time = 0.50, size = 52, normalized size = 1.44

$$-\frac{1}{2}\,a\left(\frac{\log\left(-\frac{\sqrt{a}-\sqrt{\frac{a}{\sin(x)^2}}}{\sqrt{a}+\sqrt{\frac{a}{\sin(x)^2}}}\right)}{a^{\frac{3}{2}}}+\frac{2}{a\sqrt{\frac{a}{\sin(x)^2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+a*cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*a*(log(-(sqrt(a) - sqrt(a/sin(x)^2))/(sqrt(a) + sqrt(a/sin(x)^2)))/a^(
3/2) + 2/(a*sqrt(a/sin(x)^2)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(28) = 56$.
time = 2.95, size = 78, normalized size = 2.17

$$\frac{(\tan(x)^2+1)\sqrt{a}\,\log\left(2\,a\tan(x)^2+2\sqrt{a}\sqrt{\frac{a\tan(x)^2+a}{\tan(x)^2}}\,\tan(x)^2+a\right)-2\sqrt{\frac{a\tan(x)^2+a}{\tan(x)^2}}\,\tan(x)^2}{2\left(a\tan(x)^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+a*cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*((tan(x)^2 + 1)*sqrt(a)*log(2*a*tan(x)^2 + 2*sqrt(a)*sqrt((a*tan(x)^2 +
 a)/tan(x)^2)*tan(x)^2 + a) - 2*sqrt((a*tan(x)^2 + a)/tan(x)^2)*tan(x)^2)/(
a*tan(x)^2 + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{\sqrt{a(\cot^2(x)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+a*cot(x)**2)**(1/2),x)
[Out] Integral(tan(x)/sqrt(a*(cot(x)**2 + 1)), x)
```

Giac [A]
time = 0.46, size = 12, normalized size = 0.33

$$-\frac{\sin(x)}{\sqrt{a}\,\mathrm{sgn}(\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+a*cot(x)^2)^(1/2),x, algorithm="giac")
[Out] -sin(x)/(sqrt(a)*sgn(sin(x)))
```

Mupad [B]
time = 0.42, size = 20, normalized size = 0.56

$$\frac{\operatorname{atanh}\left(\sqrt{\frac{1}{\sin(x)^2}}\right)-\sqrt{\sin(x)^2}}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a + a*cot(x)^2)^(1/2),x)
[Out] (atanh((1/sin(x)^2)^(1/2)) - (sin(x)^2)^(1/2))/a^(1/2)
```

3.18 $\int \frac{\tan^2(x)}{\sqrt{a+a\cot^2(x)}}\,dx$

Optimal. Leaf size=29

$$\frac{\cot(x)}{\sqrt{a\csc^2(x)}}+\frac{\csc(x)\sec(x)}{\sqrt{a\csc^2(x)}}$$

```
[Out] cot(x)/(a*csc(x)^2)^(1/2)+csc(x)*sec(x)/(a*csc(x)^2)^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3738, 4210, 2670, 14}

$$\frac{\cot(x)}{\sqrt{a\csc^2(x)}}+\frac{\csc(x)\sec(x)}{\sqrt{a\csc^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^2/Sqrt[a + a*Cot[x]^2],x]
```

```
[Out] Cot[x]/Sqrt[a*Csc[x]^2] + (Csc[x]*Sec[x])/Sqrt[a*Csc[x]^2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4210

```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sec[e + f*x]^
n)^FracPart[p]/(Sec[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sec
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
```

```
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^2(x)}{\sqrt{a+a\cot^2(x)}}\,dx &= \int \frac{\tan^2(x)}{\sqrt{a\csc^2(x)}}\,dx\\ &= \frac{\csc(x)\int \sin(x)\tan^2(x)\,dx}{\sqrt{a\csc^2(x)}}\\ &= -\frac{\csc(x)\mathrm{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, \cos(x)\right)}{\sqrt{a\csc^2(x)}}\\ &= -\frac{\csc(x)\mathrm{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)\,dx, x, \cos(x)\right)}{\sqrt{a\csc^2(x)}}\\ &= \frac{\cot(x)}{\sqrt{a\csc^2(x)}}+\frac{\csc(x)\sec(x)}{\sqrt{a\csc^2(x)}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 19, normalized size = 0.66

$$\frac{\cot(x)+\csc(x)\sec(x)}{\sqrt{a\csc^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^2/Sqrt[a + a*Cot[x]^2],x]
[Out] (Cot[x] + Csc[x]*Sec[x])/Sqrt[a*Csc[x]^2]
```

Maple [A]
time = 0.42, size = 33, normalized size = 1.14

method	result	size
default	$\frac{(\sin^3(x))\sqrt{4}}{2\sqrt{-\frac{a}{\cos^2(x)-1}}\,\cos(x)(-1+\cos(x))^2}$	33
risch	$\frac{i e^{2ix}}{2\sqrt{-\frac{a\,e^{2ix}}{(e^{2ix}-1)^2}}\,(e^{2ix}-1)}+\frac{i}{2(e^{2ix}-1)\sqrt{-\frac{a\,e^{2ix}}{(e^{2ix}-1)^2}}}+\frac{2i e^{2ix}}{\sqrt{-\frac{a\,e^{2ix}}{(e^{2ix}-1)^2}}\,(e^{2ix}-1)(e^{2ix}+1)}$	114

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(a+a*cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sin(x)^3/(-1/(cos(x)^2-1)*a)^(1/2)/cos(x)/(-1+cos(x))^2*4^(1/2)
```

Maxima [A]
time = 0.53, size = 18, normalized size = 0.62

$$\frac{\tan\left(x\right)^{2}+2}{\sqrt{\tan\left(x\right)^{2}+1}\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+a*cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] (tan(x)^2 + 2)/(sqrt(tan(x)^2 + 1)*sqrt(a))
```

Fricas [A]
time = 2.40, size = 35, normalized size = 1.21

$$\frac{\left(\tan\left(x\right)^{3}+2\,\tan\left(x\right)\right)\sqrt{\frac{a\tan\left(x\right)^{2}+a}{\tan\left(x\right)^{2}}}}{a\tan\left(x\right)^{2}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+a*cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (tan(x)^3 + 2*tan(x))*sqrt((a*tan(x)^2 + a)/tan(x)^2)/(a*tan(x)^2 + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan^{2}\left(x\right)}{\sqrt{a\left(\cot^{2}\left(x\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**2/(a+a*cot(x)**2)**(1/2),x)
```

```
[Out] Integral(tan(x)**2/sqrt(a*(cot(x)**2 + 1)), x)
```

Giac [A]
time = 0.42, size = 25, normalized size = 0.86

$$-\frac{2\,\mathrm{sgn}(\sin\left(x\right))}{\sqrt{a}}+\frac{\frac{1}{\cos\left(x\right)}+\cos\left(x\right)}{\sqrt{a}\,\mathrm{sgn}\left(\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+a*cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sgn(sin(x))/sqrt(a) + (1/cos(x) + cos(x))/(sqrt(a)*sgn(sin(x)))
```

Mupad [B]
time = 0.73, size = 34, normalized size = 1.17

$$\frac{\tan(x)^3\sqrt{\frac{1}{\tan(x)^2}}+2\tan(x)\sqrt{\frac{1}{\tan(x)^2}}}{\sqrt{a}\sqrt{\tan(x)^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(a + a*cot(x)^2)^(1/2),x)
```

```
[Out] (tan(x)^3*(1/tan(x)^2)^(1/2) + 2*tan(x)*(1/tan(x)^2)^(1/2))/(a^(1/2)*(tan(x
)^2 + 1)^(1/2))
```

3.19 $\int \cot^3(x)\sqrt{a+b\cot^2(x)}\,dx$

Optimal. Leaf size=66

$$-\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)+\sqrt{a+b\cot^2(x)}-\frac{\left(a+b\cot^2(x)\right)^{3/2}}{3b}$$

```
[Out] -1/3*(a+b*cot(x)^2)^(3/2)/b-arctanh((a+b*cot(x)^2)^(1/2)/(a-b)^(1/2))*(a-b)
^(1/2)+(a+b*cot(x)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3751, 457, 81, 52, 65, 214}

$$-\frac{\left(a+b\cot^2(x)\right)^{3/2}}{3b}+\sqrt{a+b\cot^2(x)}-\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^3*Sqrt[a + b*Cot[x]^2],x]
```

```
[Out] -(Sqrt[a - b]*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]]) + Sqrt[a + b*Cot[x
]^2] - (a + b*Cot[x]^2)^(3/2)/(3*b)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \cot^3(x)\sqrt{a+b\cot^2(x)}\,dx &= -\mathrm{Subst}\left(\int \frac{x^3\sqrt{a+bx^2}}{1+x^2}\,dx, x, \cot(x)\right)\\
&= -\left(\frac{1}{2}\mathrm{Subst}\left(\int \frac{x\sqrt{a+bx}}{1+x}\,dx, x, \cot^2(x)\right)\right)\\
&= -\frac{\left(a+b\cot^2(x)\right)^{3/2}}{3b}+\frac{1}{2}\mathrm{Subst}\left(\int \frac{\sqrt{a+bx}}{1+x}\,dx, x, \cot^2(x)\right)\\
&= \sqrt{a+b\cot^2(x)}-\frac{\left(a+b\cot^2(x)\right)^{3/2}}{3b}+\frac{1}{2}(a-b)\mathrm{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\right.\\
&= \sqrt{a+b\cot^2(x)}-\frac{\left(a+b\cot^2(x)\right)^{3/2}}{3b}+\frac{(a-b)\mathrm{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a}\right.}{b}\\
&= -\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)+\sqrt{a+b\cot^2(x)}-\frac{(a+b\cot^2(x)}{3b}
\end{aligned}
$$

Mathematica [A]
time = 0.20, size = 65, normalized size = 0.98

$$-\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)-\frac{\sqrt{a+b\cot^2(x)}\,(a-3b+b\cot^2(x))}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3*Sqrt[a + b*Cot[x]^2],x]
```

```
[Out] -(Sqrt[a - b]*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]]) - (Sqrt[a + b*Cot[
x]^2]*(a - 3*b + b*Cot[x]^2))/(3*b)
```

Maple [A]
time = 0.22, size = 90, normalized size = 1.36

method	result
derivativedivides	$-\frac{(a+b(\cot^2(x)))^{\frac{3}{2}}}{3b}+b\left(\frac{\sqrt{a+b\left(\cot^2(x)\right)}}{b}-\frac{\arctan\left(\frac{\sqrt{a+b\left(\cot^2(x)\right)}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}\right)+\frac{a\arctan\left(\frac{\sqrt{a-}}{}\right.}{\sqrt{-}}$
default	$-\frac{(a+b(\cot^2(x)))^{\frac{3}{2}}}{3b}+b\left(\frac{\sqrt{a+b\left(\cot^2(x)\right)}}{b}-\frac{\arctan\left(\frac{\sqrt{a+b\left(\cot^2(x)\right)}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}\right)+\frac{a\arctan\left(\frac{\sqrt{a-}}{}\right.}{\sqrt{-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3*(a+b*cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(a+b*cot(x)^2)^(3/2)/b+b*((a+b*cot(x)^2)^(1/2)/b-1/(-a+b)^(1/2)*arctan
((a+b*cot(x)^2)^(1/2)/(-a+b)^(1/2)))+a/(-a+b)^(1/2)*arctan((a+b*cot(x)^2)^(
1/2)/(-a+b)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*(a+b*cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(54) = 108$.
time = 3.40, size = 330, normalized size = 5.00

$$\left[\frac{3\,\left(b\cos\left(2\,x\right)-b\right)\sqrt{a-b}\,\log\left(-2\,\left(a^2-2\,ab+b^2\right)\cos\left(2\,x\right)^2-2\,a^2+b^2+2\,\left(\left(a-b\right)\cos\left(2\,x\right)^2-\left(2\,a-b\right)\cos\left(2\,x\right)+a\right)\sqrt{a-b}\,\sqrt{\frac{\left(a-b\right)\cos\left(2\,x\right)-a-b}{\cos\left(2\,x\right)-1}}+4\,\left(a^2-ab\right)\cos\left(2\,x\right)\right)-4\,\left(\left(a-4\,b\right)\cos\left(2\,x\right)-a+2\,b\right)\sqrt{\frac{\left(a-b\right)\cos\left(2\,x\right)-a-b}{\cos\left(2\,x\right)-1}}}{12\,\left(b\cos\left(2\,x\right)-b\right)},\,-\frac{3\,\left(b\cos\left(2\,x\right)-b\right)\sqrt{-a+b}\,\arctan\left(-\frac{\sqrt{-a+b}\,\sqrt{\frac{\left(a-b\right)\cos\left(2\,x\right)-a-b}{\cos\left(2\,x\right)-1}}\,\left(\cos\left(2\,x\right)-1\right)}{\left(a-b\right)\cos\left(2\,x\right)-a}\right)+2\,\left(\left(a-4\,b\right)\cos\left(2\,x\right)-a+2\,b\right)\sqrt{\frac{\left(a-b\right)\cos\left(2\,x\right)-a-b}{\cos\left(2\,x\right)-1}}}{6\,\left(b\cos\left(2\,x\right)-b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*(a+b*cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b*cos(2*x) - b)*sqrt(a - b)*log(-2*(a^2 - 2*a*b + b^2)*cos(2*x)^2
 - 2*a^2 + b^2 + 2*((a - b)*cos(2*x)^2 - (2*a - b)*cos(2*x) + a)*sqrt(a - b
)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)) + 4*(a^2 - a*b)*cos(2*x))
 - 4*((a - 4*b)*cos(2*x) - a + 2*b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*
x) - 1)))/(b*cos(2*x) - b), -1/6*(3*(b*cos(2*x) - b)*sqrt(-a + b)*arctan(-s
qrt(-a + b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*(cos(2*x) - 1)/
((a - b)*cos(2*x) - a)) + 2*((a - 4*b)*cos(2*x) - a + 2*b)*sqrt(((a - b)*co
s(2*x) - a - b)/(cos(2*x) - 1)))/(b*cos(2*x) - b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \cot^{2}\left(x\right)}\, \cot^{3}\left(x\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**3*(a+b*cot(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cot(x)**2)*cot(x)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*(a+b*cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

Mupad [B]
time = 3.16, size = 66, normalized size = 1.00

$$\sqrt{b\,{\mathrm{cot}\left(x\right)}^2+a}-\frac{{\left(b\,{\mathrm{cot}\left(x\right)}^2+a\right)}^{3/2}}{3\,b}+2\,\mathrm{atan}\left(\frac{2\,\sqrt{b\,{\mathrm{cot}\left(x\right)}^2+a}\,\sqrt{\frac{b}{4}-\frac{a}{4}}}{a-b}\right)\,\sqrt{\frac{b}{4}-\frac{a}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3*(a + b*cot(x)^2)^(1/2),x)
```

```
[Out] (a + b*cot(x)^2)^(1/2) - (a + b*cot(x)^2)^(3/2)/(3*b) + 2*atan((2*(a + b*co
t(x)^2)^(1/2)*(b/4 - a/4)^(1/2))/(a - b))*(b/4 - a/4)^(1/2)
```

3.20 $\int \cot(x)\sqrt{a+b\cot^2(x)}\,dx$

Optimal. Leaf size=48

$$\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)-\sqrt{a+b\cot^2(x)}$$

```
[Out] arctanh((a+b*cot(x)^2)^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)-(a+b*cot(x)^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{3751, 455, 52, 65, 214\}$

$$\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)-\sqrt{a+b\cot^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*Sqrt[a + b*Cot[x]^2],x]
```

```
[Out] Sqrt[a - b]*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]] - Sqrt[a + b*Cot[x]^2
]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \cot(x)\sqrt{a+b\cot^2(x)}\,dx &= -\text{Subst}\left(\int \frac{x\sqrt{a+bx^2}}{1+x^2}\,dx, x, \cot(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1+x}\,dx, x, \cot^2(x)\right)\right)\\
&= -\sqrt{a+b\cot^2(x)} - \frac{1}{2}(a-b)\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \cot^2(x)\right)\\
&= -\sqrt{a+b\cot^2(x)} - \frac{(a-b)\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cot^2(x)}\right)}{b}\\
&= \sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right) - \sqrt{a+b\cot^2(x)}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 48, normalized size = 1.00

$$
\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right) - \sqrt{a+b\cot^2(x)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*Sqrt[a + b*Cot[x]^2],x]
```

```
[Out] Sqrt[a - b]*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]] - Sqrt[a + b*Cot[x]^2
]
```

Maple [A]
time = 0.14, size = 77, normalized size = 1.60

method	result
derivativedivides	$-b\left(\frac{\sqrt{a+b\left(\cot^{2}\left(x\right)\right)}}{b}-\frac{\arctan\left(\frac{\sqrt{a+b\left(\cot^{2}\left(x\right)\right)}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}\right)-\frac{a\arctan\left(\frac{\sqrt{a+b\left(\cot^{2}\left(x\right)\right)}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$
default	$-b\left(\frac{\sqrt{a+b\left(\cot^{2}\left(x\right)\right)}}{b}-\frac{\arctan\left(\frac{\sqrt{a+b\left(\cot^{2}\left(x\right)\right)}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}\right)-\frac{a\arctan\left(\frac{\sqrt{a+b\left(\cot^{2}\left(x\right)\right)}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a+b*cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -b*((a+b*cot(x)^2)^(1/2)/b-1/(-a+b)^(1/2)*arctan((a+b*cot(x)^2)^(1/2)/(-a+b
)^(1/2)))-a/(-a+b)^(1/2)*arctan((a+b*cot(x)^2)^(1/2)/(-a+b)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(40) = 80.
time = 3.19, size = 248, normalized size = 5.17

$$\left[\frac{1}{4}\sqrt{a-b}\log\left(-2\left(a^{2}-2ab+b^{2}\right)\cos\left(2x\right)^{2}-2a^{2}+b^{2}-2\left(\left(a-b\right)\cos\left(2x\right)^{2}-\left(2a-b\right)\cos\left(2x\right)+a\right)\sqrt{a-b}\sqrt{\frac{\left(a-b\right)\cos\left(2x\right)-a-b}{\cos\left(2x\right)-1}}+4\left(a^{2}-ab\right)\cos\left(2x\right)\right)-\sqrt{\frac{\left(a-b\right)\cos\left(2x\right)-a-b}{\cos\left(2x\right)-1}},\ \frac{1}{2}\sqrt{-a+b}\arctan\left(-\frac{\sqrt{-a+b}\sqrt{\frac{\left(a-b\right)\cos\left(2x\right)-a-b}{\cos\left(2x\right)-1}}\left(\cos\left(2x\right)-1\right)}{\left(a-b\right)\cos\left(2x\right)-a}\right)-\sqrt{\frac{\left(a-b\right)\cos\left(2x\right)-a-b}{\cos\left(2x\right)-1}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a - b)*log(-2*(a^2 - 2*a*b + b^2)*cos(2*x)^2 - 2*a^2 + b^2 - 2*((
a - b)*cos(2*x)^2 - (2*a - b)*cos(2*x) + a)*sqrt(a - b)*sqrt(((a - b)*cos(2
*x) - a - b)/(cos(2*x) - 1)) + 4*(a^2 - a*b)*cos(2*x)) - sqrt(((a - b)*cos(
2*x) - a - b)/(cos(2*x) - 1)), 1/2*sqrt(-a + b)*arctan(-sqrt(-a + b)*sqrt((
(a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*(cos(2*x) - 1)/((a - b)*cos(2*x)
- a)) - sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\cot^2(x)}\,\cot(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*cot(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cot(x)**2)*cot(x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(40) = 80.
time = 0.48, size = 95, normalized size = 1.98

$$-\frac{1}{2}\left(\sqrt{a-b}\,\log\left(\left(\sqrt{a-b}\,\sin(x)-\sqrt{a\sin(x)^2-b\sin(x)^2+b}\right)^2\right)-\frac{4\sqrt{a-b}\,b}{\left(\sqrt{a-b}\,\sin(x)-\sqrt{a\sin(x)^2-b\sin(x)^2+b}\right)^2-b}\right)\mathrm{sgn}(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(a - b)*log((sqrt(a - b)*sin(x) - sqrt(a*sin(x)^2 - b*sin(x)^2 +
b))^2) - 4*sqrt(a - b)*b/((sqrt(a - b)*sin(x) - sqrt(a*sin(x)^2 - b*sin(x)^
2 + b))^2 - b))*sgn(sin(x))
```

Mupad [B]
time = 1.17, size = 53, normalized size = 1.10

$$-\sqrt{b\cot(x)^2+a}-2\,\mathrm{atan}\left(\frac{2\sqrt{b\cot(x)^2+a}\,\sqrt{\frac{b}{4}-\frac{a}{4}}}{a-b}\right)\sqrt{\frac{b}{4}-\frac{a}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a + b*cot(x)^2)^(1/2),x)
```

```
[Out] - (a + b*cot(x)^2)^(1/2) - 2*atan((2*(a + b*cot(x)^2)^(1/2)*(b/4 - a/4)^(1/
2))/(a - b))*(b/4 - a/4)^(1/2)
```

3.21 $\int \sqrt{a+b\cot^2(x)} \tan(x)\,dx$

Optimal. Leaf size=60

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)-\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)$$

```
[Out] arctanh((a+b*cot(x)^2)^(1/2)/a^(1/2))*a^(1/2)-arctanh((a+b*cot(x)^2)^(1/2)/
(a-b)^(1/2))*(a-b)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3751, 457, 85, 65, 214}

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)-\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cot[x]^2]*Tan[x],x]
```

```
[Out] Sqrt[a]*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a]] - Sqrt[a - b]*ArcTanh[Sqrt[a
+ b*Cot[x]^2]/Sqrt[a - b]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x),
 x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{a+b\cot^2(x)}\,\tan(x)\,dx &= -\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x\left(1+x^2\right)}\,dx, x, \cot(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x(1+x)}\,dx, x, \cot^2(x)\right)\right)\\
&= -\left(\frac{1}{2}a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \cot^2(x)\right)\right) - \frac{1}{2}(-a+b)\text{Subst}\left(\int \frac{}{(1+x)}\right.\\
&= -\frac{a\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cot^2(x)}\right)}{b} - \frac{(-a+b)\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx,\right.}{b}\\
&= \sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right) - \sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 60, normalized size = 1.00

$$\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right) - \sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cot[x]^2]*Tan[x],x]
```

[Out] Sqrt[a]*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a]] - Sqrt[a - b]*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]]

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.83, size = 591, normalized size = 9.85

method	result
default	$\frac{\left(\mathrm{EllipticF}\left(\frac{(-1+\cos(x))\sqrt{\frac{2\sqrt{a}\,\sqrt{a-b}\,-2a+b}{b}}}{\sin(x)}, \sqrt{\frac{8a^{\frac{3}{2}}\sqrt{a-b}\,-4\sqrt{a}\,\sqrt{a-b}\,b+8a^2-8ab+b^2}{b^2}}\right)b-2\,\mathrm{EllipticPi}\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cot(x)^2)^(1/2)*tan(x),x,method=_RETURNVERBOSE)

[Out] 1/2*(EllipticF((-1+cos(x))*((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/sin(x),((8*a^(3/2)*(a-b)^(1/2)-4*a^(1/2)*(a-b)^(1/2)*b+8*a^2-8*a*b+b^2)/b^2)^(1/2))*b-2*EllipticPi((-1+cos(x))*((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/sin(x),1/(2*a^(1/2)*(a-b)^(1/2)-2*a+b)*b,(-(2*a^(1/2)*(a-b)^(1/2)+2*a-b)/b)^(1/2)/((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2))*a+2*EllipticPi((-1+cos(x))*((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/sin(x),-1/(2*a^(1/2)*(a-b)^(1/2)-2*a+b)*b,(-(2*a^(1/2)*(a-b)^(1/2)+2*a-b)/b)^(1/2)/((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2))*a-2*EllipticPi((-1+cos(x))*((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/sin(x),-1/(2*a^(1/2)*(a-b)^(1/2)-2*a+b)*b,(-(2*a^(1/2)*(a-b)^(1/2)+2*a-b)/b)^(1/2)/((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2))*b)*sin(x)^3*(-2*(cos(x)*a^(1/2)*(a-b)^(1/2)+a*cos(x)-b*cos(x)-a^(1/2)*(a-b)^(1/2)-a)/(cos(x)+1)/b)^(1/2)*2^(1/2)*((cos(x)*a^(1/2)*(a-b)^(1/2)-a*cos(x)+b*cos(x)-a^(1/2)*(a-b)^(1/2)+a)/(cos(x)+1)/b)^(1/2)*((cos(x)^2*a-b*cos(x)^2-a)/(cos(x)^2-1))^(1/2)/(-1+cos(x))/(cos(x)^2*a-b*cos(x)^2-a)*4^(1/2)/((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cot(x)^2)^(1/2)*tan(x),x, algorithm="maxima")

[Out] integrate(sqrt(b*cot(x)^2 + a)*tan(x), x)

Fricas [A]
time = 2.72, size = 351, normalized size = 5.85

$$\left[\frac{1}{2}\sqrt{a}\log\left(2a\tan(x)^2+2\sqrt{a}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan(x)^2+b\right)+\frac{1}{2}\sqrt{a-b}\log\left(\frac{(2a-b)\tan(x)^2-2\sqrt{a-b}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan(x)^2+b}{\tan(x)^2+1}\right), -\sqrt{-a+b}\arctan\left(-\frac{\sqrt{-a+b}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}}{a-b}\right)+\frac{1}{2}\sqrt{a}\log\left(2a\tan(x)^2+2\sqrt{a}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan(x)^2+b\right), -\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}}{a}\right)+\frac{1}{2}\sqrt{a-b}\log\left(\frac{(2a-b)\tan(x)^2-2\sqrt{a-b}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan(x)^2+b}{\tan(x)^2+1}\right), -\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}}{a}\right)-\sqrt{-a+b}\arctan\left(-\frac{\sqrt{-a+b}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}}{a-b}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)^2)^(1/2)*tan(x),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log(2*a*tan(x)^2 + 2*sqrt(a)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*t
an(x)^2 + b) + 1/2*sqrt(a - b)*log(((2*a - b)*tan(x)^2 - 2*sqrt(a - b)*sqrt
((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2 + b)/(tan(x)^2 + 1)), -sqrt(-a + b)*ar
ctan(-sqrt(-a + b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)/(a - b)) + 1/2*sqrt(a)*l
og(2*a*tan(x)^2 + 2*sqrt(a)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2 + b),
-sqrt(-a)*arctan(sqrt(-a)*sqrt((a*tan(x)^2 + b)/tan(x)^2)/a) + 1/2*sqrt(a -
 b)*log(((2*a - b)*tan(x)^2 - 2*sqrt(a - b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)
*tan(x)^2 + b)/(tan(x)^2 + 1)), -sqrt(-a)*arctan(sqrt(-a)*sqrt((a*tan(x)^2
+ b)/tan(x)^2)/a) - sqrt(-a + b)*arctan(-sqrt(-a + b)*sqrt((a*tan(x)^2 + b)
/tan(x)^2)/(a - b))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\cot^{2}(x)}\,\tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)**2)**(1/2)*tan(x),x)
```

```
[Out] Integral(sqrt(a + b*cot(x)**2)*tan(x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(48) = 96.
time = 0.50, size = 187, normalized size = 3.12

$$\frac{1}{2}\left(\frac{2\sqrt{a-b}\,a\arctan\left(\frac{\left(\sqrt{a-b}\sin(x)-\sqrt{a\sin(x)^{2}-b\sin(x)^{2}+b}\right)^{2}-2a+b}{2\sqrt{-a^{2}+ab}}\right)}{\sqrt{-a^{2}+ab}}+\sqrt{a-b}\log\left(\left(\sqrt{a-b}\sin(x)-\sqrt{a\sin(x)^{2}-b\sin(x)^{2}+b}\right)^{2}\right)\right)\operatorname{sgn}(\sin(x))-\frac{\left(2\sqrt{a-b}\,a\arctan\left(-\frac{a-b}{\sqrt{-a^{2}+ab}}\right)+\sqrt{-a^{2}+ab}\sqrt{a-b}\log(b)\right)\operatorname{sgn}(\sin(x))}{2\sqrt{-a^{2}+ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)^2)^(1/2)*tan(x),x, algorithm="giac")
```

```
[Out] 1/2*(2*sqrt(a - b)*a*arctan(1/2*((sqrt(a - b)*sin(x) - sqrt(a*sin(x)^2 - b*
sin(x)^2 + b))^2 - 2*a + b)/sqrt(-a^2 + a*b))/sqrt(-a^2 + a*b) + sqrt(a - b
)*log((sqrt(a - b)*sin(x) - sqrt(a*sin(x)^2 - b*sin(x)^2 + b))^2))*sgn(sin(
x)) - 1/2*(2*sqrt(a - b)*a*arctan(-(a - b)/sqrt(-a^2 + a*b)) + sqrt(-a^2 +
a*b)*sqrt(a - b)*log(b))*sgn(sin(x))/sqrt(-a^2 + a*b)
```

Mupad [B]

time = 0.48, size = 69, normalized size = 1.15

$$\operatorname{atanh}\left(\frac{2\,a\,b^3\,\sqrt{a-b}\,\sqrt{a+\frac{b}{\tan\left(x\right)^2}}}{2\,a\,b^4-2\,a^2\,b^3}\right)\,\sqrt{a-b}+\sqrt{a}\,\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\tan\left(x\right)^2}}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(a + b*cot(x)^2)^(1/2),x)
```

```
[Out] atanh((2*a*b^3*(a - b)^(1/2)*(a + b/tan(x)^2)^(1/2))/(2*a*b^4 - 2*a^2*b^3))
*(a - b)^(1/2) + a^(1/2)*atanh((a + b/tan(x)^2)^(1/2)/a^(1/2))
```

3.22 $\int \cot^2(x)\sqrt{a+b\cot^2(x)}\,dx$

Optimal. Leaf size=89

$$\sqrt{a-b}\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)-\frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{2\sqrt{b}}-\frac{1}{2}\cot(x)\sqrt{a+b\cot^2(x)}$$

```
[Out] arctan(cot(x)*(a-b)^(1/2)/(a+b*cot(x)^2)^(1/2))*(a-b)^(1/2)-1/2*(a-2*b)*arc
tanh(cot(x)*b^(1/2)/(a+b*cot(x)^2)^(1/2))/b^(1/2)-1/2*cot(x)*(a+b*cot(x)^2)
^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3751, 489, 537, 223, 212, 385, 209}

$$\sqrt{a-b}\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)-\frac{1}{2}\cot(x)\sqrt{a+b\cot^2(x)}-\frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2*Sqrt[a + b*Cot[x]^2],x]
```

```
[Out] Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Cot[x])/Sqrt[a + b*Cot[x]^2]] - ((a - 2*b)*
ArcTanh[(Sqrt[b]*Cot[x])/Sqrt[a + b*Cot[x]^2]])/(2*Sqrt[b]) - (Cot[x]*Sqrt[
a + b*Cot[x]^2])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^2(x)\sqrt{a+b\cot^2(x)}\,dx = -\text{Subst}\left(\int \frac{x^2\sqrt{a+bx^2}}{1+x^2}\,dx, x, \cot(x)\right)$$

$$= -\frac{1}{2}\cot(x)\sqrt{a+b\cot^2(x)} + \frac{1}{2}\text{Subst}\left(\int \frac{a+(-a+2b)x^2}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \cot(x)\right)$$

$$= -\frac{1}{2}\cot(x)\sqrt{a+b\cot^2(x)} + (a-b)\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \cot($$

$$= -\frac{1}{2}\cot(x)\sqrt{a+b\cot^2(x)} + (a-b)\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\text{co}}{\sqrt{a+}}\right.$$

$$= \sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right) - \frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{2\sqrt{b}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2105 vs. $2(89) = 178$.
time = 22.86, size = 2105, normalized size = 23.65

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[x]^2*Sqrt[a + b*Cot[x]^2],x]
```

```
[Out] -1/2*(Sqrt[(-a - b + a*Cos[2*x] - b*Cos[2*x])/(-1 + Cos[2*x])]*Cot[x]) + ((
-4*Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b]*(-1 + Tan[x/2]^2))/Sqrt[b*Cos[x]
^2*Sec[x/2]^4 + 4*a*Tan[x/2]^2]] - (a - 2*b)*ArcTanh[(Sqrt[2]*(a + (-a + b)
*Cos[x])*Sec[x/2]^2)/(Sqrt[b]*Sqrt[(a + b + (-a + b)*Cos[2*x])*Sec[x/2]^4])
] + (a - 2*b)*ArcTanh[(2*a + b*(-1 + Tan[x/2]^2))/(Sqrt[b]*Sqrt[b*Cos[x]^2*
Sec[x/2]^4 + 4*a*Tan[x/2]^2])])*((b*Sqrt[-(a/(-1 + Cos[2*x])) - b/(-1 + Cos
[2*x]) + (a*Cos[2*x])/(-1 + Cos[2*x]) - (b*Cos[2*x])/(-1 + Cos[2*x])])/(-a
- b + a*Cos[2*x] - b*Cos[2*x]) - (a*Cos[2*x]*Sqrt[-(a/(-1 + Cos[2*x])) - b/
(-1 + Cos[2*x]) + (a*Cos[2*x])/(-1 + Cos[2*x]) - (b*Cos[2*x])/(-1 + Cos[2*x
])])/(-a - b + a*Cos[2*x] - b*Cos[2*x]) + (b*Cos[2*x]*Sqrt[-(a/(-1 + Cos[2*
x])) - b/(-1 + Cos[2*x]) + (a*Cos[2*x])/(-1 + Cos[2*x]) - (b*Cos[2*x])/(-1
+ Cos[2*x])])/(-a - b + a*Cos[2*x] - b*Cos[2*x]))*Sqrt[a + b*Cot[x]^2]*Tan[
x/2])/(Sqrt[2]*Sqrt[b]*Sqrt[(a + b + (-a + b)*Cos[2*x])*Sec[x/2]^4]*(((-4*S
qrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b]*(-1 + Tan[x/2]^2))/Sqrt[b*Cos[x]^2*S
ec[x/2]^4 + 4*a*Tan[x/2]^2]] - (a - 2*b)*ArcTanh[(Sqrt[2]*(a + (-a + b)*Cos
```

```
[x])*Sec[x/2]^2)/(Sqrt[b]*Sqrt[(a + b + (-a + b)*Cos[2*x])*Sec[x/2]^4])] +
(a - 2*b)*ArcTanh[(2*a + b*(-1 + Tan[x/2]^2))/(Sqrt[b]*Sqrt[b*Cos[x]^2*Sec[
x/2]^4 + 4*a*Tan[x/2]^2])])*Sqrt[a + b*Cot[x]^2]*Sec[x/2]^2)/(2*Sqrt[2]*Sqr
t[b]*Sqrt[(a + b + (-a + b)*Cos[2*x])*Sec[x/2]^4]) - (Sqrt[b]*(-4*Sqrt[a -
b]*Sqrt[b]*ArcTan[(Sqrt[a - b]*(-1 + Tan[x/2]^2))/Sqrt[b*Cos[x]^2*Sec[x/2]^
4 + 4*a*Tan[x/2]^2]] - (a - 2*b)*ArcTanh[(Sqrt[2]*(a + (-a + b)*Cos[x])*Sec
[x/2]^2)/(Sqrt[b]*Sqrt[(a + b + (-a + b)*Cos[2*x])*Sec[x/2]^4])] + (a - 2*b
)*ArcTanh[(2*a + b*(-1 + Tan[x/2]^2))/(Sqrt[b]*Sqrt[b*Cos[x]^2*Sec[x/2]^4 +
 4*a*Tan[x/2]^2])])*Cot[x]*Csc[x]^2*Tan[x/2])/(Sqrt[2]*Sqrt[a + b*Cot[x]^2]
*Sqrt[(a + b + (-a + b)*Cos[2*x])*Sec[x/2]^4]) - ((-4*Sqrt[a - b]*Sqrt[b]*A
rcTan[(Sqrt[a - b]*(-1 + Tan[x/2]^2))/Sqrt[b*Cos[x]^2*Sec[x/2]^4 + 4*a*Tan[
x/2]^2]] - (a - 2*b)*ArcTanh[(Sqrt[2]*(a + (-a + b)*Cos[x])*Sec[x/2]^2)/(Sq
rt[b]*Sqrt[(a + b + (-a + b)*Cos[2*x])*Sec[x/2]^4])] + (a - 2*b)*ArcTanh[(2
*a + b*(-1 + Tan[x/2]^2))/(Sqrt[b]*Sqrt[b*Cos[x]^2*Sec[x/2]^4 + 4*a*Tan[x/2
]^2])])*Sqrt[a + b*Cot[x]^2]*Tan[x/2]*(-2*(-a + b)*Sec[x/2]^4*Sin[2*x] + 2*
(a + b + (-a + b)*Cos[2*x])*Sec[x/2]^4*Tan[x/2]))/(2*Sqrt[2]*Sqrt[b]*((a +
b + (-a + b)*Cos[2*x])*Sec[x/2]^4)^(3/2)) + (Sqrt[a + b*Cot[x]^2]*Tan[x/2]*
(-(((a - 2*b)*(-((Sqrt[2]*(-a + b)*Sec[x/2]^2*Sin[x])/(Sqrt[b]*Sqrt[(a + b
+ (-a + b)*Cos[2*x])*Sec[x/2]^4])) + (Sqrt[2]*(a + (-a + b)*Cos[x])*Sec[x/2
]^2*Tan[x/2])/(Sqrt[b]*Sqrt[(a + b + (-a + b)*Cos[2*x])*Sec[x/2]^4]) - ((a
+ (-a + b)*Cos[x])*Sec[x/2]^2*(-2*(-a + b)*Sec[x/2]^4*Sin[2*x] + 2*(a + b +
 (-a + b)*Cos[2*x])*Sec[x/2]^4*Tan[x/2]))/(Sqrt[2]*Sqrt[b]*((a + b + (-a +
b)*Cos[2*x])*Sec[x/2]^4)^(3/2))))/(1 - (2*(a + (-a + b)*Cos[x])^2)/(b*(a +
b + (-a + b)*Cos[2*x])))) - (4*Sqrt[a - b]*Sqrt[b]*(-1/2*(Sqrt[a - b]*(-2*b
*Cos[x]*Sec[x/2]^4*Sin[x] + 4*a*Sec[x/2]^2*Tan[x/2] + 2*b*Cos[x]^2*Sec[x/2]
^4*Tan[x/2])*(-1 + Tan[x/2]^2))/(b*Cos[x]^2*Sec[x/2]^4 + 4*a*Tan[x/2]^2)^(3
/2) + (Sqrt[a - b]*Sec[x/2]^2*Tan[x/2])/Sqrt[b*Cos[x]^2*Sec[x/2]^4 + 4*a*Ta
n[x/2]^2]))/(1 + ((a - b)*(-1 + Tan[x/2]^2)^2)/(b*Cos[x]^2*Sec[x/2]^4 + 4*a
*Tan[x/2]^2)) + ((a - 2*b)*((Sqrt[b]*Sec[x/2]^2*Tan[x/2])/Sqrt[b*Cos[x]^2*S
ec[x/2]^4 + 4*a*Tan[x/2]^2] - ((-2*b*Cos[x]*Sec[x/2]^4*Sin[x] + 4*a*Sec[x/2
]^2*Tan[x/2] + 2*b*Cos[x]^2*Sec[x/2]^4*Tan[x/2])*(2*a + b*(-1 + Tan[x/2]^2)
))/(2*Sqrt[b]*(b*Cos[x]^2*Sec[x/2]^4 + 4*a*Tan[x/2]^2)^(3/2))))/(1 - (2*a +
 b*(-1 + Tan[x/2]^2))^2/(b*(b*Cos[x]^2*Sec[x/2]^4 + 4*a*Tan[x/2]^2)))))/(Sq
rt[2]*Sqrt[b]*Sqrt[(a + b + (-a + b)*Cos[2*x])*Sec[x/2]^4]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(71) = 142$.

time = 0.15, size = 177, normalized size = 1.99

method	result
derivativedivides	$-\frac{\cot(x)\sqrt{a+b(\cot^2(x))}}{2}-\frac{a\ln\left(\sqrt{b}\,\cot(x)+\sqrt{a+b(\cot^2(x))}\right)}{2\sqrt{b}}+b\left(\frac{\ln\left(\sqrt{b}\,\cot(x)+\sqrt{a+\right.}}{\sqrt{b}}\right.$

default	$-\frac{\cot(x)\sqrt{a+b\left(\cot^2(x)\right)}}{2}-\frac{a\ln\left(\sqrt{b}\,\cot(x)+\sqrt{a+b\left(\cot^2(x)\right)}\right)}{2\sqrt{b}}+b\left(\frac{\ln\left(\sqrt{b}\,\cot(x)+\sqrt{a+\right.}}{\sqrt{b}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2*(a+b*cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*cot(x)*(a+b*cot(x)^2)^(1/2)-1/2*a/b^(1/2)*ln(b^(1/2)*cot(x)+(a+b*cot(x
)^2)^(1/2))+b*(ln(b^(1/2)*cot(x)+(a+b*cot(x)^2)^(1/2))/b^(1/2)-(b^4*(a-b))^
(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*cot(x)^2)^(1/2)*cot
(x)))+a*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b
*cot(x)^2)^(1/2)*cot(x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*(a+b*cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cot(x)^2 + a)*cot(x)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(71) = 142$.
time = 2.49, size = 768, normalized size = 8.63

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*(a+b*cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-a + b)*b*log(-(a - b)*cos(2*x) - sqrt(-a + b)*sqrt(((a - b)*c
os(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x) + b)*sin(2*x) - (a - 2*b)*sqrt(b)
*log(((a - 2*b)*cos(2*x) - 2*sqrt(b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2
*x) - 1))*sin(2*x) - a - 2*b)/(cos(2*x) - 1))*sin(2*x) - 2*(b*cos(2*x) + b)
*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)))/(b*sin(2*x)), 1/4*(4*sqrt
(a - b)*b*arctan(-sqrt(a - b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1
))*sin(2*x)/((a - b)*cos(2*x) + a - b))*sin(2*x) - (a - 2*b)*sqrt(b)*log(((
a - 2*b)*cos(2*x) - 2*sqrt(b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1
))*sin(2*x) - a - 2*b)/(cos(2*x) - 1))*sin(2*x) - 2*(b*cos(2*x) + b)*sqrt((
(a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)))/(b*sin(2*x)), 1/2*((a - 2*b)*sqr
t(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2
```

```
*x)/(b*cos(2*x) + b))*sin(2*x) + sqrt(-a + b)*b*log(-(a - b)*cos(2*x) - sqr
t(-a + b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x) + b)*sin
(2*x) - (b*cos(2*x) + b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)))/(
b*sin(2*x)), 1/2*(2*sqrt(a - b)*b*arctan(-sqrt(a - b)*sqrt(((a - b)*cos(2*x
) - a - b)/(cos(2*x) - 1))*sin(2*x)/((a - b)*cos(2*x) + a - b))*sin(2*x) +
(a - 2*b)*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x
) - 1))*sin(2*x)/(b*cos(2*x) + b))*sin(2*x) - (b*cos(2*x) + b)*sqrt(((a - b
)*cos(2*x) - a - b)/(cos(2*x) - 1)))/(b*sin(2*x))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\cot^2{(x)}}\cot^2{(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2*(a+b*cot(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cot(x)**2)*cot(x)**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*(a+b*cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{cot}(x)^2\,\sqrt{b\,{\mathrm{cot}\left(x\right)}^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2*(a + b*cot(x)^2)^(1/2),x)
```

```
[Out] int(cot(x)^2*(a + b*cot(x)^2)^(1/2), x)
```

3.23 $\int \sqrt{a+b\cot^2(x)}\,dx$

Optimal. Leaf size=65

$$-\sqrt{a-b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)-\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)$$

```
[Out] -arctan(cot(x)*(a-b)^(1/2)/(a+b*cot(x)^2)^(1/2))*(a-b)^(1/2)-arctanh(cot(x)
*b^(1/2)/(a+b*cot(x)^2)^(1/2))*b^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3742, 399, 223, 212, 385, 209}

$$-\sqrt{a-b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)-\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cot[x]^2],x]
```

```
[Out] -(Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Cot[x])/Sqrt[a + b*Cot[x]^2]]) - Sqrt[b]*
ArcTanh[(Sqrt[b]*Cot[x])/Sqrt[a + b*Cot[x]^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\cot^2(x)}\,dx &= -\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{1+x^2}\,dx, x, \cot(x)\right)\\ &= -\left(b\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \cot(x)\right)\right) + (-a+b)\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\right.\\ &= -\left(b\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)\right) + (-a+b)\text{Subst}\left(\int \frac{1}{1-(-a}\right.\\ &= -\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right) - \sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)\end{aligned}$$

Mathematica [A]
time = 0.17, size = 78, normalized size = 1.20

$$\sqrt{a-b}\,\text{ArcTan}\left(\frac{-\cot(x)\sqrt{a+b\cot^2(x)}+\sqrt{b}\,\csc^2(x)}{\sqrt{a-b}}\right) + \sqrt{b}\,\log\left(-\sqrt{b}\,\cot(x) + \sqrt{a+b\cot^2(x)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cot[x]^2],x]
```

```
[Out] Sqrt[a - b]*ArcTan[(-(Cot[x]*Sqrt[a + b*Cot[x]^2]) + Sqrt[b]*Csc[x]^2)/Sqrt
[a - b]] + Sqrt[b]*Log[-(Sqrt[b]*Cot[x]) + Sqrt[a + b*Cot[x]^2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(53) = 106$.
time = 0.18, size = 141, normalized size = 2.17

method	result
derivativedivides	$-b\left(\frac{\ln\left(\sqrt{b}\,\cot(x)+\sqrt{a+b\left(\cot^{2}(x)\right)}\right)}{\sqrt{b}}-\frac{\sqrt{b^{4}(a-b)}\,\arctan\left(\frac{b^{2}(a-b)\cot(x)}{\sqrt{b^{4}(a-b)}\,\sqrt{a+b\left(\cot^{2}(\right.}}\right.}{b^{2}(a-b)}\right.$
default	$-b\left(\frac{\ln\left(\sqrt{b}\,\cot(x)+\sqrt{a+b\left(\cot^{2}(x)\right)}\right)}{\sqrt{b}}-\frac{\sqrt{b^{4}(a-b)}\,\arctan\left(\frac{b^{2}(a-b)\cot(x)}{\sqrt{b^{4}(a-b)}\,\sqrt{a+b\left(\cot^{2}(\right.}}\right.}{b^{2}(a-b)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -b*(ln(b^(1/2)*cot(x)+(a+b*cot(x)^2)^(1/2))/b^(1/2)-(b^4*(a-b))^(1/2)/b^2/(
a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*cot(x)^2)^(1/2)*cot(x)))-a*(b^
4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*cot(x)^2)^
(1/2)*cot(x))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(53) = 106$.
time = 2.59, size = 515, normalized size = 7.92

$$\left[\frac{1}{2}\sqrt{-a+b}\log\left(-(a-b)\cos(2x)+\sqrt{-a+b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\sin(2x)+b\right)+\frac{1}{2}\sqrt{b}\log\left(\frac{(a-2b)\cos(2x)+2\sqrt{b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\sin(2x)-a-2b}{\cos(2x)-1}\right), -\sqrt{a-b}\arctan\left(-\frac{\sqrt{a-b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\sin(2x)}{(a-b)\cos(2x)+a-b}\right)+\frac{1}{2}\sqrt{b}\log\left(\frac{(a-2b)\cos(2x)+2\sqrt{b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\sin(2x)-a-2b}{\cos(2x)-1}\right), \sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\sin(2x)}{b\cos(2x)+b}\right)+\frac{1}{2}\sqrt{-a+b}\log\left(-(a-b)\cos(2x)+\sqrt{-a+b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\sin(2x)+b\right), -\sqrt{a-b}\arctan\left(-\frac{\sqrt{a-b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\sin(2x)}{(a-b)\cos(2x)+a-b}\right)+\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\sin(2x)}{b\cos(2x)+b}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-a + b)*log(-(a - b)*cos(2*x) + sqrt(-a + b)*sqrt(((a - b)*cos(2*
x) - a - b)/(cos(2*x) - 1))*sin(2*x) + b) + 1/2*sqrt(b)*log(((a - 2*b)*cos(
2*x) + 2*sqrt(b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x) -
 a - 2*b)/(cos(2*x) - 1)), -sqrt(a - b)*arctan(-sqrt(a - b)*sqrt(((a - b)*c
os(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x)/((a - b)*cos(2*x) + a - b)) + 1/2
*sqrt(b)*log(((a - 2*b)*cos(2*x) + 2*sqrt(b)*sqrt(((a - b)*cos(2*x) - a - b
)/(cos(2*x) - 1))*sin(2*x) - a - 2*b)/(cos(2*x) - 1)), sqrt(-b)*arctan(sqrt
(-b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x)/(b*cos(2*x) +
 b)) + 1/2*sqrt(-a + b)*log(-(a - b)*cos(2*x) + sqrt(-a + b)*sqrt(((a - b)*
cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x) + b), -sqrt(a - b)*arctan(-sqrt(
a - b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x)/((a - b)*co
s(2*x) + a - b)) + sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(2*x) - a - b)
/(cos(2*x) - 1))*sin(2*x)/(b*cos(2*x) + b))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\cot^2(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cot(x)**2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(53) = 106.
time = 1.72, size = 210, normalized size = 3.23

$$-\frac{1}{2}\left(\frac{2\sqrt{-a+b}\,b\arctan\left(\frac{\left(\sqrt{-a+b}\cos(x)-\sqrt{-a\cos(x)^2+b\cos(x)^2+a}\right)^2+a-2b}{2\sqrt{ab-b^2}}\right)}{\sqrt{ab-b^2}}+\sqrt{-a+b}\log\left(\left(\sqrt{-a+b}\cos(x)-\sqrt{-a\cos(x)^2+b\cos(x)^2+a}\right)^2\right)\right)\operatorname{sgn}(\sin(x))-\frac{\left(2\sqrt{-a+b}\,b\arctan\left(\frac{\sqrt{-a+b}\sqrt{b}}{\sqrt{ab-b^2}}\right)-\sqrt{ab-b^2}\sqrt{-a+b}\log\left(-a-2\sqrt{-a+b}\sqrt{b}+2b\right)\right)\operatorname{sgn}(\sin(x))}{2\sqrt{ab-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(2*sqrt(-a + b)*b*arctan(1/2*((sqrt(-a + b)*cos(x) - sqrt(-a*cos(x)^2
+ b*cos(x)^2 + a))^2 + a - 2*b)/sqrt(a*b - b^2))/sqrt(a*b - b^2) + sqrt(-a
+ b)*log((sqrt(-a + b)*cos(x) - sqrt(-a*cos(x)^2 + b*cos(x)^2 + a))^2))*sgn
(sin(x)) - 1/2*(2*sqrt(-a + b)*b*arctan(sqrt(-a + b)*sqrt(b)/sqrt(a*b - b^2
)) - sqrt(a*b - b^2)*sqrt(-a + b)*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b))*s
gn(sin(x))/sqrt(a*b - b^2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{b\cot(x)^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(x)^2)^(1/2),x)
```

```
[Out] int((a + b*cot(x)^2)^(1/2), x)
```

3.24 $\int \sqrt{a+b\cot^2(x)}\,\tan^2(x)\,dx$

Optimal. Leaf size=51

$$\sqrt{a-b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)+\sqrt{a+b\cot^2(x)}\,\tan(x)$$

```
[Out] arctan(cot(x)*(a-b)^(1/2)/(a+b*cot(x)^2)^(1/2))*(a-b)^(1/2)+(a+b*cot(x)^2)^
(1/2)*tan(x)
```

Rubi [A]
time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3751, 486, 12, 385, 209}

$$\sqrt{a-b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)+\tan(x)\sqrt{a+b\cot^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cot[x]^2]*Tan[x]^2,x]
```

```
[Out] Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Cot[x])/Sqrt[a + b*Cot[x]^2]] + Sqrt[a + b*
Cot[x]^2]*Tan[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
```

```
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a+b\cot^2(x)}\,\tan^2(x)\,dx &= -\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2\,(1+x^2)}\,dx, x, \cot(x)\right)\\
&= \sqrt{a+b\cot^2(x)}\,\tan(x) - \text{Subst}\left(\int \frac{-a+b}{(1+x^2)\,\sqrt{a+bx^2}}\,dx, x, \cot(x)\right)\\
&= \sqrt{a+b\cot^2(x)}\,\tan(x) - (-a+b)\text{Subst}\left(\int \frac{1}{(1+x^2)\,\sqrt{a+bx^2}}\,dx, x, \cot(\ldots\right.\\
&= \sqrt{a+b\cot^2(x)}\,\tan(x) - (-a+b)\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\cot\ldots}{\sqrt{a+b\ldots}}\right.\\
&= \sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right) + \sqrt{a+b\cot^2(x)}\,\tan(x)
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.11, size = 44, normalized size = 0.86

$$\sqrt{a+b\cot^2(x)}\;{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{(a-b)\cot^2(x)}{a+b\cot^2(x)}\right)\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cot[x]^2]*Tan[x]^2,x]
```

```
[Out] Sqrt[a + b*Cot[x]^2]*Hypergeometric2F1[-1/2, 1, 1/2, -(((a - b)*Cot[x]^2)/(
a + b*Cot[x]^2))]*Tan[x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 751 vs. $2(43) = 86$.
time = 0.89, size = 752, normalized size = 14.75

method	result	size
default	Expression too large to display	752

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(x)^2)^(1/2)*tan(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-1+cos(x))*(cos(x)*ln(4*cos(x)*(-a+b)^(1/2)*(-(cos(x)^2*a-b*cos(x)^2-a
)/(cos(x)+1)^2)^(1/2)-4*a*cos(x)+4*b*cos(x)+4*(-a+b)^(1/2)*(-(cos(x)^2*a-b*
cos(x)^2-a)/(cos(x)+1)^2)^(1/2))*b^(3/2)-cos(x)*ln(4*cos(x)*(-a+b)^(1/2)*(-
(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)-4*a*cos(x)+4*b*cos(x)+4*(-a+b
)^(1/2)*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2))*b^(1/2)*a-cos(x)*l
n(-2*(-1+cos(x))*(cos(x)*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*b^
(1/2)+a*cos(x)-b*cos(x)+(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*b^(
1/2)+a)/sin(x)^2/b^(1/2))*(-a+b)^(1/2)*a+cos(x)*ln(-2*(-1+cos(x))*(cos(x)*(
-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*b^(1/2)+a*cos(x)-b*cos(x)+(-
(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*b^(1/2)+a)/sin(x)^2/b^(1/2))*
(-a+b)^(1/2)*b+cos(x)*ln(-4*(-1+cos(x))*(cos(x)*(-(cos(x)^2*a-b*cos(x)^2-a)
/(cos(x)+1)^2)^(1/2)*b^(1/2)+a*cos(x)-b*cos(x)+(-(cos(x)^2*a-b*cos(x)^2-a)/
(cos(x)+1)^2)^(1/2)*b^(1/2)+a)/sin(x)^2/b^(1/2))*(-a+b)^(1/2)*a-cos(x)*ln(-
4*(-1+cos(x))*(cos(x)*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*b^(1/
2)+a*cos(x)-b*cos(x)+(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*b^(1/2
)+a)/sin(x)^2/b^(1/2))*(-a+b)^(1/2)*b-cos(x)*(-a+b)^(1/2)*(-(cos(x)^2*a-b*c
os(x)^2-a)/(cos(x)+1)^2)^(1/2)*b^(1/2)-(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+
1)^2)^(1/2)*b^(1/2)*(-a+b)^(1/2))*((cos(x)^2*a-b*cos(x)^2-a)/(cos(x)^2-1))^
(1/2)/cos(x)/sin(x)/(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*4^(1/2)
/b^(1/2)/(-a+b)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)^2)^(1/2)*tan(x)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cot(x)^2 + a)*tan(x)^2, x)
```

Fricas [A]
time = 3.50, size = 193, normalized size = 3.78

$$\left[\frac{1}{4}\,\sqrt{-a+b}\,\log\left(-\frac{a^{2}\tan\left(x\right)^{4}-2\,\left(3\,a^{2}-4\,ab\right)\tan\left(x\right)^{2}+a^{2}-8\,ab+8\,b^{2}-4\,\left(a\tan\left(x\right)^{3}-\left(a-2\,b\right)\tan\left(x\right)\right)\sqrt{-a+b}\,\sqrt{\frac{a\tan\left(x\right)^{2}+b}{\tan\left(x\right)^{2}}}}{\tan\left(x\right)^{4}+2\,\tan\left(x\right)^{2}+1}\right)+\sqrt{\frac{a\tan\left(x\right)^{2}+b}{\tan\left(x\right)^{2}}}\,\tan\left(x\right),\ \frac{1}{2}\,\sqrt{a-b}\,\arctan\left(\frac{2\,\sqrt{a-b}\,\sqrt{\frac{a\tan\left(x\right)^{2}+b}{\tan\left(x\right)^{2}}}\,\tan\left(x\right)}{a\tan\left(x\right)^{2}-a+2\,b}\right)+\sqrt{\frac{a\tan\left(x\right)^{2}+b}{\tan\left(x\right)^{2}}}\,\tan\left(x\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)^2)^(1/2)*tan(x)^2,x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(-a + b)*log(-(a^2*tan(x)^4 - 2*(3*a^2 - 4*a*b)*tan(x)^2 + a^2 - 8
*a*b + 8*b^2 - 4*(a*tan(x)^3 - (a - 2*b)*tan(x))*sqrt(-a + b)*sqrt((a*tan(x
)^2 + b)/tan(x)^2))/(tan(x)^4 + 2*tan(x)^2 + 1)) + sqrt((a*tan(x)^2 + b)/ta
n(x)^2)*tan(x), 1/2*sqrt(a - b)*arctan(2*sqrt(a - b)*sqrt((a*tan(x)^2 + b)/
tan(x)^2)*tan(x)/(a*tan(x)^2 - a + 2*b)) + sqrt((a*tan(x)^2 + b)/tan(x)^2)*
tan(x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\cot^{2}(x)}\,\tan^{2}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)**2)**(1/2)*tan(x)**2,x)
```

```
[Out] Integral(sqrt(a + b*cot(x)**2)*tan(x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(43) = 86.
time = 0.47, size = 239, normalized size = 4.69

$$\frac{1}{2}\left(\sqrt{-a+b}\log\left(\left(\sqrt{-a+b}\cos(x)-\sqrt{-a\cos(x)^{2}+b\cos(x)^{2}+a}\right)^{2}\right)-\frac{4a\sqrt{-a+b}}{\left(\sqrt{-a+b}\cos(x)-\sqrt{-a\cos(x)^{2}+b\cos(x)^{2}+a}\right)^{2}-a}\right)\mathrm{sgn}(\sin(x))-\frac{\left(a\sqrt{-a+b}\log\left(-a-2\sqrt{-a+b}\sqrt{b}+2b\right)-a\sqrt{b}\log\left(-a-2\sqrt{-a+b}\sqrt{b}+2b\right)-\sqrt{-a+b}\,b\log\left(-a-2\sqrt{-a+b}\sqrt{b}+2b\right)+b^{\frac{3}{2}}\log\left(-a-2\sqrt{-a+b}\sqrt{b}+2b\right)+2a\sqrt{-a+b}\right)\mathrm{sgn}(\sin(x))}{2\left(a+\sqrt{-a+b}\sqrt{b}-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)^2)^(1/2)*tan(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(-a + b)*log((sqrt(-a + b)*cos(x) - sqrt(-a*cos(x)^2 + b*cos(x)^2
+ a))^2) - 4*a*sqrt(-a + b)/((sqrt(-a + b)*cos(x) - sqrt(-a*cos(x)^2 + b*co
s(x)^2 + a))^2 - a))*sgn(sin(x)) - 1/2*(a*sqrt(-a + b)*log(-a - 2*sqrt(-a +
 b)*sqrt(b) + 2*b) - a*sqrt(b)*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b) - sqr
t(-a + b)*b*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b) + b^(3/2)*log(-a - 2*sqr
t(-a + b)*sqrt(b) + 2*b) + 2*a*sqrt(-a + b))*sgn(sin(x))/(a + sqrt(-a + b)*
sqrt(b) - b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\mathrm{tan}\left(x\right)}^2\,\sqrt{b\,{\mathrm{cot}\left(x\right)}^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2*(a + b*cot(x)^2)^(1/2),x)
```

```
[Out] int(tan(x)^2*(a + b*cot(x)^2)^(1/2), x)
```

3.25 $\int \sqrt{a+b\cot^2(x)}\,\tan^4(x)\,dx$

Optimal. Leaf size=85

$$-\sqrt{a-b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)-\frac{(3a-b)\sqrt{a+b\cot^2(x)}\,\tan(x)}{3a}+\frac{1}{3}\sqrt{a+b\cot^2(x)}\,\tan^3(x)$$

```
[Out] -arctan(cot(x)*(a-b)^(1/2)/(a+b*cot(x)^2)^(1/2))*(a-b)^(1/2)-1/3*(3*a-b)*(a
+b*cot(x)^2)^(1/2)*tan(x)/a+1/3*(a+b*cot(x)^2)^(1/2)*tan(x)^3
```

Rubi [A]
time = 0.10, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3751, 486, 597, 12, 385, 209}

$$-\sqrt{a-b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)+\frac{1}{3}\tan^3(x)\sqrt{a+b\cot^2(x)}-\frac{(3a-b)\tan(x)\sqrt{a+b\cot^2(x)}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cot[x]^2]*Tan[x]^4,x]
```

```
[Out] -(Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Cot[x])/Sqrt[a + b*Cot[x]^2]]) - ((3*a -
b)*Sqrt[a + b*Cot[x]^2]*Tan[x])/(3*a) + (Sqrt[a + b*Cot[x]^2]*Tan[x]^3)/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
```

```
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a+b\cot^2(x)}\,\tan^4(x)\,dx &= -\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^4\,(1+x^2)}\,dx, x, \cot(x)\right)\\
&= \frac{1}{3}\sqrt{a+b\cot^2(x)}\,\tan^3(x) - \frac{1}{3}\text{Subst}\left(\int \frac{-3a+b-2bx^2}{x^2\,(1+x^2)\,\sqrt{a+bx^2}}\,dx, x, \cot(\right.\\
&= -\frac{(3a-b)\sqrt{a+b\cot^2(x)}\,\tan(x)}{3a} + \frac{1}{3}\sqrt{a+b\cot^2(x)}\,\tan^3(x) + \frac{\text{Subst}\left(\int\right.}{}\\
&= -\frac{(3a-b)\sqrt{a+b\cot^2(x)}\,\tan(x)}{3a} + \frac{1}{3}\sqrt{a+b\cot^2(x)}\,\tan^3(x) + (-a+b)\\
&= -\frac{(3a-b)\sqrt{a+b\cot^2(x)}\,\tan(x)}{3a} + \frac{1}{3}\sqrt{a+b\cot^2(x)}\,\tan^3(x) + (-a+b)\\
&= -\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right) - \frac{(3a-b)\sqrt{a+b\cot^2(x)}\,\tan(x)}{3a} +
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.68, size = 174, normalized size = 2.05

$$\frac{1}{3}\sqrt{a+b\cot^2(x)}\left(1+\frac{b\cot^2(x)}{a}\right)\sin^2(x)\left(-\frac{4(a-b)\cos^2(x)\,(a+b\cot^2(x))\,{}_2F_1\left(2,2;\frac{3}{2};\frac{(a-b)\cos^2(x)}{a}\right)}{a^2}+\frac{(a-2b\cot^2(x))\csc^2(x)\left(\text{ArcSin}\left(\sqrt{\frac{(a-b)\cos^2(x)}{a}}\right)\sqrt{\frac{(a-b)\cos^2(x)}{a}}+\sqrt{\frac{b\cos^2(x)}{a}+\sin^2(x)}\right)}{(a+b\cot^2(x))\sqrt{\frac{b\cos^2(x)}{a}+\sin^2(x)}}\right)\tan^3(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cot[x]^2]*Tan[x]^4,x]
```

```
[Out] (Sqrt[a + b*Cot[x]^2]*(1 + (b*Cot[x]^2)/a)*Sin[x]^2*((-4*(a - b)*Cos[x]^2*(
a + b*Cot[x]^2)*Hypergeometric2F1[2, 2, 3/2, ((a - b)*Cos[x]^2)/a])/a^2 + (
(a - 2*b*Cot[x]^2)*Csc[x]^2*(ArcSin[Sqrt[((a - b)*Cos[x]^2)/a]]*Sqrt[((a -
b)*Cos[x]^2)/a] + Sqrt[(b*Cos[x]^2)/a + Sin[x]^2]))/((a + b*Cot[x]^2)*Sqrt[
(b*Cos[x]^2)/a + Sin[x]^2]))*Tan[x]^3)/3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 950 vs. 2(71) = 142.
time = 0.64, size = 951, normalized size = 11.19

method	result	size
default	Expression too large to display	951

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(x)^2)^(1/2)*tan(x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(-1+cos(x))*(3*cos(x)^3*ln(4*cos(x)*(-a+b)^(1/2)*(-(cos(x)^2*a-b*cos(x
)^2-a)/(cos(x)+1)^2)^(1/2)-4*a*cos(x)+4*b*cos(x)+4*(-a+b)^(1/2)*(-(cos(x)^2
*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2))*b^(3/2)*a+cos(x)^3*(-(cos(x)^2*a-b*co
s(x)^2-a)/(cos(x)+1)^2)^(1/2)*(-a+b)^(1/2)*b^(3/2)+3*cos(x)^3*ln(-4*(-1+cos
(x))*(cos(x)*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*b^(1/2)+a*cos(
x)-b*cos(x)+(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*b^(1/2)+a)/sin(
x)^2/b^(1/2))*(-a+b)^(1/2)*a^2-3*cos(x)^3*ln(-4*(-1+cos(x))*(cos(x)*(-(cos(
x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*b^(1/2)+a*cos(x)-b*cos(x)+(-(cos(x
)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*b^(1/2)+a)/sin(x)^2/b^(1/2))*(-a+b)
^(1/2)*a*b-3*cos(x)^3*ln(4*cos(x)*(-a+b)^(1/2)*(-(cos(x)^2*a-b*cos(x)^2-a)/
(cos(x)+1)^2)^(1/2)-4*a*cos(x)+4*b*cos(x)+4*(-a+b)^(1/2)*(-(cos(x)^2*a-b*co
s(x)^2-a)/(cos(x)+1)^2)^(1/2))*b^(1/2)*a^2-4*cos(x)^3*(-(cos(x)^2*a-b*cos(x
)^2-a)/(cos(x)+1)^2)^(1/2)*(-a+b)^(1/2)*b^(1/2)*a-3*cos(x)^3*ln(-2*(-1+cos(
x))*(cos(x)*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*b^(1/2)+a*cos(x
)-b*cos(x)+(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*b^(1/2)+a)/sin(x
)^2/b^(1/2))*(-a+b)^(1/2)*a^2+3*cos(x)^3*ln(-2*(-1+cos(x))*(cos(x)*(-(cos(x
)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*b^(1/2)+a*cos(x)-b*cos(x)+(-(cos(x)
^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*b^(1/2)+a)/sin(x)^2/b^(1/2))*(-a+b)^
(1/2)*a*b+cos(x)^2*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*(-a+b)^(
1/2)*b^(3/2)-4*cos(x)^2*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*(-a
+b)^(1/2)*b^(1/2)*a+cos(x)*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*
(-a+b)^(1/2)*b^(1/2)*a+(-a+b)^(1/2)*b^(1/2)*(-(cos(x)^2*a-b*cos(x)^2-a)/(co
s(x)+1)^2)^(1/2)*a)*((cos(x)^2*a-b*cos(x)^2-a)/(cos(x)^2-1))^(1/2)/cos(x)^3
/sin(x)/(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*4^(1/2)/(-a+b)^(1/2
)/a/b^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)^2)^(1/2)*tan(x)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cot(x)^2 + a)*tan(x)^4, x)
```

Fricas [A]
time = 2.79, size = 239, normalized size = 2.81

$$\left[\frac{3\,a\sqrt{-a+b}\,\log\left(-\frac{a^2\tan(x)^4-2\,(3\,a^2-4\,ab)\tan(x)^2+a^2-8\,ab+8\,b^2+4\left(a\tan(x)^3-(a-2\,b)\tan(x)\right)\sqrt{-a+b}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}}{\tan(x)^4+2\,\tan(x)^2+1}\right)+4\left(a\tan(x)^3-(3\,a-b)\tan(x)\right)\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}}{12\,a},\ -\frac{3\,\sqrt{a-b}\,a\arctan\left(\frac{2\,\sqrt{a-b}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\,\tan(x)}{a\tan(x)^2-a+2\,b}\right)-2\left(a\tan(x)^3-(3\,a-b)\tan(x)\right)\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}}{6\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)^2)^(1/2)*tan(x)^4,x, algorithm="fricas")
```

```
[Out] [1/12*(3*a*sqrt(-a + b)*log(-(a^2*tan(x)^4 - 2*(3*a^2 - 4*a*b)*tan(x)^2 + a
^2 - 8*a*b + 8*b^2 + 4*(a*tan(x)^3 - (a - 2*b)*tan(x))*sqrt(-a + b)*sqrt((a
*tan(x)^2 + b)/tan(x)^2))/(tan(x)^4 + 2*tan(x)^2 + 1)) + 4*(a*tan(x)^3 - (3
*a - b)*tan(x))*sqrt((a*tan(x)^2 + b)/tan(x)^2))/a, -1/6*(3*sqrt(a - b)*a*a
rctan(2*sqrt(a - b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)/(a*tan(x)^2 - a
+ 2*b)) - 2*(a*tan(x)^3 - (3*a - b)*tan(x))*sqrt((a*tan(x)^2 + b)/tan(x)^2)
)/a]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\cot^2(x)}\,\tan^4(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)**2)**(1/2)*tan(x)**4,x)
```

```
[Out] Integral(sqrt(a + b*cot(x)**2)*tan(x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 476 vs. 2(71) = 142.
time = 0.49, size = 476, normalized size = 5.60

$$-\frac{1}{6}\left(3\sqrt{-a+b}\log\left(\left(\sqrt{-a+b}\cos(x)-\sqrt{-a\cos(x)^2+b\cos(x)^2+a}\right)^2\right)-\frac{4\left(3\left(\sqrt{-a+b}\cos(x)-\sqrt{-a\cos(x)^2+b\cos(x)^2+a}\right)^4(2a-b)\sqrt{-a+b}-6\left(\sqrt{-a+b}\cos(x)-\sqrt{-a\cos(x)^2+b\cos(x)^2+a}\right)^2a^2\sqrt{-a+b}+(4a^3-a^2b)\sqrt{-a+b}\right)}{\left(\left(\sqrt{-a+b}\cos(x)-\sqrt{-a\cos(x)^2+b\cos(x)^2+a}\right)^2-a\right)^3}\right)\mathrm{sgn}(\sin(x))+\frac{\left(3a^2\sqrt{-a+b}\log\left(-a-2\sqrt{-a+b}\sqrt{b}+2b\right)-9a^2\sqrt{b}\log\left(-a-2\sqrt{-a+b}\sqrt{b}+2b\right)-15a\sqrt{-a+b}\,b\log\left(-a-2\sqrt{-a+b}\sqrt{b}+2b\right)+21ab^{\frac{3}{2}}\log\left(-a-2\sqrt{-a+b}\sqrt{b}+2b\right)+12\sqrt{-a+b}\,b^2\log\left(-a-2\sqrt{-a+b}\sqrt{b}+2b\right)-12b^{\frac{5}{2}}\log\left(-a-2\sqrt{-a+b}\sqrt{b}+2b\right)+8a^2\sqrt{-a+b}-18a^2\sqrt{b}-24a\sqrt{-a+b}\,b+30ab^{\frac{3}{2}}+12\sqrt{-a+b}\,b^2-12b^{\frac{5}{2}}\right)\mathrm{sgn}(\sin(x))}{6\left(a^2+3a\sqrt{-a+b}\sqrt{b}-5ab-4\sqrt{-a+b}\,b^{\frac{3}{2}}+4b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)^2)^(1/2)*tan(x)^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*sqrt(-a + b)*log((sqrt(-a + b)*cos(x) - sqrt(-a*cos(x)^2 + b*cos(x)
^2 + a))^2) - 4*(3*(sqrt(-a + b)*cos(x) - sqrt(-a*cos(x)^2 + b*cos(x)^2 + a
))^4*(2*a - b)*sqrt(-a + b) - 6*(sqrt(-a + b)*cos(x) - sqrt(-a*cos(x)^2 + b
*cos(x)^2 + a))^2*a^2*sqrt(-a + b) + (4*a^3 - a^2*b)*sqrt(-a + b))/((sqrt(-
a + b)*cos(x) - sqrt(-a*cos(x)^2 + b*cos(x)^2 + a))^2 - a)^3)*sgn(sin(x)) +
 1/6*(3*a^2*sqrt(-a + b)*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b) - 9*a^2*sqr
t(b)*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b) - 15*a*sqrt(-a + b)*b*log(-a -
2*sqrt(-a + b)*sqrt(b) + 2*b) + 21*a*b^(3/2)*log(-a - 2*sqrt(-a + b)*sqrt(b
) + 2*b) + 12*sqrt(-a + b)*b^2*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b) - 12*
b^(5/2)*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b) + 8*a^2*sqrt(-a + b) - 18*a^
2*sqrt(b) - 24*a*sqrt(-a + b)*b + 30*a*b^(3/2) + 12*sqrt(-a + b)*b^2 - 12*b
^(5/2))*sgn(sin(x))/(a^2 + 3*a*sqrt(-a + b)*sqrt(b) - 5*a*b - 4*sqrt(-a + b
)*b^(3/2) + 4*b^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(x)^4 \sqrt{b\cot(x)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^4*(a + b*cot(x)^2)^(1/2),x)
[Out] int(tan(x)^4*(a + b*cot(x)^2)^(1/2), x)
```

3.26 $\int \cot^3(x)\left(a+b\cot^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=88

$$-(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)+(a-b)\sqrt{a+b\cot^2(x)}+\frac{1}{3}\left(a+b\cot^2(x)\right)^{3/2}-\frac{\left(a+b\cot^2(x)\right)^{5/2}}{5b}$$

```
[Out] -(a-b)^(3/2)*arctanh((a+b*cot(x)^2)^(1/2)/(a-b)^(1/2))+1/3*(a+b*cot(x)^2)^(
3/2)-1/5*(a+b*cot(x)^2)^(5/2)/b+(a-b)*(a+b*cot(x)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3751, 457, 81, 52, 65, 214}

$$-\frac{\left(a+b\cot^2(x)\right)^{5/2}}{5b}+\frac{1}{3}\left(a+b\cot^2(x)\right)^{3/2}+(a-b)\sqrt{a+b\cot^2(x)}-(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^3*(a + b*Cot[x]^2)^(3/2),x]
```

```
[Out] -((a - b)^(3/2)*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]]) + (a - b)*Sqrt[a
 + b*Cot[x]^2] + (a + b*Cot[x]^2)^(3/2)/3 - (a + b*Cot[x]^2)^(5/2)/(5*b)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^3(x)\left(a+b\cot^2(x)\right)^{3/2}\,dx = -\text{Subst}\left(\int \frac{x^3(a+bx^2)^{3/2}}{1+x^2}\,dx, x, \cot(x)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{x(a+bx)^{3/2}}{1+x}\,dx, x, \cot^2(x)\right)\right)$$
$$= -\frac{\left(a+b\cot^2(x)\right)^{5/2}}{5b} + \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{1+x}\,dx, x, \cot^2(x)\right)$$
$$= \frac{1}{3}\left(a+b\cot^2(x)\right)^{3/2} - \frac{\left(a+b\cot^2(x)\right)^{5/2}}{5b} + \frac{1}{2}(a-b)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1+x}\right.$$
$$= (a-b)\sqrt{a+b\cot^2(x)} + \frac{1}{3}\left(a+b\cot^2(x)\right)^{3/2} - \frac{\left(a+b\cot^2(x)\right)^{5/2}}{5b} + \frac{1}{2}(a$$
$$= (a-b)\sqrt{a+b\cot^2(x)} + \frac{1}{3}\left(a+b\cot^2(x)\right)^{3/2} - \frac{\left(a+b\cot^2(x)\right)^{5/2}}{5b} + \frac{(a-}{}$$
$$= -(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right) + (a-b)\sqrt{a+b\cot^2(x)} + \frac{1}{3}(a$$

Mathematica [A]
time = 0.55, size = 91, normalized size = 1.03

$$-(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right) - \frac{\sqrt{a+b\cot^2(x)}\,\left(3a^2-20ab+15b^2+(6a-5b)b\cot^2(x)+3b^2\cot^4(x)\right)}{15b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3*(a + b*Cot[x]^2)^(3/2),x]
```

```
[Out] -((a - b)^(3/2)*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]]) - (Sqrt[a + b*Co
t[x]^2]*(3*a^2 - 20*a*b + 15*b^2 + (6*a - 5*b)*b*Cot[x]^2 + 3*b^2*Cot[x]^4)
)/(15*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(72) = 144$.
time = 0.16, size = 176, normalized size = 2.00

method	result

derivativedivides	$-\frac{(a+b(\cot^2(x)))^{\frac{5}{2}}}{5b}+b^2\left(\frac{(\cot^2(x))\sqrt{a+b(\cot^2(x))}}{3b}-\frac{2a\sqrt{a+b(\cot^2(x))}}{3b^2}-\frac{\sqrt{a+b(\cot^2}}{b}\right.$
default	$-\frac{(a+b(\cot^2(x)))^{\frac{5}{2}}}{5b}+b^2\left(\frac{(\cot^2(x))\sqrt{a+b(\cot^2(x))}}{3b}-\frac{2a\sqrt{a+b(\cot^2(x))}}{3b^2}-\frac{\sqrt{a+b(\cot^2}}{b}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3*(a+b*cot(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*(a+b*cot(x)^2)^(5/2)/b+b^2*(1/3*cot(x)^2/b*(a+b*cot(x)^2)^(1/2)-2/3*a/
b^2*(a+b*cot(x)^2)^(1/2)-(a+b*cot(x)^2)^(1/2)/b+1/(-a+b)^(1/2)*arctan((a+b*
cot(x)^2)^(1/2)/(-a+b)^(1/2)))+2*a*b*((a+b*cot(x)^2)^(1/2)/b-1/(-a+b)^(1/2)
*arctan((a+b*cot(x)^2)^(1/2)/(-a+b)^(1/2)))+a^2/(-a+b)^(1/2)*arctan((a+b*co
t(x)^2)^(1/2)/(-a+b)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*(a+b*cot(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(72) = 144$.
time = 3.50, size = 486, normalized size = 5.52

$$\left[-\frac{15\left((ab-b^2)\cos(2x)^2+ab-b^2-2(ab-b^2)\cos(2x)\right)\sqrt{a-b}\log\left(-2(a^2-2ab+b^2)\cos(2x)^2-2a^2+b^2-2\left((a-b)\cos(2x)^2-(2a-b)\cos(2x)+a\right)\sqrt{a-b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}+4(a^2-ab)\cos(2x)\right)+4\left((3a^2-26ab+23b^2)\cos(2x)^2+3a^2-14ab+13b^2-2(3a^2-20ab+12b^2)\cos(2x)\right)\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}}{60\left(b\cos(2x)^2-2b\cos(2x)+b\right)}, -\frac{15\left((ab-b^2)\cos(2x)^2+ab-b^2-2(ab-b^2)\cos(2x)\right)\sqrt{-a+b}\arctan\left(-\frac{\sqrt{-a+b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}(\cos(2x)-1)}{(a-b)\cos(2x)-a}\right)+2\left((3a^2-26ab+23b^2)\cos(2x)^2+3a^2-14ab+13b^2-2(3a^2-20ab+12b^2)\cos(2x)\right)\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}}{30\left(b\cos(2x)^2-2b\cos(2x)+b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*(a+b*cot(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/60*(15*((a*b - b^2)*cos(2*x)^2 + a*b - b^2 - 2*(a*b - b^2)*cos(2*x))*sq
rt(a - b)*log(-2*(a^2 - 2*a*b + b^2)*cos(2*x)^2 - 2*a^2 + b^2 - 2*((a - b)*
cos(2*x)^2 - (2*a - b)*cos(2*x) + a)*sqrt(a - b)*sqrt(((a - b)*cos(2*x) - a
```

```
 - b)/(cos(2*x) - 1)) + 4*(a^2 - a*b)*cos(2*x)) + 4*((3*a^2 - 26*a*b + 23*b
^2)*cos(2*x)^2 + 3*a^2 - 14*a*b + 13*b^2 - 2*(3*a^2 - 20*a*b + 12*b^2)*cos(
2*x))*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)))/(b*cos(2*x)^2 - 2*b*
cos(2*x) + b), -1/30*(15*((a*b - b^2)*cos(2*x)^2 + a*b - b^2 - 2*(a*b - b^2
)*cos(2*x))*sqrt(-a + b)*arctan(-sqrt(-a + b)*sqrt(((a - b)*cos(2*x) - a -
b)/(cos(2*x) - 1))*(cos(2*x) - 1)/((a - b)*cos(2*x) - a)) + 2*((3*a^2 - 26*
a*b + 23*b^2)*cos(2*x)^2 + 3*a^2 - 14*a*b + 13*b^2 - 2*(3*a^2 - 20*a*b + 12
*b^2)*cos(2*x))*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)))/(b*cos(2*x
)^2 - 2*b*cos(2*x) + b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\cot^{2}\left(x\right)\right)^{\frac{3}{2}}\cot^{3}\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**3*(a+b*cot(x)**2)**(3/2),x)
```

```
[Out] Integral((a + b*cot(x)**2)**(3/2)*cot(x)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*(a+b*cot(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

Mupad [B]
time = 11.13, size = 120, normalized size = 1.36

$$\left(\frac{a}{3\,b}-\frac{a-b}{3\,b}\right)\,{\left(b\,{\cot\left(x\right)}^2+a\right)}^{3/2}-\frac{{\left(b\,{\cot\left(x\right)}^2+a\right)}^{5/2}}{5\,b}+\left(a-b\right)\,\left(\frac{a}{b}-\frac{a-b}{b}\right)\,\sqrt{b\,{\cot\left(x\right)}^2+a}+\mathrm{atan}\left(\frac{{\left(a-b\right)}^{3/2}\,\sqrt{b\,{\cot\left(x\right)}^2+a}\,1{}\mathrm{i}}{a^2-2\,a\,b+b^2}\right)\,{\left(a-b\right)}^{3/2}\,1{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3*(a + b*cot(x)^2)^(3/2),x)
```

```
[Out] atan(((a - b)^(3/2)*(a + b*cot(x)^2)^(1/2)*1i)/(a^2 - 2*a*b + b^2))*(a - b)
^(3/2)*1i - (a + b*cot(x)^2)^(5/2)/(5*b) + (a/(3*b) - (a - b)/(3*b))*(a + b
*cot(x)^2)^(3/2) + (a - b)*(a/b - (a - b)/b)*(a + b*cot(x)^2)^(1/2)
```

3.27 $\int \cot^2(x)\left(a+b\cot^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=127

$$(a-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)-\frac{(3a^2-12ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{8\sqrt{b}}-\frac{1}{8}(5a-4b)\cot(x)\sqrt{a$$

```
[Out] (a-b)^(3/2)*arctan(cot(x)*(a-b)^(1/2)/(a+b*cot(x)^2)^(1/2))-1/8*(3*a^2-12*a
*b+8*b^2)*arctanh(cot(x)*b^(1/2)/(a+b*cot(x)^2)^(1/2))/b^(1/2)-1/8*(5*a-4*b
)*cot(x)*(a+b*cot(x)^2)^(1/2)-1/4*b*cot(x)^3*(a+b*cot(x)^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3751, 488, 596, 537, 223, 212, 385, 209}

$$-\frac{(3a^2-12ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{8\sqrt{b}}+(a-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)-\frac{1}{8}(5a-4b)\cot(x)\sqrt{a+b\cot^2(x)}-\frac{1}{4}b\cot^3(x)\sqrt{a+b\cot^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2*(a + b*Cot[x]^2)^(3/2),x]
```

```
[Out] (a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Cot[x])/Sqrt[a + b*Cot[x]^2]] - ((3*a^2 -
 12*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Cot[x])/Sqrt[a + b*Cot[x]^2]])/(8*Sqrt[b]
) - ((5*a - 4*b)*Cot[x]*Sqrt[a + b*Cot[x]^2])/8 - (b*Cot[x]^3*Sqrt[a + b*Co
t[x]^2])/4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 488

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^2(x)\left(a+b\cot^2(x)\right)^{3/2}\,dx = -\text{Subst}\left(\int \frac{x^2(a+bx^2)^{3/2}}{1+x^2}\,dx, x, \cot(x)\right)$$

$$= -\frac{1}{4}b\cot^3(x)\sqrt{a+b\cot^2(x)} - \frac{1}{4}\text{Subst}\left(\int \frac{x^2(a(4a-3b)+(5a-4b)bx^2)}{(1+x^2)\sqrt{a+bx^2}}\right.$$

$$= -\frac{1}{8}(5a-4b)\cot(x)\sqrt{a+b\cot^2(x)} - \frac{1}{4}b\cot^3(x)\sqrt{a+b\cot^2(x)} + \frac{\text{Subst}}{}$$

$$= -\frac{1}{8}(5a-4b)\cot(x)\sqrt{a+b\cot^2(x)} - \frac{1}{4}b\cot^3(x)\sqrt{a+b\cot^2(x)} + (a-b$$

$$= -\frac{1}{8}(5a-4b)\cot(x)\sqrt{a+b\cot^2(x)} - \frac{1}{4}b\cot^3(x)\sqrt{a+b\cot^2(x)} + (a-b$$

$$= (a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right) - \frac{(3a^2-12ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{a-}}\right.}{8\sqrt{b}}$$

Mathematica [A]
time = 1.29, size = 253, normalized size = 1.99

$$\frac{\sqrt{-a-b+(a-b)\cos(2x)}\csc(x)\left(8\sqrt{2}(a-b)^2\sqrt{-b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a-b}\cos(x)}{\sqrt{-a-b+(a-b)\cos(2x)}}\right)+\sqrt{a-b}\left(-\sqrt{2}(3a^2-12ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{-b}\cos(x)}{\sqrt{-a-b+(a-b)\cos(2x)}}\right)+\sqrt{-b}\sqrt{-a-b+(a-b)\cos(2x)}\cot(x)\csc(x)(5a-6b+2b\csc^2(x))\right)\right)}{8\sqrt{2}\sqrt{a-b}\sqrt{-b}\sqrt{-((-a-b+(a-b)\cos(2x))\csc^2(x))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2*(a + b*Cot[x]^2)^(3/2),x]
```

```
[Out] (Sqrt[-a - b + (a - b)*Cos[2*x]]*Csc[x]*(8*Sqrt[2]*(a - b)^2*Sqrt[-b]*ArcTa
nh[(Sqrt[2]*Sqrt[a - b]*Cos[x])/Sqrt[-a - b + (a - b)*Cos[2*x]]] + Sqrt[a -
 b]*(-(Sqrt[2]*(3*a^2 - 12*a*b + 8*b^2)*ArcTanh[(Sqrt[2]*Sqrt[-b]*Cos[x])/S
qrt[-a - b + (a - b)*Cos[2*x]]]) + Sqrt[-b]*Sqrt[-a - b + (a - b)*Cos[2*x]]
*Cot[x]*Csc[x]*(5*a - 6*b + 2*b*Csc[x]^2))))/(8*Sqrt[2]*Sqrt[a - b]*Sqrt[-b
]*Sqrt[-((-a - b + (a - b)*Cos[2*x])*Csc[x]^2)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(105) = 210.
time = 0.12, size = 323, normalized size = 2.54

method	result

derivativedivides	$-\frac{\cot(x)\left(a+b\left(\cot^2(x)\right)\right)^{\frac{3}{2}}}{4}-\frac{3a\left(\frac{\cot(x)\sqrt{a+b\left(\cot^2(x)\right)}}{2}+\frac{a\ln\left(\sqrt{b}\,\cot(x)+\sqrt{a+b\left(\cot^2(x)\right)}\right)}{2\sqrt{b}}\right)}{4}+$
default	$-\frac{\cot(x)\left(a+b\left(\cot^2(x)\right)\right)^{\frac{3}{2}}}{4}-\frac{3a\left(\frac{\cot(x)\sqrt{a+b\left(\cot^2(x)\right)}}{2}+\frac{a\ln\left(\sqrt{b}\,\cot(x)+\sqrt{a+b\left(\cot^2(x)\right)}\right)}{2\sqrt{b}}\right)}{4}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2*(a+b*cot(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*cot(x)*(a+b*cot(x)^2)^(3/2)-3/4*a*(1/2*cot(x)*(a+b*cot(x)^2)^(1/2)+1/2
*a/b^(1/2)*ln(b^(1/2)*cot(x)+(a+b*cot(x)^2)^(1/2)))+b^2*(1/2*cot(x)/b*(a+b*
cot(x)^2)^(1/2)-1/2*a/b^(3/2)*ln(b^(1/2)*cot(x)+(a+b*cot(x)^2)^(1/2))-ln(b^
(1/2)*cot(x)+(a+b*cot(x)^2)^(1/2))/b^(1/2)+(b^4*(a-b))^(1/2)/b^2/(a-b)*arct
an(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*cot(x)^2)^(1/2)*cot(x)))+2*a*b*(ln(b^(1
/2)*cot(x)+(a+b*cot(x)^2)^(1/2))/b^(1/2)-(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan
(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*cot(x)^2)^(1/2)*cot(x)))+a^2*(b^4*(a-b))^
(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*cot(x)^2)^(1/2)*cot
(x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*(a+b*cot(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cot(x)^2 + a)^(3/2)*cot(x)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(105) = 210$.
time = 3.43, size = 1134, normalized size = 8.93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*(a+b*cot(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*(a*b - b^2 - (a*b - b^2)*cos(2*x))*sqrt(-a + b)*log(-(a - b)*cos(2
*x) + sqrt(-a + b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x)
```

```
 + b)*sin(2*x) - (3*a^2 - 12*a*b + 8*b^2 - (3*a^2 - 12*a*b + 8*b^2)*cos(2*x
))*sqrt(b)*log(((a - 2*b)*cos(2*x) + 2*sqrt(b)*sqrt(((a - b)*cos(2*x) - a -
 b)/(cos(2*x) - 1))*sin(2*x) - a - 2*b)/(cos(2*x) - 1))*sin(2*x) + 2*(4*b^2
*cos(2*x) - (5*a*b - 6*b^2)*cos(2*x)^2 + 5*a*b - 2*b^2)*sqrt(((a - b)*cos(2
*x) - a - b)/(cos(2*x) - 1)))/((b*cos(2*x) - b)*sin(2*x)), -1/8*((3*a^2 - 1
2*a*b + 8*b^2 - (3*a^2 - 12*a*b + 8*b^2)*cos(2*x))*sqrt(-b)*arctan(sqrt(-b)
*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x)/(b*cos(2*x) + b))
*sin(2*x) - 4*(a*b - b^2 - (a*b - b^2)*cos(2*x))*sqrt(-a + b)*log(-(a - b)*
cos(2*x) + sqrt(-a + b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin
(2*x) + b)*sin(2*x) - (4*b^2*cos(2*x) - (5*a*b - 6*b^2)*cos(2*x)^2 + 5*a*b
- 2*b^2)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)))/((b*cos(2*x) - b)
*sin(2*x)), -1/16*(16*(a*b - b^2 - (a*b - b^2)*cos(2*x))*sqrt(a - b)*arctan
(-sqrt(a - b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x)/((a
- b)*cos(2*x) + a - b))*sin(2*x) + (3*a^2 - 12*a*b + 8*b^2 - (3*a^2 - 12*a*
b + 8*b^2)*cos(2*x))*sqrt(b)*log(((a - 2*b)*cos(2*x) + 2*sqrt(b)*sqrt(((a -
 b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x) - a - 2*b)/(cos(2*x) - 1))*s
in(2*x) - 2*(4*b^2*cos(2*x) - (5*a*b - 6*b^2)*cos(2*x)^2 + 5*a*b - 2*b^2)*s
qrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)))/((b*cos(2*x) - b)*sin(2*x))
, -1/8*(8*(a*b - b^2 - (a*b - b^2)*cos(2*x))*sqrt(a - b)*arctan(-sqrt(a - b
)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x)/((a - b)*cos(2*x
) + a - b))*sin(2*x) + (3*a^2 - 12*a*b + 8*b^2 - (3*a^2 - 12*a*b + 8*b^2)*c
os(2*x))*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x)
 - 1))*sin(2*x)/(b*cos(2*x) + b))*sin(2*x) - (4*b^2*cos(2*x) - (5*a*b - 6*b
^2)*cos(2*x)^2 + 5*a*b - 2*b^2)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) -
 1)))/((b*cos(2*x) - b)*sin(2*x))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\cot^{2}\left(x\right)\right)^{\frac{3}{2}} \cot^{2}\left(x\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2*(a+b*cot(x)**2)**(3/2),x)
```

```
[Out] Integral((a + b*cot(x)**2)**(3/2)*cot(x)**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*(a+b*cot(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
```

```
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(x)^2 \left(b\cot(x)^2+a\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2*(a + b*cot(x)^2)^(3/2),x)
```

```
[Out] int(cot(x)^2*(a + b*cot(x)^2)^(3/2), x)
```

3.28 $\int \cot(x) \left(a+b\cot^2(x)\right)^{3/2} \, dx$

Optimal. Leaf size=69

$$(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)-(a-b)\sqrt{a+b\cot^2(x)}-\frac{1}{3}\left(a+b\cot^2(x)\right)^{3/2}$$

```
[Out] (a-b)^(3/2)*arctanh((a+b*cot(x)^2)^(1/2)/(a-b)^(1/2))-1/3*(a+b*cot(x)^2)^(3
/2)-(a-b)*(a+b*cot(x)^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3751, 455, 52, 65, 214}

$$-(a-b)\sqrt{a+b\cot^2(x)}-\frac{1}{3}\left(a+b\cot^2(x)\right)^{3/2}+(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*(a + b*Cot[x]^2)^(3/2),x]
```

```
[Out] (a - b)^(3/2)*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]] - (a - b)*Sqrt[a +
b*Cot[x]^2] - (a + b*Cot[x]^2)^(3/2)/3
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \cot(x)\left(a+b\cot^2(x)\right)^{3/2}\,dx &= -\mathrm{Subst}\left(\int \frac{x(a+bx^2)^{3/2}}{1+x^2}\,dx, x, \cot(x)\right)\\
&= -\left(\frac{1}{2}\mathrm{Subst}\left(\int \frac{(a+bx)^{3/2}}{1+x}\,dx, x, \cot^2(x)\right)\right)\\
&= -\frac{1}{3}\left(a+b\cot^2(x)\right)^{3/2}-\frac{1}{2}(a-b)\mathrm{Subst}\left(\int \frac{\sqrt{a+bx}}{1+x}\,dx, x, \cot^2(x)\right)\\
&= -(a-b)\sqrt{a+b\cot^2(x)}-\frac{1}{3}\left(a+b\cot^2(x)\right)^{3/2}-\frac{1}{2}(a-b)^2\mathrm{Subst}\left(\int \frac{}{(1}\right.\\
&= -(a-b)\sqrt{a+b\cot^2(x)}-\frac{1}{3}\left(a+b\cot^2(x)\right)^{3/2}-\frac{(a-b)^2\mathrm{Subst}\left(\int \frac{1}{1-\frac{a}{b}+}\right.}{}\\
&= (a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)-(a-b)\sqrt{a+b\cot^2(x)}-\frac{1}{3}(a+
\end{aligned}
$$

Mathematica [A]

time = 0.18, size = 63, normalized size = 0.91

$$(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)-\frac{1}{3}\sqrt{a+b\cot^2(x)}\,(4a-3b+b\cot^2(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*(a + b*Cot[x]^2)^(3/2),x]
```

```
[Out] (a - b)^(3/2)*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]] - (Sqrt[a + b*Cot[x
]^2]*(4*a - 3*b + b*Cot[x]^2))/3
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(57) = 114$.
time = 0.12, size = 163, normalized size = 2.36

method	result
derivativedivides	$-b^2\left(\frac{(\cot^2(x))\sqrt{a+b(\cot^2(x))}}{3b}-\frac{2a\sqrt{a+b(\cot^2(x))}}{3b^2}-\frac{\sqrt{a+b(\cot^2(x))}}{b}+\frac{\arctan\left(\frac{\sqrt{}}{}\right.}{}\right.$
default	$-b^2\left(\frac{(\cot^2(x))\sqrt{a+b(\cot^2(x))}}{3b}-\frac{2a\sqrt{a+b(\cot^2(x))}}{3b^2}-\frac{\sqrt{a+b(\cot^2(x))}}{b}+\frac{\arctan\left(\frac{\sqrt{}}{}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a+b*cot(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -b^2*(1/3*cot(x)^2/b*(a+b*cot(x)^2)^(1/2)-2/3*a/b^2*(a+b*cot(x)^2)^(1/2)-(a
+b*cot(x)^2)^(1/2)/b+1/(-a+b)^(1/2)*arctan((a+b*cot(x)^2)^(1/2)/(-a+b)^(1/2
)))-2*a*b*((a+b*cot(x)^2)^(1/2)/b-1/(-a+b)^(1/2)*arctan((a+b*cot(x)^2)^(1/2
)/(-a+b)^(1/2)))-a^2/(-a+b)^(1/2)*arctan((a+b*cot(x)^2)^(1/2)/(-a+b)^(1/2))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*cot(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(57) = 114$.
time = 4.17, size = 330, normalized size = 4.78

$$\left[-\frac{3\left((a-b)\cos\left(2\,x\right)-a+b\right)\sqrt{a-b}\log\left(-2\left(a^2-2\,ab+b^2\right)\cos\left(2\,x\right)^2-2\,a^2+b^2+2\left((a-b)\cos\left(2\,x\right)^2-(2\,a-b)\cos\left(2\,x\right)+a\right)\sqrt{a-b}\sqrt{\frac{(a-b)\cos\left(2\,x\right)-a-b}{\cos\left(2\,x\right)-1}}+4\left(a^2-ab\right)\cos\left(2\,x\right)\right)+8\left(2\left(a-b\right)\cos\left(2\,x\right)-2\,a+b\right)\sqrt{\frac{(a-b)\cos\left(2\,x\right)-a-b}{\cos\left(2\,x\right)-1}}}{12\left(\cos\left(2\,x\right)-1\right)}, \frac{3\left((a-b)\cos\left(2\,x\right)-a+b\right)\sqrt{-a+b}\arctan\left(-\frac{\sqrt{-a+b}\sqrt{\frac{(a-b)\cos\left(2\,x\right)-a-b}{\cos\left(2\,x\right)-1}}\left(\cos\left(2\,x\right)-1\right)}{(a-b)\cos\left(2\,x\right)-a}\right)-4\left(2\left(a-b\right)\cos\left(2\,x\right)-2\,a+b\right)\sqrt{\frac{(a-b)\cos\left(2\,x\right)-a-b}{\cos\left(2\,x\right)-1}}}{6\left(\cos\left(2\,x\right)-1\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*cot(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*((a - b)*cos(2*x) - a + b)*sqrt(a - b)*log(-2*(a^2 - 2*a*b + b^2)
*cos(2*x)^2 - 2*a^2 + b^2 + 2*((a - b)*cos(2*x)^2 - (2*a - b)*cos(2*x) + a)
*sqrt(a - b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)) + 4*(a^2 - a*b
)*cos(2*x)) + 8*(2*(a - b)*cos(2*x) - 2*a + b)*sqrt(((a - b)*cos(2*x) - a -
 b)/(cos(2*x) - 1)))/(cos(2*x) - 1), 1/6*(3*((a - b)*cos(2*x) - a + b)*sqrt
(-a + b)*arctan(-sqrt(-a + b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1
))*(cos(2*x) - 1)/((a - b)*cos(2*x) - a)) - 4*(2*(a - b)*cos(2*x) - 2*a + b
)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)))/(cos(2*x) - 1)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\cot^{2}\left(x\right)\right)^{\frac{3}{2}}\cot\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*cot(x)**2)**(3/2),x)
```

```
[Out] Integral((a + b*cot(x)**2)**(3/2)*cot(x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*cot(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

Mupad [B]
time = 3.54, size = 70, normalized size = 1.01

$$\operatorname{atanh}\left(\frac{(a-b)^{3/2}\,\sqrt{b\cot\left(x\right)^2+a}}{a^2-2\,a\,b+b^2}\right)\,(a-b)^{3/2}-\frac{{\left(b\cot(x)^2+a\right)}^{3/2}}{3}-(a-b)\,\sqrt{b\cot\left(x\right)^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a + b*cot(x)^2)^(3/2),x)
```

```
[Out] atanh(((a - b)^(3/2)*(a + b*cot(x)^2)^(1/2))/(a^2 - 2*a*b + b^2))*(a - b)^(
3/2) - (a + b*cot(x)^2)^(3/2)/3 - (a - b)*(a + b*cot(x)^2)^(1/2)
```

3.29 $\int (a+b\cot^2(x))^{3/2}\tan(x)\,dx$

Optimal. Leaf size=75

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)-(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)-b\sqrt{a+b\cot^2(x)}$$

```
[Out] a^(3/2)*arctanh((a+b*cot(x)^2)^(1/2)/a^(1/2))-(a-b)^(3/2)*arctanh((a+b*cot(
x)^2)^(1/2)/(a-b)^(1/2))-b*(a+b*cot(x)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3751, 457, 86, 162, 65, 214}

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)-b\sqrt{a+b\cot^2(x)}-(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[x]^2)^(3/2)*Tan[x],x]
```

```
[Out] a^(3/2)*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a]] - (a - b)^(3/2)*ArcTanh[Sqrt[
a + b*Cot[x]^2]/Sqrt[a - b]] - b*Sqrt[a + b*Cot[x]^2]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Dist[1/(b*d), I
nt[(b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/(
(a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
```

```
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \left(a+b\cot^2(x)\right)^{3/2}\tan(x)\,dx &= -\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x\left(1+x^2\right)}\,dx,x,\cot(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x(1+x)}\,dx,x,\cot^2(x)\right)\right)\\
&= -b\sqrt{a+b\cot^2(x)}-\frac{1}{2}\text{Subst}\left(\int \frac{a^2+(2a-b)bx}{x(1+x)\sqrt{a+bx}}\,dx,x,\cot^2(x)\right)\\
&= -b\sqrt{a+b\cot^2(x)}-\frac{1}{2}a^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,\cot^2(x)\right)+\frac{1}{2}(a-b)^2\\
&= -b\sqrt{a+b\cot^2(x)}-\frac{a^2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\cot^2(x)}\right)}{b}+\frac{(a-b)}{}\\
&= a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)-(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)-
\end{aligned}
$$

Mathematica [**A**]
time = 0.08, size = 75, normalized size = 1.00

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)-(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)-b\sqrt{a+b\cot^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[x]^2)^(3/2)*Tan[x],x]
```

```
[Out] a^(3/2)*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a]] - (a - b)^(3/2)*ArcTanh[Sqrt[
a + b*Cot[x]^2]/Sqrt[a - b]] - b*Sqrt[a + b*Cot[x]^2]
```

Maple [**C**] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.73, size = 2628, normalized size = 35.04

method	result	size
default	Expression too large to display	2628

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(x)^2)^(3/2)*tan(x),x,method=_RETURNVERBOSE)
```

```
[Out] sin(x)^2*((cos(x)^2*a-b*cos(x)^2-a)/(cos(x)^2-1))^(3/2)*(2*2^(1/2)*((cos(x)
*a^(1/2)*(a-b)^(1/2)-a*cos(x)+b*cos(x)-a^(1/2)*(a-b)^(1/2)+a)/(cos(x)+1)/b)
^(1/2)*(-2*(cos(x)*a^(1/2)*(a-b)^(1/2)+a*cos(x)-b*cos(x)-a^(1/2)*(a-b)^(1/2
)-a)/(cos(x)+1)/b)^(1/2)*EllipticF((-1+cos(x))*((2*a^(1/2)*(a-b)^(1/2)-2*a+
b)/b)^(1/2)/sin(x),((8*a^(3/2)*(a-b)^(1/2)-4*a^(1/2)*(a-b)^(1/2)*b+8*a^2-8*
a*b+b^2)/b^2)^(1/2))*a*b*sin(x)*cos(x)-2^(1/2)*((cos(x)*a^(1/2)*(a-b)^(1/2)
-a*cos(x)+b*cos(x)-a^(1/2)*(a-b)^(1/2)+a)/(cos(x)+1)/b)^(1/2)*(-2*(cos(x)*a
^(1/2)*(a-b)^(1/2)+a*cos(x)-b*cos(x)-a^(1/2)*(a-b)^(1/2)-a)/(cos(x)+1)/b)^(
1/2)*EllipticF((-1+cos(x))*((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/sin(x),(
(8*a^(3/2)*(a-b)^(1/2)-4*a^(1/2)*(a-b)^(1/2)*b+8*a^2-8*a*b+b^2)/b^2)^(1/2))
*b^2*sin(x)*cos(x)+2*2^(1/2)*((cos(x)*a^(1/2)*(a-b)^(1/2)-a*cos(x)+b*cos(x)
-a^(1/2)*(a-b)^(1/2)+a)/(cos(x)+1)/b)^(1/2)*(-2*(cos(x)*a^(1/2)*(a-b)^(1/2)
+a*cos(x)-b*cos(x)-a^(1/2)*(a-b)^(1/2)-a)/(cos(x)+1)/b)^(1/2)*EllipticPi((-
1+cos(x))*((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/sin(x),-1/(2*a^(1/2)*(a-b
)^(1/2)-2*a+b)*b,(-(2*a^(1/2)*(a-b)^(1/2)+2*a-b)/b)^(1/2)/((2*a^(1/2)*(a-b)
^(1/2)-2*a+b)/b)^(1/2))*a^2*sin(x)*cos(x)-4*2^(1/2)*((cos(x)*a^(1/2)*(a-b)^
(1/2)-a*cos(x)+b*cos(x)-a^(1/2)*(a-b)^(1/2)+a)/(cos(x)+1)/b)^(1/2)*(-2*(cos
(x)*a^(1/2)*(a-b)^(1/2)+a*cos(x)-b*cos(x)-a^(1/2)*(a-b)^(1/2)-a)/(cos(x)+1)
/b)^(1/2)*EllipticPi((-1+cos(x))*((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/si
n(x),-1/(2*a^(1/2)*(a-b)^(1/2)-2*a+b)*b,(-(2*a^(1/2)*(a-b)^(1/2)+2*a-b)/b)^
(1/2)/((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2))*a*b*sin(x)*cos(x)+2*2^(1/2)*
((cos(x)*a^(1/2)*(a-b)^(1/2)-a*cos(x)+b*cos(x)-a^(1/2)*(a-b)^(1/2)+a)/(cos(
x)+1)/b)^(1/2)*(-2*(cos(x)*a^(1/2)*(a-b)^(1/2)+a*cos(x)-b*cos(x)-a^(1/2)*(a
```

```
-b)^(1/2)-a)/(cos(x)+1)/b)^(1/2)*EllipticPi((-1+cos(x))*((2*a^(1/2)*(a-b)^(
1/2)-2*a+b)/b)^(1/2)/sin(x),-1/(2*a^(1/2)*(a-b)^(1/2)-2*a+b)*b,(-(2*a^(1/2)
*(a-b)^(1/2)+2*a-b)/b)^(1/2)/((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2))*b^2*s
in(x)*cos(x)-2*2^(1/2)*((cos(x)*a^(1/2)*(a-b)^(1/2)-a*cos(x)+b*cos(x)-a^(1/
2)*(a-b)^(1/2)+a)/(cos(x)+1)/b)^(1/2)*(-2*(cos(x)*a^(1/2)*(a-b)^(1/2)+a*cos
(x)-b*cos(x)-a^(1/2)*(a-b)^(1/2)-a)/(cos(x)+1)/b)^(1/2)*EllipticPi((-1+cos(
x))*((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/sin(x),1/(2*a^(1/2)*(a-b)^(1/2)
-2*a+b)*b,(-(2*a^(1/2)*(a-b)^(1/2)+2*a-b)/b)^(1/2)/((2*a^(1/2)*(a-b)^(1/2)-
2*a+b)/b)^(1/2))*a^2*sin(x)*cos(x)+2*2^(1/2)*((cos(x)*a^(1/2)*(a-b)^(1/2)-a
*cos(x)+b*cos(x)-a^(1/2)*(a-b)^(1/2)+a)/(cos(x)+1)/b)^(1/2)*(-2*(cos(x)*a^(
1/2)*(a-b)^(1/2)+a*cos(x)-b*cos(x)-a^(1/2)*(a-b)^(1/2)-a)/(cos(x)+1)/b)^(1/
2)*EllipticF((-1+cos(x))*((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/sin(x),((8
*a^(3/2)*(a-b)^(1/2)-4*a^(1/2)*(a-b)^(1/2)*b+8*a^2-8*a*b+b^2)/b^2)^(1/2))*a
*b*sin(x)-2^(1/2)*((cos(x)*a^(1/2)*(a-b)^(1/2)-a*cos(x)+b*cos(x)-a^(1/2)*(a
-b)^(1/2)+a)/(cos(x)+1)/b)^(1/2)*(-2*(cos(x)*a^(1/2)*(a-b)^(1/2)+a*cos(x)-b
*cos(x)-a^(1/2)*(a-b)^(1/2)-a)/(cos(x)+1)/b)^(1/2)*EllipticF((-1+cos(x))*((
2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/sin(x),((8*a^(3/2)*(a-b)^(1/2)-4*a^(1
/2)*(a-b)^(1/2)*b+8*a^2-8*a*b+b^2)/b^2)^(1/2))*b^2*sin(x)+2*2^(1/2)*((cos(x
)*a^(1/2)*(a-b)^(1/2)-a*cos(x)+b*cos(x)-a^(1/2)*(a-b)^(1/2)+a)/(cos(x)+1)/b
)^(1/2)*(-2*(cos(x)*a^(1/2)*(a-b)^(1/2)+a*cos(x)-b*cos(x)-a^(1/2)*(a-b)^(1/
2)-a)/(cos(x)+1)/b)^(1/2)*EllipticPi((-1+cos(x))*((2*a^(1/2)*(a-b)^(1/2)-2*
a+b)/b)^(1/2)/sin(x),-1/(2*a^(1/2)*(a-b)^(1/2)-2*a+b)*b,(-(2*a^(1/2)*(a-b)^
(1/2)+2*a-b)/b)^(1/2)/((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2))*a^2*sin(x)-4
*2^(1/2)*((cos(x)*a^(1/2)*(a-b)^(1/2)-a*cos(x)+b*cos(x)-a^(1/2)*(a-b)^(1/2)
+a)/(cos(x)+1)/b)^(1/2)*(-2*(cos(x)*a^(1/2)*(a-b)^(1/2)+a*cos(x)-b*cos(x)-a
^(1/2)*(a-b)^(1/2)-a)/(cos(x)+1)/b)^(1/2)*EllipticPi((-1+cos(x))*((2*a^(1/2
)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/sin(x),-1/(2*a^(1/2)*(a-b)^(1/2)-2*a+b)*b,(-(
2*a^(1/2)*(a-b)^(1/2)+2*a-b)/b)^(1/2)/((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/
2))*a*b*sin(x)+2*2^(1/2)*((cos(x)*a^(1/2)*(a-b)^(1/2)-a*cos(x)+b*cos(x)-a^(
1/2)*(a-b)^(1/2)+a)/(cos(x)+1)/b)^(1/2)*(-2*(cos(x)*a^(1/2)*(a-b)^(1/2)+a*c
os(x)-b*cos(x)-a^(1/2)*(a-b)^(1/2)-a)/(cos(x)+1)/b)^(1/2)*EllipticPi((-1+co
s(x))*((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/sin(x),-1/(2*a^(1/2)*(a-b)^(1
/2)-2*a+b)*b,(-(2*a^(1/2)*(a-b)^(1/2)+2*a-b)/b)^(1/2)/((2*a^(1/2)*(a-b)^(1/
2)-2*a+b)/b)^(1/2))*b^2*sin(x)-2*2^(1/2)*((cos(x)*a^(1/2)*(a-b)^(1/2)-a*cos
(x)+b*cos(x)-a^(1/2)*(a-b)^(1/2)+a)/(cos(x)+1)/b)^(1/2)*(-2*(cos(x)*a^(1/2)
*(a-b)^(1/2)+a*cos(x)-b*cos(x)-a^(1/2)*(a-b)^(1/2)-a)/(cos(x)+1)/b)^(1/2)*E
llipticPi((-1+cos(x))*((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/sin(x),1/(2*a
^(1/2)*(a-b)^(1/2)-2*a+b)*b,(-(2*a^(1/2)*(a-b)^(1/2)+2*a-b)/b)^(1/2)/((2*a^
(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2))*a^2*sin(x)+cos(x)^2*((2*a^(1/2)*(a-b)^(1
/2)-2*a+b)/b)^(1/2)*a*b-cos(x)^2*((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)*b^
2-((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)*a*b)/(cos(x)^2*a-b*cos(x)^2-a)^2/
((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)^2)^(3/2)*tan(x),x, algorithm="maxima")
```

```
[Out] integrate((b*cot(x)^2 + a)^(3/2)*tan(x), x)
```

Fricas [A]
time = 5.08, size = 565, normalized size = 7.53

$$\left[\frac{1}{2} a^{\frac{3}{2}} \log\left(2 a \tan(x)^2 + 2 \sqrt{a} \sqrt{\frac{a \tan(x)^2 + b}{\tan(x)^2}} \tan(x)^2 + b\right) - \frac{1}{4} (a-b)^{\frac{3}{2}} \log\left(-\frac{(8 a^2 - 8 a b + b^2) \tan(x)^4 + 2 (4 a b - 3 b^2) \tan(x)^2 + b^2 + 4 \left((2 a - b) \tan(x)^4 + b \tan(x)^2\right) \sqrt{a-b} \sqrt{\frac{a \tan(x)^2 + b}{\tan(x)^2}}}{\tan(x)^4 + 2 \tan(x)^2 + 1}\right) - b \sqrt{\frac{a \tan(x)^2 + b}{\tan(x)^2}}, -\sqrt{-a}\, a \arctan\left(\frac{\sqrt{-a} \sqrt{\frac{a \tan(x)^2 + b}{\tan(x)^2}} \tan(x)^2}{a \tan(x)^2 + b}\right) - \frac{1}{4} (a-b)^{\frac{3}{2}} \log\left(-\frac{(8 a^2 - 8 a b + b^2) \tan(x)^4 + 2 (4 a b - 3 b^2) \tan(x)^2 + b^2 + 4 \left((2 a - b) \tan(x)^4 + b \tan(x)^2\right) \sqrt{a-b} \sqrt{\frac{a \tan(x)^2 + b}{\tan(x)^2}}}{\tan(x)^4 + 2 \tan(x)^2 + 1}\right) - b \sqrt{\frac{a \tan(x)^2 + b}{\tan(x)^2}}, \frac{1}{2} (-a+b)^{\frac{3}{2}} \arctan\left(-\frac{2 \sqrt{-a+b} \sqrt{\frac{a \tan(x)^2 + b}{\tan(x)^2}} \tan(x)^2}{(2 a - b) \tan(x)^2 + b}\right) + \frac{1}{2} a^{\frac{3}{2}} \log\left(2 a \tan(x)^2 + 2 \sqrt{a} \sqrt{\frac{a \tan(x)^2 + b}{\tan(x)^2}} \tan(x)^2 + b\right) - b \sqrt{\frac{a \tan(x)^2 + b}{\tan(x)^2}}, -\sqrt{-a}\, a \arctan\left(\frac{\sqrt{-a} \sqrt{\frac{a \tan(x)^2 + b}{\tan(x)^2}} \tan(x)^2}{a \tan(x)^2 + b}\right) + \frac{1}{2} (-a+b)^{\frac{3}{2}} \arctan\left(-\frac{2 \sqrt{-a+b} \sqrt{\frac{a \tan(x)^2 + b}{\tan(x)^2}} \tan(x)^2}{(2 a - b) \tan(x)^2 + b}\right) - b \sqrt{\frac{a \tan(x)^2 + b}{\tan(x)^2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)^2)^(3/2)*tan(x),x, algorithm="fricas")
```

```
[Out] [1/2*a^(3/2)*log(2*a*tan(x)^2 + 2*sqrt(a)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*t
an(x)^2 + b) - 1/4*(a - b)^(3/2)*log(-((8*a^2 - 8*a*b + b^2)*tan(x)^4 + 2*(
4*a*b - 3*b^2)*tan(x)^2 + b^2 + 4*((2*a - b)*tan(x)^4 + b*tan(x)^2)*sqrt(a
- b)*sqrt((a*tan(x)^2 + b)/tan(x)^2))/(tan(x)^4 + 2*tan(x)^2 + 1)) - b*sqrt
((a*tan(x)^2 + b)/tan(x)^2), -sqrt(-a)*a*arctan(sqrt(-a)*sqrt((a*tan(x)^2 +
 b)/tan(x)^2)*tan(x)^2/(a*tan(x)^2 + b)) - 1/4*(a - b)^(3/2)*log(-((8*a^2 -
 8*a*b + b^2)*tan(x)^4 + 2*(4*a*b - 3*b^2)*tan(x)^2 + b^2 + 4*((2*a - b)*ta
n(x)^4 + b*tan(x)^2)*sqrt(a - b)*sqrt((a*tan(x)^2 + b)/tan(x)^2))/(tan(x)^4
 + 2*tan(x)^2 + 1)) - b*sqrt((a*tan(x)^2 + b)/tan(x)^2), 1/2*(-a + b)^(3/2)
*arctan(-2*sqrt(-a + b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2/((2*a - b)
*tan(x)^2 + b)) + 1/2*a^(3/2)*log(2*a*tan(x)^2 + 2*sqrt(a)*sqrt((a*tan(x)^2
 + b)/tan(x)^2)*tan(x)^2 + b) - b*sqrt((a*tan(x)^2 + b)/tan(x)^2), -sqrt(-a
)*a*arctan(sqrt(-a)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2/(a*tan(x)^2 +
b)) + 1/2*(-a + b)^(3/2)*arctan(-2*sqrt(-a + b)*sqrt((a*tan(x)^2 + b)/tan(x
)^2)*tan(x)^2/((2*a - b)*tan(x)^2 + b)) - b*sqrt((a*tan(x)^2 + b)/tan(x)^2)
]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \cot^{2}{\left(x \right)}\right)^{\frac{3}{2}} \tan{\left(x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)**2)**(3/2)*tan(x),x)
```

```
[Out] Integral((a + b*cot(x)**2)**(3/2)*tan(x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)^2)^(3/2)*tan(x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

Mupad [B]
time = 0.54, size = 506, normalized size = 6.75

$$\operatorname{atanh}\left(\frac{2\,b^6\,\sqrt{a^3}\,\sqrt{a+\frac{b}{{\mathrm{tan}\left(x\right)}^2}}}{-6\,a^5\,b^3+12\,a^4\,b^4-8\,a^3\,b^5+2\,a^2\,b^6}-\frac{8\,a\,b^5\,\sqrt{a^3}\,\sqrt{a+\frac{b}{{\mathrm{tan}\left(x\right)}^2}}}{-6\,a^5\,b^3+12\,a^4\,b^4-8\,a^3\,b^5+2\,a^2\,b^6}+\frac{12\,a^2\,b^4\,\sqrt{a^3}\,\sqrt{a+\frac{b}{{\mathrm{tan}\left(x\right)}^2}}}{-6\,a^5\,b^3+12\,a^4\,b^4-8\,a^3\,b^5+2\,a^2\,b^6}-\frac{6\,a^3\,b^3\,\sqrt{a^3}\,\sqrt{a+\frac{b}{{\mathrm{tan}\left(x\right)}^2}}}{-6\,a^5\,b^3+12\,a^4\,b^4-8\,a^3\,b^5+2\,a^2\,b^6}\right)\,\sqrt{a^3}-\operatorname{atanh}\left(\frac{2\,a\,b^5\,\sqrt{a+\frac{b}{{\mathrm{tan}\left(x\right)}^2}}\,\sqrt{a^3-3\,a^2\,b+3\,a\,b^2-b^3}}{6\,a^5\,b^3-18\,a^4\,b^4+20\,a^3\,b^5-10\,a^2\,b^6+2\,a\,b^7}-\frac{6\,a^2\,b^4\,\sqrt{a+\frac{b}{{\mathrm{tan}\left(x\right)}^2}}\,\sqrt{a^3-3\,a^2\,b+3\,a\,b^2-b^3}}{6\,a^5\,b^3-18\,a^4\,b^4+20\,a^3\,b^5-10\,a^2\,b^6+2\,a\,b^7}+\frac{6\,a^3\,b^3\,\sqrt{a+\frac{b}{{\mathrm{tan}\left(x\right)}^2}}\,\sqrt{a^3-3\,a^2\,b+3\,a\,b^2-b^3}}{6\,a^5\,b^3-18\,a^4\,b^4+20\,a^3\,b^5-10\,a^2\,b^6+2\,a\,b^7}\right)\,\sqrt{{\left(a-b\right)}^3}-b\,\sqrt{a+\frac{b}{{\mathrm{tan}\left(x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(a + b*cot(x)^2)^(3/2),x)
```

```
[Out] atanh((2*b^6*(a^3)^(1/2)*(a + b/tan(x)^2)^(1/2))/(2*a^2*b^6 - 8*a^3*b^5 + 1
2*a^4*b^4 - 6*a^5*b^3) - (8*a*b^5*(a^3)^(1/2)*(a + b/tan(x)^2)^(1/2))/(2*a^
2*b^6 - 8*a^3*b^5 + 12*a^4*b^4 - 6*a^5*b^3) + (12*a^2*b^4*(a^3)^(1/2)*(a +
b/tan(x)^2)^(1/2))/(2*a^2*b^6 - 8*a^3*b^5 + 12*a^4*b^4 - 6*a^5*b^3) - (6*a^
3*b^3*(a^3)^(1/2)*(a + b/tan(x)^2)^(1/2))/(2*a^2*b^6 - 8*a^3*b^5 + 12*a^4*b
^4 - 6*a^5*b^3))*(a^3)^(1/2) - atanh((2*a*b^5*(a + b/tan(x)^2)^(1/2)*(3*a*b
^2 - 3*a^2*b + a^3 - b^3)^(1/2))/(2*a*b^7 - 10*a^2*b^6 + 20*a^3*b^5 - 18*a^
4*b^4 + 6*a^5*b^3) - (6*a^2*b^4*(a + b/tan(x)^2)^(1/2)*(3*a*b^2 - 3*a^2*b +
 a^3 - b^3)^(1/2))/(2*a*b^7 - 10*a^2*b^6 + 20*a^3*b^5 - 18*a^4*b^4 + 6*a^5*
b^3) + (6*a^3*b^3*(a + b/tan(x)^2)^(1/2)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)^(1
/2))/(2*a*b^7 - 10*a^2*b^6 + 20*a^3*b^5 - 18*a^4*b^4 + 6*a^5*b^3))*((a - b)
^3)^(1/2) - b*(a + b/tan(x)^2)^(1/2)
```

3.30 $\int (a+b\cot^2(x))^{3/2}\tan^2(x)\,dx$

Optimal. Leaf size=80

$$(a-b)^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)-b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)+a\sqrt{a+b\cot^2(x)}\,\tan(x)$$

```
[Out] (a-b)^(3/2)*arctan(cot(x)*(a-b)^(1/2)/(a+b*cot(x)^2)^(1/2))-b^(3/2)*arctanh
(cot(x)*b^(1/2)/(a+b*cot(x)^2)^(1/2))+a*(a+b*cot(x)^2)^(1/2)*tan(x)
```

Rubi [A]
time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3751, 485, 537, 223, 212, 385, 209}

$$(a-b)^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)-b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)+a\tan(x)\sqrt{a+b\cot^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[x]^2)^(3/2)*Tan[x]^2,x]
```

```
[Out] (a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Cot[x])/Sqrt[a + b*Cot[x]^2]] - b^(3/2)*A
rcTanh[(Sqrt[b]*Cot[x])/Sqrt[a + b*Cot[x]^2]] + a*Sqrt[a + b*Cot[x]^2]*Tan[
x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \left(a+b\cot^2(x)\right)^{3/2}\tan^2(x)\,dx &= -\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x^2\left(1+x^2\right)}\,dx, x, \cot(x)\right)\\
&= a\sqrt{a+b\cot^2(x)}\,\tan(x) - \text{Subst}\left(\int \frac{-a(a-2b)+b^2x^2}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx, x, \cot(x)\right)\\
&= a\sqrt{a+b\cot^2(x)}\,\tan(x) + (a-b)^2\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx, x, \text{c}\right.\\
&= a\sqrt{a+b\cot^2(x)}\,\tan(x) + (a-b)^2\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{}{\sqrt{a}}\right.\\
&= (a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right) - b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 222 vs. $2(80) = 160$.
time = 0.79, size = 222, normalized size = 2.78

$$
\frac{\sqrt{-\left((-a-b+(a-b)\cos(2x))\csc^2(x)\right)}\left(-\sqrt{2}\,(a-b)^2\sqrt{-b}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a-b}\,\cos(x)}{\sqrt{-a-b+(a-b)\cos(2x)}}\right)+\sqrt{a-b}\left(\sqrt{2}\,b^2\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{-b}\,\cos(x)}{\sqrt{-a-b+(a-b)\cos(2x)}}\right)+a\sqrt{-b}\,\sqrt{-a-b+(a-b)\cos(2x)}\,\sec(x)\right)\right)\sin(x)}{\sqrt{2}\,\sqrt{a-b}\,\sqrt{-b}\,\sqrt{-a-b+(a-b)\cos(2x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[x]^2)^(3/2)*Tan[x]^2,x]
```

```
[Out] (Sqrt[-((-a - b + (a - b)*Cos[2*x])*Csc[x]^2)]*(-(Sqrt[2]*(a - b)^2*Sqrt[-b
]*ArcTanh[(Sqrt[2]*Sqrt[a - b]*Cos[x])/Sqrt[-a - b + (a - b)*Cos[2*x]]]) +
Sqrt[a - b]*(Sqrt[2]*b^2*ArcTanh[(Sqrt[2]*Sqrt[-b]*Cos[x])/Sqrt[-a - b + (a
 - b)*Cos[2*x]]] + a*Sqrt[-b]*Sqrt[-a - b + (a - b)*Cos[2*x]]*Sec[x]))*Sin[
x])/(Sqrt[2]*Sqrt[a - b]*Sqrt[-b]*Sqrt[-a - b + (a - b)*Cos[2*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1275 vs. $2(66) = 132$.
time = 0.64, size = 1276, normalized size = 15.95

method	result	size
default	Expression too large to display	1276

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(x)^2)^(3/2)*tan(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((cos(x)^2*a-b*cos(x)^2-a)/(cos(x)^2-1))^(3/2)*(-1+cos(x))^3*(-2*cos(x)
*b^(9/2)*ln(4*cos(x)*(-a+b)^(1/2)*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)
^(1/2)-4*a*cos(x)+4*b*cos(x)+4*(-a+b)^(1/2)*(-(cos(x)^2*a-b*cos(x)^2-a)/(co
s(x)+1)^2)^(1/2))+4*cos(x)*b^(7/2)*ln(4*cos(x)*(-a+b)^(1/2)*(-(cos(x)^2*a-b
*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)-4*a*cos(x)+4*b*cos(x)+4*(-a+b)^(1/2)*(-(co
s(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2))*a-2*cos(x)*b^(5/2)*ln(4*cos(x)*
(-a+b)^(1/2)*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)-4*a*cos(x)+4*b
*cos(x)+4*(-a+b)^(1/2)*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2))*a^2
-2*cos(x)*b^(5/2)*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*(-a+b)^(1
/2)*a-2*a*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*(-a+b)^(1/2)*b^(5
/2)+cos(x)*ln(-4*(cos(x)*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*b^
(1/2)-a*cos(x)+b*cos(x)+(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*b^(
1/2)+a)/(-1+cos(x)))*(-a+b)^(1/2)*b^4+3*cos(x)*ln(-2*(-1+cos(x))*(cos(x)*(-
(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*b^(1/2)+a*cos(x)-b*cos(x)+(-(
cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*b^(1/2)+a)/sin(x)^2/b^(1/2))*(
-a+b)^(1/2)*a^3*b-6*cos(x)*ln(-2*(-1+cos(x))*(cos(x)*(-(cos(x)^2*a-b*cos(x)
^2-a)/(cos(x)+1)^2)^(1/2)*b^(1/2)+a*cos(x)-b*cos(x)+(-(cos(x)^2*a-b*cos(x)^
2-a)/(cos(x)+1)^2)^(1/2)*b^(1/2)+a)/sin(x)^2/b^(1/2))*(-a+b)^(1/2)*a^2*b^2+
3*cos(x)*ln(-2*(-1+cos(x))*(cos(x)*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2
)^(1/2)*b^(1/2)+a*cos(x)-b*cos(x)+(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)
^(1/2)*b^(1/2)+a)/sin(x)^2/b^(1/2))*(-a+b)^(1/2)*a*b^3-cos(x)*ln(-2*(-1+cos
(x))*(cos(x)*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*b^(1/2)+a*cos(
x)-b*cos(x)+(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*b^(1/2)+a)/sin(
x)^2/b^(1/2))*(-a+b)^(1/2)*b^4-3*cos(x)*ln(-4*(-1+cos(x))*(cos(x)*(-(cos(x)
^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*b^(1/2)+a*cos(x)-b*cos(x)+(-(cos(x)^
2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*b^(1/2)+a)/sin(x)^2/b^(1/2))*(-a+b)^(
1/2)*a^3*b+6*cos(x)*ln(-4*(-1+cos(x))*(cos(x)*(-(cos(x)^2*a-b*cos(x)^2-a)/(
cos(x)+1)^2)^(1/2)*b^(1/2)+a*cos(x)-b*cos(x)+(-(cos(x)^2*a-b*cos(x)^2-a)/(c
os(x)+1)^2)^(1/2)*b^(1/2)+a)/sin(x)^2/b^(1/2))*(-a+b)^(1/2)*a^2*b^2-3*cos(x
)*ln(-4*(-1+cos(x))*(cos(x)*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)
*b^(1/2)+a*cos(x)-b*cos(x)+(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*
b^(1/2)+a)/sin(x)^2/b^(1/2))*(-a+b)^(1/2)*a*b^3)/cos(x)/sin(x)^3/(-(cos(x)^
2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(3/2)/b^(5/2)/(-a+b)^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)^2)^(3/2)*tan(x)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*cot(x)^2 + a)^(3/2)*tan(x)^2, x)
```

Fricas [**A**]

time = 6.17, size = 543, normalized size = 6.79

$$\left[\frac{1}{4}\,{\left(-a + b\right)}^{\frac{3}{2}} \log\left(-\frac{a^{2} \tan\left(x\right)^{4} - 2\,{\left(3\,a^{2} - 4\,a b\right)} \tan\left(x\right)^{2} + a^{2} - 8\,a b + 8\,b^{2} + 4\,{\left(a \tan\left(x\right)^{3} - {\left(a - 2\,b\right)} \tan\left(x\right)\right)} \sqrt{-a + b} \sqrt{\frac{a \tan\left(x\right)^{2} + b}{\tan\left(x\right)^{2}}}}{\tan\left(x\right)^{4} + 2\,\tan\left(x\right)^{2} + 1}\right) + \frac{1}{2}\,b^{\frac{3}{2}} \log\left(\frac{a \tan\left(x\right)^{2} - 2\,\sqrt{b} \sqrt{\frac{a \tan\left(x\right)^{2} + b}{\tan\left(x\right)^{2}}} \tan\left(x\right) + 2\,b}{\tan\left(x\right)^{2}}\right) + a \sqrt{\frac{a \tan\left(x\right)^{2} + b}{\tan\left(x\right)^{2}}} \tan\left(x\right),\ \sqrt{-b} b \arctan\left(\frac{\sqrt{-b} \sqrt{\frac{a \tan\left(x\right)^{2} + b}{\tan\left(x\right)^{2}}} \tan\left(x\right)}{b}\right) + \frac{1}{4}\,{\left(-a + b\right)}^{\frac{3}{2}} \log\left(-\frac{a^{2} \tan\left(x\right)^{4} - 2\,{\left(3\,a^{2} - 4\,a b\right)} \tan\left(x\right)^{2} + a^{2} - 8\,a b + 8\,b^{2} + 4\,{\left(a \tan\left(x\right)^{3} - {\left(a - 2\,b\right)} \tan\left(x\right)\right)} \sqrt{-a + b} \sqrt{\frac{a \tan\left(x\right)^{2} + b}{\tan\left(x\right)^{2}}}}{\tan\left(x\right)^{4} + 2\,\tan\left(x\right)^{2} + 1}\right) + a \sqrt{\frac{a \tan\left(x\right)^{2} + b}{\tan\left(x\right)^{2}}} \tan\left(x\right),\ \frac{1}{2}\,{\left(a - b\right)}^{\frac{3}{2}} \arctan\left(\frac{2\,\sqrt{a - b} \sqrt{\frac{a \tan\left(x\right)^{2} + b}{\tan\left(x\right)^{2}}} \tan\left(x\right)}{a \tan\left(x\right)^{2} - a + 2\,b}\right) + \frac{1}{2}\,b^{\frac{3}{2}} \log\left(\frac{a \tan\left(x\right)^{2} - 2\,\sqrt{b} \sqrt{\frac{a \tan\left(x\right)^{2} + b}{\tan\left(x\right)^{2}}} \tan\left(x\right) + 2\,b}{\tan\left(x\right)^{2}}\right) + a \sqrt{\frac{a \tan\left(x\right)^{2} + b}{\tan\left(x\right)^{2}}} \tan\left(x\right),\ \frac{1}{2}\,{\left(a - b\right)}^{\frac{3}{2}} \arctan\left(\frac{2\,\sqrt{a - b} \sqrt{\frac{a \tan\left(x\right)^{2} + b}{\tan\left(x\right)^{2}}} \tan\left(x\right)}{a \tan\left(x\right)^{2} - a + 2\,b}\right) + \sqrt{-b} b \arctan\left(\frac{\sqrt{-b} \sqrt{\frac{a \tan\left(x\right)^{2} + b}{\tan\left(x\right)^{2}}} \tan\left(x\right)}{b}\right) + a \sqrt{\frac{a \tan\left(x\right)^{2} + b}{\tan\left(x\right)^{2}}} \tan\left(x\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)^2)^(3/2)*tan(x)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(-a + b)^(3/2)*log(-(a^2*tan(x)^4 - 2*(3*a^2 - 4*a*b)*tan(x)^2 + a^2 -
 8*a*b + 8*b^2 + 4*(a*tan(x)^3 - (a - 2*b)*tan(x))*sqrt(-a + b)*sqrt((a*tan
(x)^2 + b)/tan(x)^2))/(tan(x)^4 + 2*tan(x)^2 + 1)) + 1/2*b^(3/2)*log((a*tan
(x)^2 - 2*sqrt(b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x) + 2*b)/tan(x)^2) +
 a*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x), sqrt(-b)*b*arctan(sqrt(-b)*sqrt(
(a*tan(x)^2 + b)/tan(x)^2)*tan(x)/b) + 1/4*(-a + b)^(3/2)*log(-(a^2*tan(x)^
4 - 2*(3*a^2 - 4*a*b)*tan(x)^2 + a^2 - 8*a*b + 8*b^2 + 4*(a*tan(x)^3 - (a -
 2*b)*tan(x))*sqrt(-a + b)*sqrt((a*tan(x)^2 + b)/tan(x)^2))/(tan(x)^4 + 2*t
an(x)^2 + 1)) + a*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x), 1/2*(a - b)^(3/2)
*arctan(2*sqrt(a - b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)/(a*tan(x)^2 -
a + 2*b)) + 1/2*b^(3/2)*log((a*tan(x)^2 - 2*sqrt(b)*sqrt((a*tan(x)^2 + b)/t
an(x)^2)*tan(x) + 2*b)/tan(x)^2) + a*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)
, 1/2*(a - b)^(3/2)*arctan(2*sqrt(a - b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*ta
n(x)/(a*tan(x)^2 - a + 2*b)) + sqrt(-b)*b*arctan(sqrt(-b)*sqrt((a*tan(x)^2
+ b)/tan(x)^2)*tan(x)/b) + a*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \cot^{2}{\left(x \right)}\right)^{\frac{3}{2}} \tan^{2}{\left(x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)**2)**(3/2)*tan(x)**2,x)
```

```
[Out] Integral((a + b*cot(x)**2)**(3/2)*tan(x)**2, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x)^2)^(3/2)*tan(x)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{tan}\left(x\right)}^2\,{\left(b\,{\mathrm{cot}\left(x\right)}^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2*(a + b*cot(x)^2)^(3/2),x)
```

```
[Out] int(tan(x)^2*(a + b*cot(x)^2)^(3/2), x)
```

3.31 $\int \left(a+b\cot^2(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=171

$$-\frac{(a-b)^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d}-\frac{\sqrt{b}\,(15a^2-20ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{8d}-$$

```
[Out] -(a-b)^(5/2)*arctan(cot(d*x+c)*(a-b)^(1/2)/(a+b*cot(d*x+c)^2)^(1/2))/d-1/4*
b*cot(d*x+c)*(a+b*cot(d*x+c)^2)^(3/2)/d-1/8*(15*a^2-20*a*b+8*b^2)*arctanh(c
ot(d*x+c)*b^(1/2)/(a+b*cot(d*x+c)^2)^(1/2))*b^(1/2)/d-1/8*(7*a-4*b)*b*cot(d
*x+c)*(a+b*cot(d*x+c)^2)^(1/2)/d
```

Rubi [A]
time = 0.13, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3742, 427, 542, 537, 223, 212, 385, 209}

$$-\frac{\sqrt{b}\,(15a^2-20ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{8d}-\frac{(a-b)^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d}-\frac{b\cot(c+dx)\left(a+b\cot^2(c+dx)\right)^{3/2}}{4d}-\frac{b(7a-4b)\cot(c+dx)\sqrt{a+b\cot^2(c+dx)}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[c + d*x]^2)^(5/2),x]
```

```
[Out] -(((a - b)^(5/2)*ArcTan[(Sqrt[a - b]*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]^
2]])/d) - (Sqrt[b]*(15*a^2 - 20*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Cot[c + d*x])
/Sqrt[a + b*Cot[c + d*x]^2]])/(8*d) - ((7*a - 4*b)*b*Cot[c + d*x]*Sqrt[a +
b*Cot[c + d*x]^2])/(8*d) - (b*Cot[c + d*x]*(a + b*Cot[c + d*x]^2)^(3/2))/(4
*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \left(a+b\cot^2(c+dx)\right)^{5/2}\,dx = -\frac{\text{Subst}\left(\int \frac{(a+bx^2)^{5/2}}{1+x^2}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{b\cot(c+dx)\left(a+b\cot^2(c+dx)\right)^{3/2}}{4d} - \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}\,(a(4a-b)+(7a-4b}{1+x^2}\right.}{4d}$$

$$= -\frac{(7a-4b)b\cot(c+dx)\sqrt{a+b\cot^2(c+dx)}}{8d} - \frac{b\cot(c+dx)\left(a+b\cot^2(c\right.}{4d}$$

$$= -\frac{(7a-4b)b\cot(c+dx)\sqrt{a+b\cot^2(c+dx)}}{8d} - \frac{b\cot(c+dx)\left(a+b\cot^2(c\right.}{4d}$$

$$= -\frac{(7a-4b)b\cot(c+dx)\sqrt{a+b\cot^2(c+dx)}}{8d} - \frac{b\cot(c+dx)\left(a+b\cot^2(c\right.}{4d}$$

$$= -\frac{(a-b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a-b}\,\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d} - \frac{\sqrt{b}\,(15a^2-20ab+8b^2)\tan}{}$$

Mathematica [A]
time = 0.88, size = 169, normalized size = 0.99

$$\frac{8(a-b)^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}+\sqrt{b}\,\cot^2(c+dx)-\cot(c+dx)\sqrt{a+b\cot^2(c+dx)}}{\sqrt{a-b}}\right) - b\cot(c+dx)\sqrt{a+b\cot^2(c+dx)}\,(9a-4b+2b\cot^2(c+dx)) + \sqrt{b}\,(15a^2-20ab+8b^2)\log\left(-\sqrt{b}\,\cot(c+dx)+\sqrt{a+b\cot^2(c+dx)}\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[c + d*x]^2)^(5/2),x]
```

```
[Out] (8*(a - b)^(5/2)*ArcTan[(Sqrt[b] + Sqrt[b]*Cot[c + d*x]^2 - Cot[c + d*x]*Sq
rt[a + b*Cot[c + d*x]^2])/Sqrt[a - b]] - b*Cot[c + d*x]*Sqrt[a + b*Cot[c +
d*x]^2]*(9*a - 4*b + 2*b*Cot[c + d*x]^2) + Sqrt[b]*(15*a^2 - 20*a*b + 8*b^2
)*Log[-(Sqrt[b]*Cot[c + d*x]) + Sqrt[a + b*Cot[c + d*x]^2]])/(8*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 579 vs. $2(149) = 298$.
time = 0.33, size = 580, normalized size = 3.39

method	result

derivativedivides	$\frac{-b^3\left(\frac{\left(\cot^3(dx+c)\right)\sqrt{a+b\left(\cot^2(dx+c)\right)}}{4b}-\frac{3a\left(\frac{\cot(dx+c)\sqrt{a+b\left(\cot^2(dx+c)\right)}}{2b}-\frac{a\ln\left(\sqrt{b}\cot(dx+c)\right.}{}\right.}{4b}\right.}{}$
default	$\frac{-b^3\left(\frac{\left(\cot^3(dx+c)\right)\sqrt{a+b\left(\cot^2(dx+c)\right)}}{4b}-\frac{3a\left(\frac{\cot(dx+c)\sqrt{a+b\left(\cot^2(dx+c)\right)}}{2b}-\frac{a\ln\left(\sqrt{b}\cot(dx+c)\right.}{}\right.}{4b}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(d*x+c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b^3*(1/4*cot(d*x+c)^3/b*(a+b*cot(d*x+c)^2)^(1/2)-3/4*a/b*(1/2*cot(d*x
+c)/b*(a+b*cot(d*x+c)^2)^(1/2)-1/2*a/b^(3/2)*ln(b^(1/2)*cot(d*x+c)+(a+b*cot
(d*x+c)^2)^(1/2)))-1/2*cot(d*x+c)/b*(a+b*cot(d*x+c)^2)^(1/2)+1/2*a/b^(3/2)*
ln(b^(1/2)*cot(d*x+c)+(a+b*cot(d*x+c)^2)^(1/2))+ln(b^(1/2)*cot(d*x+c)+(a+b*
cot(d*x+c)^2)^(1/2))/b^(1/2)-(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(
b^4*(a-b))^(1/2)/(a+b*cot(d*x+c)^2)^(1/2)*cot(d*x+c)))-3*a*b^2*(1/2*cot(d*x
+c)/b*(a+b*cot(d*x+c)^2)^(1/2)-1/2*a/b^(3/2)*ln(b^(1/2)*cot(d*x+c)+(a+b*cot
(d*x+c)^2)^(1/2))-ln(b^(1/2)*cot(d*x+c)+(a+b*cot(d*x+c)^2)^(1/2))/b^(1/2)+(
b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*cot(d*x+
c)^2)^(1/2)*cot(d*x+c)))-3*a^2*b*(ln(b^(1/2)*cot(d*x+c)+(a+b*cot(d*x+c)^2)^
(1/2))/b^(1/2)-(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/
2)/(a+b*cot(d*x+c)^2)^(1/2)*cot(d*x+c)))-a^3*(b^4*(a-b))^(1/2)/b^2/(a-b)*ar
ctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*cot(d*x+c)^2)^(1/2)*cot(d*x+c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cot(d*x + c)^2 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(149) = 298$.
time = 3.82, size = 1520, normalized size = 8.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*(a^2 - 2*a*b + b^2 - (a^2 - 2*a*b + b^2)*cos(2*d*x + 2*c))*sqrt(-
a + b)*log(-(a - b)*cos(2*d*x + 2*c) + sqrt(-a + b)*sqrt(((a - b)*cos(2*d*x
 + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c) + b)*sin(2*d*x +
2*c) + (15*a^2 - 20*a*b + 8*b^2 - (15*a^2 - 20*a*b + 8*b^2)*cos(2*d*x + 2*c
))*sqrt(b)*log(((a - 2*b)*cos(2*d*x + 2*c) + 2*sqrt(b)*sqrt(((a - b)*cos(2*
d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c) - a - 2*b)/(co
s(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c) - 2*(4*b^2*cos(2*d*x + 2*c) - 3*(3*a*
b - 2*b^2)*cos(2*d*x + 2*c)^2 + 9*a*b - 2*b^2)*sqrt(((a - b)*cos(2*d*x + 2*
c) - a - b)/(cos(2*d*x + 2*c) - 1)))/((d*cos(2*d*x + 2*c) - d)*sin(2*d*x +
2*c)), 1/16*(16*(a^2 - 2*a*b + b^2 - (a^2 - 2*a*b + b^2)*cos(2*d*x + 2*c))*
sqrt(a - b)*arctan(-sqrt(a - b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(co
s(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c)/((a - b)*cos(2*d*x + 2*c) + a - b))*s
in(2*d*x + 2*c) - (15*a^2 - 20*a*b + 8*b^2 - (15*a^2 - 20*a*b + 8*b^2)*cos(
2*d*x + 2*c))*sqrt(b)*log(((a - 2*b)*cos(2*d*x + 2*c) + 2*sqrt(b)*sqrt(((a
- b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c) - a
 - 2*b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c) + 2*(4*b^2*cos(2*d*x + 2*c
) - 3*(3*a*b - 2*b^2)*cos(2*d*x + 2*c)^2 + 9*a*b - 2*b^2)*sqrt(((a - b)*cos
(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1)))/((d*cos(2*d*x + 2*c) - d)*s
in(2*d*x + 2*c)), -1/8*((15*a^2 - 20*a*b + 8*b^2 - (15*a^2 - 20*a*b + 8*b^2
)*cos(2*d*x + 2*c))*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(2*d*x + 2*c)
 - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c)/(b*cos(2*d*x + 2*c) + b)
)*sin(2*d*x + 2*c) + 4*(a^2 - 2*a*b + b^2 - (a^2 - 2*a*b + b^2)*cos(2*d*x +
 2*c))*sqrt(-a + b)*log(-(a - b)*cos(2*d*x + 2*c) + sqrt(-a + b)*sqrt(((a -
 b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c) + b)
*sin(2*d*x + 2*c) - (4*b^2*cos(2*d*x + 2*c) - 3*(3*a*b - 2*b^2)*cos(2*d*x +
 2*c)^2 + 9*a*b - 2*b^2)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x
 + 2*c) - 1)))/((d*cos(2*d*x + 2*c) - d)*sin(2*d*x + 2*c)), 1/8*(8*(a^2 - 2
*a*b + b^2 - (a^2 - 2*a*b + b^2)*cos(2*d*x + 2*c))*sqrt(a - b)*arctan(-sqrt
(a - b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin
(2*d*x + 2*c)/((a - b)*cos(2*d*x + 2*c) + a - b))*sin(2*d*x + 2*c) - (15*a^
2 - 20*a*b + 8*b^2 - (15*a^2 - 20*a*b + 8*b^2)*cos(2*d*x + 2*c))*sqrt(-b)*a
rctan(sqrt(-b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) -
1))*sin(2*d*x + 2*c)/(b*cos(2*d*x + 2*c) + b))*sin(2*d*x + 2*c) + (4*b^2*co
s(2*d*x + 2*c) - 3*(3*a*b - 2*b^2)*cos(2*d*x + 2*c)^2 + 9*a*b - 2*b^2)*sqrt
(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1)))/((d*cos(2*d*x
+ 2*c) - d)*sin(2*d*x + 2*c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\cot^2\left(c + dx\right)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)**2)**(5/2),x)
```

```
[Out] Integral((a + b*cot(c + d*x)**2)**(5/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,{\mathrm{cot}\left(c+d\,x\right)}^2+a\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(c + d*x)^2)^(5/2),x)
```

```
[Out] int((a + b*cot(c + d*x)^2)^(5/2), x)
```

3.32 $\int \left(a+b\cot^2(c+dx)\right)^{3/2}\,dx$

Optimal. Leaf size=126

$$-\frac{(a-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{a-b}\,\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d}-\frac{(3a-2b)\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{2d}-\frac{b\cot(c+}{}$$

[Out] -(a-b)^(3/2)*arctan(cot(d*x+c)*(a-b)^(1/2)/(a+b*cot(d*x+c)^2)^(1/2))/d-1/2*(3*a-2*b)*arctanh(cot(d*x+c)*b^(1/2)/(a+b*cot(d*x+c)^2)^(1/2))*b^(1/2)/d-1/2*b*cot(d*x+c)*(a+b*cot(d*x+c)^2)^(1/2)/d

Rubi [A]
time = 0.07, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3742, 427, 537, 223, 212, 385, 209}

$$-\frac{(a-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{a-b}\,\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d}-\frac{b\cot(c+dx)\sqrt{a+b\cot^2(c+dx)}}{2d}-\frac{\sqrt{b}\,(3a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[c + d*x]^2)^(3/2),x]
```

```
[Out] -(((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]^2]])/d) - ((3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]^2]])/(2*d) - (b*Cot[c + d*x]*Sqrt[a + b*Cot[c + d*x]^2])/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \left(a+b\cot^2(c+dx)\right)^{3/2}\,dx = -\frac{\text{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{1+x^2}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{b\cot(c+dx)\sqrt{a+b\cot^2(c+dx)}}{2d} - \frac{\text{Subst}\left(\int \frac{a(2a-b)+(3a-2b)bx^2}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \cot\right.}{2d}$$

$$= -\frac{b\cot(c+dx)\sqrt{a+b\cot^2(c+dx)}}{2d} - \frac{(a-b)^2\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,d\right.}{d}$$

$$= -\frac{b\cot(c+dx)\sqrt{a+b\cot^2(c+dx)}}{2d} - \frac{(a-b)^2\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, -\right.}{d}$$

$$= -\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\,\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d} - \frac{(3a-2b)\sqrt{b}\,\tanh^{-1}\left(\frac{}{\sqrt{a}}\right.}{2d}$$

Mathematica [A]
time = 0.48, size = 143, normalized size = 1.13

$$\frac{2(a-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}+\sqrt{b}\,\cot^2(c+dx)-\cot(c+dx)\sqrt{a+b\cot^2(c+dx)}}{\sqrt{a-b}}\right) - b\cot(c+dx)\sqrt{a+b\cot^2(c+dx)} + (3a-2b)\sqrt{b}\,\log\left(-\sqrt{b}\,\cot(c+dx)+\sqrt{a+b\cot^2(c+dx)}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[c + d*x]^2)^(3/2),x]
```

```
[Out] (2*(a - b)^(3/2)*ArcTan[(Sqrt[b] + Sqrt[b]*Cot[c + d*x]^2 - Cot[c + d*x]*Sq
rt[a + b*Cot[c + d*x]^2])/Sqrt[a - b]] - b*Cot[c + d*x]*Sqrt[a + b*Cot[c +
d*x]^2] + (3*a - 2*b)*Sqrt[b]*Log[-(Sqrt[b]*Cot[c + d*x]) + Sqrt[a + b*Cot[
c + d*x]^2]])/(2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(108) = 216$.
time = 0.23, size = 329, normalized size = 2.61

method	result

derivativedivides	$\frac{-b^2\left(\frac{\cot(dx+c)\sqrt{a+b\left(\cot^2(dx+c)\right)}}{2b}-\frac{a\ln\left(\sqrt{b}\,\cot(dx+c)+\sqrt{a+b\left(\cot^2(dx+c)\right)}\right)}{2b^{\frac{3}{2}}}-\frac{\ln\left(\sqrt{b}\,\cot(\right.}{}\right.}{}$
default	$\frac{-b^2\left(\frac{\cot(dx+c)\sqrt{a+b\left(\cot^2(dx+c)\right)}}{2b}-\frac{a\ln\left(\sqrt{b}\,\cot(dx+c)+\sqrt{a+b\left(\cot^2(dx+c)\right)}\right)}{2b^{\frac{3}{2}}}-\frac{\ln\left(\sqrt{b}\,\cot(\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b^2*(1/2*cot(d*x+c)/b*(a+b*cot(d*x+c)^2)^(1/2)-1/2*a/b^(3/2)*ln(b^(1/
2)*cot(d*x+c)+(a+b*cot(d*x+c)^2)^(1/2))-ln(b^(1/2)*cot(d*x+c)+(a+b*cot(d*x+
c)^2)^(1/2))/b^(1/2)+(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b
))^(1/2)/(a+b*cot(d*x+c)^2)^(1/2)*cot(d*x+c)))-2*a*b*(ln(b^(1/2)*cot(d*x+c)
+(a+b*cot(d*x+c)^2)^(1/2))/b^(1/2)-(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(
a-b)/(b^4*(a-b))^(1/2)/(a+b*cot(d*x+c)^2)^(1/2)*cot(d*x+c)))-a^2*(b^4*(a-b)
)^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*cot(d*x+c)^2)^(1/
2)*cot(d*x+c)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cot(d*x + c)^2 + a)^(3/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(108) = 216$.
time = 3.49, size = 1071, normalized size = 8.50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(a - b)*sqrt(-a + b)*log(-(a - b)*cos(2*d*x + 2*c) - sqrt(-a + b)*
sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x +
```

```
 2*c) + b)*sin(2*d*x + 2*c) + (3*a - 2*b)*sqrt(b)*log(((a - 2*b)*cos(2*d*x
+ 2*c) - 2*sqrt(b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c
) - 1))*sin(2*d*x + 2*c) - a - 2*b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c
) + 2*(b*cos(2*d*x + 2*c) + b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos
(2*d*x + 2*c) - 1)))/(d*sin(2*d*x + 2*c)), 1/2*((3*a - 2*b)*sqrt(-b)*arctan
(sqrt(-b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*s
in(2*d*x + 2*c)/(b*cos(2*d*x + 2*c) + b))*sin(2*d*x + 2*c) - (a - b)*sqrt(-
a + b)*log(-(a - b)*cos(2*d*x + 2*c) - sqrt(-a + b)*sqrt(((a - b)*cos(2*d*x
 + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c) + b)*sin(2*d*x +
2*c) - (b*cos(2*d*x + 2*c) + b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(co
s(2*d*x + 2*c) - 1)))/(d*sin(2*d*x + 2*c)), -1/4*(4*(a - b)^(3/2)*arctan(-s
qrt(a - b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*
sin(2*d*x + 2*c)/((a - b)*cos(2*d*x + 2*c) + a - b))*sin(2*d*x + 2*c) + (3*
a - 2*b)*sqrt(b)*log(((a - 2*b)*cos(2*d*x + 2*c) - 2*sqrt(b)*sqrt(((a - b)*
cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c) - a - 2*
b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c) + 2*(b*cos(2*d*x + 2*c) + b)*sq
rt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1)))/(d*sin(2*d*x
 + 2*c)), -1/2*(2*(a - b)^(3/2)*arctan(-sqrt(a - b)*sqrt(((a - b)*cos(2*d*x
 + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c)/((a - b)*cos(2*d*
x + 2*c) + a - b))*sin(2*d*x + 2*c) - (3*a - 2*b)*sqrt(-b)*arctan(sqrt(-b)*
sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x +
 2*c)/(b*cos(2*d*x + 2*c) + b))*sin(2*d*x + 2*c) + (b*cos(2*d*x + 2*c) + b)
*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1)))/(d*sin(2*
d*x + 2*c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\cot^2\left(c + dx\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)**2)**(3/2),x)
```

```
[Out] Integral((a + b*cot(c + d*x)**2)**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
```

```
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\cot(c+d\,x)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(c + d*x)^2)^(3/2),x)
```

```
[Out] int((a + b*cot(c + d*x)^2)^(3/2), x)
```

3.33 $\int \sqrt{a+b\cot^2(c+dx)}\,dx$

Optimal. Leaf size=87

$$-\frac{\sqrt{a-b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d}-\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d}$$

```
[Out] -arctan(cot(d*x+c)*(a-b)^(1/2)/(a+b*cot(d*x+c)^2)^(1/2))*(a-b)^(1/2)/d-arct
anh(cot(d*x+c)*b^(1/2)/(a+b*cot(d*x+c)^2)^(1/2))*b^(1/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3742, 399, 223, 212, 385, 209}

$$-\frac{\sqrt{a-b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d}-\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cot[c + d*x]^2],x]
```

```
[Out] -((Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]^2]
])/d) - (Sqrt[b]*ArcTanh[(Sqrt[b]*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]^2]]
)/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{a+b\cot^2(c+dx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{1+x^2}\,dx, x, \cot(c+dx)\right)}{d}\\
&= -\frac{(a-b)\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \cot(c+dx)\right)}{d} - \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\right.}{d}\\
&= -\frac{(a-b)\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d} - \frac{b\text{Subst}\left(\int \frac{1}{1-bx^2}\right.}{}\\
&= -\frac{\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d} - \frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\cot(c+dx)}{\sqrt{a+b\cot^2(c+d}\right.}{d}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 107, normalized size = 1.23

$$
\frac{\sqrt{a-b}\,\text{ArcTan}\left(\frac{\sqrt{b}+\sqrt{b}\,\cot^2(c+dx)-\cot(c+dx)\sqrt{a+b\cot^2(c+dx)}}{\sqrt{a-b}}\right)+\sqrt{b}\,\log\left(-\sqrt{b}\,\cot(c+dx)+\sqrt{a+b\cot^2(c+dx)}\right)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cot[c + d*x]^2],x]
```

```
[Out] (Sqrt[a - b]*ArcTan[(Sqrt[b] + Sqrt[b]*Cot[c + d*x]^2 - Cot[c + d*x]*Sqrt[a
 + b*Cot[c + d*x]^2])/Sqrt[a - b]] + Sqrt[b]*Log[-(Sqrt[b]*Cot[c + d*x]) +
Sqrt[a + b*Cot[c + d*x]^2]])/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(75) = 150$.
time = 0.24, size = 169, normalized size = 1.94

method	result
derivativedivides	$\frac{-b\left(\frac{\ln\left(\sqrt{b}\,\cot(dx+c)+\sqrt{a+b\left(\cot^2(dx+c)\right)}\right)}{\sqrt{b}}-\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\cot(dx+}{\sqrt{b^4(a-b)}\,\sqrt{a+b(c}}\right.}{b^2(a-b)}\right.}{d}$
default	$\frac{-b\left(\frac{\ln\left(\sqrt{b}\,\cot(dx+c)+\sqrt{a+b\left(\cot^2(dx+c)\right)}\right)}{\sqrt{b}}-\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\cot(dx+}{\sqrt{b^4(a-b)}\,\sqrt{a+b(c}}\right.}{b^2(a-b)}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b*(ln(b^(1/2)*cot(d*x+c)+(a+b*cot(d*x+c)^2)^(1/2))/b^(1/2)-(b^4*(a-b)
)^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*cot(d*x+c)^2)^(1/
2)*cot(d*x+c)))-a*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^
(1/2)/(a+b*cot(d*x+c)^2)^(1/2)*cot(d*x+c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(75) = 150$.

time = 3.12, size = 703, normalized size = 8.08

$$\left[\frac{\sqrt{-a+b}\log\left(-(a-b)\cos\left(2\,dx+2\,c\right)+\sqrt{-a+b}\sqrt{\frac{(a-b)\cos\left(2\,dx+2\,c\right)-a-b}{\cos\left(2\,dx+2\,c\right)-1}}\sin\left(2\,dx+2\,c\right)+b\right)+\sqrt{b}\log\left(\frac{(a-2b)\cos(2dx+2c)+2\sqrt{b}\sqrt{\frac{(a-b)\cos\left(2\,dx+2\,c\right)-a-b}{\cos\left(2\,dx+2\,c\right)-1}}\sin(2dx+2c)-a-2b}{\cos(2dx+2c)-1}\right)}{2\,d}, -\frac{2\sqrt{a-b}\arctan\left(-\frac{\sqrt{a-b}\sqrt{\frac{(a-b)\cos\left(2\,dx+2\,c\right)-a-b}{\cos\left(2\,dx+2\,c\right)-1}}\sin(2\,dx+2\,c)}{(a-b)\cos(2dx+2c)+a-b}\right)-\sqrt{b}\log\left(\frac{(a-2b)\cos(2dx+2c)+2\sqrt{b}\sqrt{\frac{(a-b)\cos\left(2\,dx+2\,c\right)-a-b}{\cos\left(2\,dx+2\,c\right)-1}}\sin(2dx+2c)-a-2b}{\cos(2dx+2c)-1}\right)}{2\,d}, \frac{2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{(a-b)\cos\left(2\,dx+2\,c\right)-a-b}{\cos\left(2\,dx+2\,c\right)-1}}\sin(2\,dx+2\,c)}{b\cos(2dx+2c)+b}\right)+\sqrt{-a+b}\log\left(-(a-b)\cos\left(2\,dx+2\,c\right)+\sqrt{-a+b}\sqrt{\frac{(a-b)\cos\left(2\,dx+2\,c\right)-a-b}{\cos\left(2\,dx+2\,c\right)-1}}\sin\left(2\,dx+2\,c\right)+b\right)}{2\,d}, -\frac{\sqrt{a-b}\arctan\left(-\frac{\sqrt{a-b}\sqrt{\frac{(a-b)\cos\left(2\,dx+2\,c\right)-a-b}{\cos\left(2\,dx+2\,c\right)-1}}\sin(2\,dx+2\,c)}{(a-b)\cos(2dx+2c)+a-b}\right)-\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{(a-b)\cos\left(2\,dx+2\,c\right)-a-b}{\cos\left(2\,dx+2\,c\right)-1}}\sin(2\,dx+2\,c)}{b\cos(2dx+2c)+b}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a + b)*log(-(a - b)*cos(2*d*x + 2*c) + sqrt(-a + b)*sqrt(((a -
b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c) + b)
+ sqrt(b)*log(((a - 2*b)*cos(2*d*x + 2*c) + 2*sqrt(b)*sqrt(((a - b)*cos(2*d
*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c) - a - 2*b)/(cos
(2*d*x + 2*c) - 1)))/d, -1/2*(2*sqrt(a - b)*arctan(-sqrt(a - b)*sqrt(((a -
b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c)/((a -
 b)*cos(2*d*x + 2*c) + a - b)) - sqrt(b)*log(((a - 2*b)*cos(2*d*x + 2*c) +
2*sqrt(b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*s
in(2*d*x + 2*c) - a - 2*b)/(cos(2*d*x + 2*c) - 1)))/d, 1/2*(2*sqrt(-b)*arct
an(sqrt(-b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))
*sin(2*d*x + 2*c)/(b*cos(2*d*x + 2*c) + b)) + sqrt(-a + b)*log(-(a - b)*cos
(2*d*x + 2*c) + sqrt(-a + b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2
*d*x + 2*c) - 1))*sin(2*d*x + 2*c) + b))/d, -(sqrt(a - b)*arctan(-sqrt(a -
b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*
x + 2*c)/((a - b)*cos(2*d*x + 2*c) + a - b)) - sqrt(-b)*arctan(sqrt(-b)*sqr
t(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*
c)/(b*cos(2*d*x + 2*c) + b)))/d]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \cot^{2}{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cot(c + d*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
```

```
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{b\cot\left(c+d\,x\right)^2+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(c + d*x)^2)^(1/2),x)
[Out] int((a + b*cot(c + d*x)^2)^(1/2), x)
```

3.34 $\int \frac{1}{\sqrt{a+b\cot^2(c+dx)}}\,dx$

Optimal. Leaf size=47

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{\sqrt{a-b}\,d}$$

[Out] -arctan(cot(d*x+c)*(a-b)^(1/2)/(a+b*cot(d*x+c)^2)^(1/2))/d/(a-b)^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3742, 385, 209}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d\sqrt{a-b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a + b*Cot[c + d*x]^2],x]

[Out] -(ArcTan[(Sqrt[a - b]*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]^2]]/(Sqrt[a - b
]*d))

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
```

```
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cot^2(c+dx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{\sqrt{a-b}\,d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 111 vs. $2(47) = 94$.
time = 0.48, size = 111, normalized size = 2.36

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{-\frac{(a-b)\cot^2(c+dx)}{a}}}{\sqrt{1+\frac{b\cot^2(c+dx)}{a}}}\right)\cot(c+dx)\sqrt{1+\frac{b\cot^2(c+dx)}{a}}}{d\sqrt{-\frac{(a-b)\cot^2(c+dx)}{a}}\,\sqrt{a+b\cot^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Cot[c + d*x]^2],x]
```

```
[Out] -((ArcTanh[Sqrt[-(((a - b)*Cot[c + d*x]^2)/a)]/Sqrt[1 + (b*Cot[c + d*x]^2)/
a]]*Cot[c + d*x]*Sqrt[1 + (b*Cot[c + d*x]^2)/a])/(d*Sqrt[-(((a - b)*Cot[c +
 d*x]^2)/a)]*Sqrt[a + b*Cot[c + d*x]^2]))
```

Maple [A]
time = 0.32, size = 68, normalized size = 1.45

method	result	size

derivativedivides	$-\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\cot(dx+c)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\left(\cot^2(dx+c)\right)}}\right)}{d\,b^2(a-b)}$	68
default	$-\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\cot(dx+c)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\left(\cot^2(dx+c)\right)}}\right)}{d\,b^2(a-b)}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cot(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*co
t(d*x+c)^2)^(1/2)*cot(d*x+c))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(41) = 82$.
time = 3.94, size = 239, normalized size = 5.09

$$\left[-\frac{\sqrt{-a+b}\,\log\left(-2\,(a^2-2\,ab+b^2)\cos(2\,dx+2\,c)^2-2\,((a-b)\cos(2\,dx+2\,c)-b)\sqrt{-a+b}\,\sqrt{\frac{(a-b)\cos(2\,dx+2\,c)-a-b}{\cos(2\,dx+2\,c)-1}}\,\sin(2\,dx+2\,c)+a^2-2\,b^2+4\,(ab-b^2)\cos(2\,dx+2\,c)\right)}{4\,(a-b)d},\ -\frac{\arctan\left(-\frac{\sqrt{a-b}\,\sqrt{\frac{(a-b)\cos(2\,dx+2\,c)-a-b}{\cos(2\,dx+2\,c)-1}}\,\sin(2\,dx+2\,c)}{(a-b)\cos(2\,dx+2\,c)-b}\right)}{2\,\sqrt{a-b}\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-a + b)*log(-2*(a^2 - 2*a*b + b^2)*cos(2*d*x + 2*c)^2 - 2*((a -
b)*cos(2*d*x + 2*c) - b)*sqrt(-a + b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a -
b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c) + a^2 - 2*b^2 + 4*(a*b - b^2)*c
os(2*d*x + 2*c))/((a - b)*d), -1/2*arctan(-sqrt(a - b)*sqrt(((a - b)*cos(2*
d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c)/((a - b)*cos(2
*d*x + 2*c) - b))/(sqrt(a - b)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\cot^2(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*cot(c + d*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(41) = 82$.
time = 1.38, size = 97, normalized size = 2.06

$$\frac{2\arctan\left(-\frac{\sqrt{b}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\sqrt{b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+4a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+b}+\sqrt{b}}{2\sqrt{a-b}}\right)}{\sqrt{a-b}\,d\,\mathrm{sgn}\left(\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-1/2*(sqrt(b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(b*tan(1/2*d*x + 1/2*c)
^4 + 4*a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c)^2 + b) + sqrt(b)
)/sqrt(a - b))/(sqrt(a - b)*d*sgn(sin(d*x + c)))
```

Mupad [B]
time = 0.85, size = 41, normalized size = 0.87

$$-\frac{\mathrm{atan}\left(\frac{\cot(c+d\,x)\,\sqrt{a-b}}{\sqrt{b\cot\left(c+d\,x\right)^2+a}}\right)}{d\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cot(c + d*x)^2)^(1/2),x)
```

```
[Out] -atan((cot(c + d*x)*(a - b)^(1/2))/(a + b*cot(c + d*x)^2)^(1/2))/(d*(a - b)
^(1/2))
```

3.35 $\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=85

$$-\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\,\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{(a-b)^{3/2}d}+\frac{b\cot(c+dx)}{a(a-b)d\sqrt{a+b\cot^2(c+dx)}}$$

```
[Out] -arctan(cot(d*x+c)*(a-b)^(1/2)/(a+b*cot(d*x+c)^2)^(1/2))/(a-b)^(3/2)/d+b*co
t(d*x+c)/a/(a-b)/d/(a+b*cot(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3742, 390, 385, 209}

$$\frac{b\cot(c+dx)}{ad(a-b)\sqrt{a+b\cot^2(c+dx)}}-\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\,\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[c + d*x]^2)^(-3/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]^2]]/((a - b)^(3
/2)*d)) + (b*Cot[c + d*x])/(a*(a - b)*d*Sqrt[a + b*Cot[c + d*x]^2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
```

```
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^{3/2}}\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \cot(c+dx)\right)}{d} \\
&= \frac{b\cot(c+dx)}{a(a-b)d\sqrt{a+b\cot^2(c+dx)}} - \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \cot(c+dx)\right)}{(a-b)d} \\
&= \frac{b\cot(c+dx)}{a(a-b)d\sqrt{a+b\cot^2(c+dx)}} - \frac{\operatorname{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{(a-b)d} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{(a-b)^{3/2}d} + \frac{b\cot(c+dx)}{a(a-b)d\sqrt{a+b\cot^2(c+dx)}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 3.76, size = 231, normalized size = 2.72

$$
-\frac{\cos^2(c+dx)\cot(c+dx)\left(4(a-b)^2\cos^2(c+dx)\,{}_2F_1\left(2,2;\frac{7}{2};\frac{(a-b)\cos^2(c+dx)}{a}\right)\left(b+a\tan^2(c+dx)\right)-\frac{15a\left(2b+3a\tan^2(c+dx)\right)\left(\operatorname{ArcSin}\left(\sqrt{\frac{(a-b)\cos^2(c+dx)}{a}}\right)\left(b+a\tan^2(c+dx)\right)-a\sec^2(c+dx)\sqrt{\frac{(a-b)\cos^4(c+dx)\left(b+a\tan^2(c+dx)\right)}{a^2}}\right)}{\sqrt{\frac{(a-b)\cos^4(c+dx)\left(b+a\tan^2(c+dx)\right)}{a^2}}}\right)}{15a^3(a-b)d\sqrt{a+b\cot^2(c+dx)}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cot[c + d*x]^2)^(-3/2),x]
```

```
[Out] -1/15*(Cos[c + d*x]^2*Cot[c + d*x]*(4*(a - b)^2*Cos[c + d*x]^2*Hypergeometr
ic2F1[2, 2, 7/2, ((a - b)*Cos[c + d*x]^2)/a]*(b + a*Tan[c + d*x]^2) - (15*a
*(2*b + 3*a*Tan[c + d*x]^2)*(ArcSin[Sqrt[((a - b)*Cos[c + d*x]^2)/a]]*(b +
a*Tan[c + d*x]^2) - a*Sec[c + d*x]^2*Sqrt[((a - b)*Cos[c + d*x]^4*(b + a*Ta
n[c + d*x]^2))/a^2]))/Sqrt[((a - b)*Cos[c + d*x]^4*(b + a*Tan[c + d*x]^2))/
a^2]))/(a^3*(a - b)*d*Sqrt[a + b*Cot[c + d*x]^2])
```

Maple [A]
time = 0.20, size = 102, normalized size = 1.20

method	result
derivativedivides	$\frac{\frac{b\cot(dx+c)}{(a-b)a\sqrt{a+b\left(\cot^2(dx+c)\right)}}-\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\cot(dx+c)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\left(\cot^2(dx+c)\right)}}\right)}{(a-b)^2b^2}}{d}$
default	$\frac{\frac{b\cot(dx+c)}{(a-b)a\sqrt{a+b\left(\cot^2(dx+c)\right)}}-\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\cot(dx+c)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\left(\cot^2(dx+c)\right)}}\right)}{(a-b)^2b^2}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cot(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a-b)*b*cot(d*x+c)/a/(a+b*cot(d*x+c)^2)^(1/2)-1/(a-b)^2*(b^4*(a-b))^
(1/2)/b^2*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*cot(d*x+c)^2)^(1/2)*cot(d
*x+c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(77) = 154$.
time = 3.32, size = 526, normalized size = 6.19

$$\left[-\frac{(a^2+ab-(a^2-ab)\cos(2dx+2c))\sqrt{-a+b}\log\left(-2(a^2-2ab+b^2)\cos(2dx+2c)^2+2((a-b)\cos(2dx+2c)-b)\sqrt{-a+b}\sqrt{\frac{(a-b)\cos(2dx+2c)-a-b}{\cos(2dx+2c)-1}}\sin(2dx+2c)+a^2-2b^2+4(ab-b^2)\cos(2dx+2c)\right)+4(ab-b^2)\sqrt{\frac{(a-b)\cos(2dx+2c)-a-b}{\cos(2dx+2c)-1}}\sin(2dx+2c)}{4((a^4-3a^3b+3a^2b^2-ab^3)d\cos(2dx+2c)-(a^4-a^3b-a^2b^2+ab^3)d)}, \frac{(a^2+ab-(a^2-ab)\cos(2dx+2c))\sqrt{a-b}\arctan\left(-\frac{\sqrt{a-b}\sqrt{\frac{(a-b)\cos(2dx+2c)-a-b}{\cos(2dx+2c)-1}}\sin(2dx+2c)}{(a-b)\cos(2dx+2c)-b}\right)-2(ab-b^2)\sqrt{\frac{(a-b)\cos(2dx+2c)-a-b}{\cos(2dx+2c)-1}}\sin(2dx+2c)}{2((a^4-3a^3b+3a^2b^2-ab^3)d\cos(2dx+2c)-(a^4-a^3b-a^2b^2+ab^3)d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((a^2 + a*b - (a^2 - a*b)*cos(2*d*x + 2*c))*sqrt(-a + b)*log(-2*(a^2
- 2*a*b + b^2)*cos(2*d*x + 2*c)^2 + 2*((a - b)*cos(2*d*x + 2*c) - b)*sqrt(-
a + b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(
2*d*x + 2*c) + a^2 - 2*b^2 + 4*(a*b - b^2)*cos(2*d*x + 2*c)) + 4*(a*b - b^2
)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x
 + 2*c))/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) - (a^4 - a
^3*b - a^2*b^2 + a*b^3)*d), 1/2*((a^2 + a*b - (a^2 - a*b)*cos(2*d*x + 2*c))
*sqrt(a - b)*arctan(-sqrt(a - b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(c
os(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c)/((a - b)*cos(2*d*x + 2*c) - b)) - 2*
(a*b - b^2)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))
*sin(2*d*x + 2*c))/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c)
- (a^4 - a^3*b - a^2*b^2 + a*b^3)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\cot^{2}\left(c + dx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)**2)**(3/2),x)
```

```
[Out] Integral((a + b*cot(c + d*x)**2)**(-3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 300 vs. 2(77) = 154.
time = 0.96, size = 300, normalized size = 3.53

$$-\frac{\frac{\frac{\left(a^{2} b \operatorname{sgn}\left(\sin\left(dx+c\right)\right)-2\, a b^{2} \operatorname{sgn}\left(\sin\left(dx+c\right)\right)+b^{3} \operatorname{sgn}\left(\sin\left(dx+c\right)\right)\right) \tan\left(\frac{1}{2}\, dx+\frac{1}{2}\, c\right)^{2}}{a^{4}-3\, a^{3} b+3\, a^{2} b^{2}-a b^{3}}-\frac{a^{2} b \operatorname{sgn}\left(\sin\left(dx+c\right)\right)-2\, a b^{2} \operatorname{sgn}\left(\sin\left(dx+c\right)\right)+b^{3} \operatorname{sgn}\left(\sin\left(dx+c\right)\right)}{a^{4}-3\, a^{3} b+3\, a^{2} b^{2}-a b^{3}}}{\sqrt{b \tan\left(\frac{1}{2}\, dx+\frac{1}{2}\, c\right)^{4}+4\, a \tan\left(\frac{1}{2}\, dx+\frac{1}{2}\, c\right)^{2}-2\, b \tan\left(\frac{1}{2}\, dx+\frac{1}{2}\, c\right)^{2}+b}}-\frac{2\, \arctan\left(-\frac{\sqrt{b} \tan\left(\frac{1}{2}\, dx+\frac{1}{2}\, c\right)^{2}-\sqrt{b \tan\left(\frac{1}{2}\, dx+\frac{1}{2}\, c\right)^{4}+4\, a \tan\left(\frac{1}{2}\, dx+\frac{1}{2}\, c\right)^{2}-2\, b \tan\left(\frac{1}{2}\, dx+\frac{1}{2}\, c\right)^{2}+b}+\sqrt{b}}{2\, \sqrt{a-b}}\right)}{\left(a \operatorname{sgn}\left(\sin\left(dx+c\right)\right)-b \operatorname{sgn}\left(\sin\left(dx+c\right)\right)\right) \sqrt{a-b}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(((a^2*b*sgn(sin(d*x + c)) - 2*a*b^2*sgn(sin(d*x + c)) + b^3*sgn(sin(d*x +
 c)))*tan(1/2*d*x + 1/2*c)^2/(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3) - (a^2*b*s
gn(sin(d*x + c)) - 2*a*b^2*sgn(sin(d*x + c)) + b^3*sgn(sin(d*x + c)))/(a^4
- 3*a^3*b + 3*a^2*b^2 - a*b^3))/sqrt(b*tan(1/2*d*x + 1/2*c)^4 + 4*a*tan(1/2
*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c)^2 + b) - 2*arctan(-1/2*(sqrt(b)*
tan(1/2*d*x + 1/2*c)^2 - sqrt(b*tan(1/2*d*x + 1/2*c)^4 + 4*a*tan(1/2*d*x +
```

```
1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c)^2 + b) + sqrt(b))/sqrt(a - b))/((a*sgn(
sin(d*x + c)) - b*sgn(sin(d*x + c)))*sqrt(a - b)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\cot\left(c+d\,x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cot(c + d*x)^2)^(3/2),x)
[Out] int(1/(a + b*cot(c + d*x)^2)^(3/2), x)
```

3.36 $\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=135

$$-\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{(a-b)^{5/2}d}+\frac{b\cot(c+dx)}{3a(a-b)d\left(a+b\cot^2(c+dx)\right)^{3/2}}+\frac{(5a-2b)b\cot(c+dx)}{3a^2(a-b)^2d\sqrt{a+b\cot^2(c+dx)}}$$

```
[Out] -arctan(cot(d*x+c)*(a-b)^(1/2)/(a+b*cot(d*x+c)^2)^(1/2))/(a-b)^(5/2)/d+1/3*
b*cot(d*x+c)/a/(a-b)/d/(a+b*cot(d*x+c)^2)^(3/2)+1/3*(5*a-2*b)*b*cot(d*x+c)/
a^2/(a-b)^2/d/(a+b*cot(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3742, 425, 541, 12, 385, 209}

$$\frac{b(5a-2b)\cot(c+dx)}{3a^2d(a-b)^2\sqrt{a+b\cot^2(c+dx)}}-\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d(a-b)^{5/2}}+\frac{b\cot(c+dx)}{3ad(a-b)\left(a+b\cot^2(c+dx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[c + d*x]^2)^(-5/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]^2]]/((a - b)^(5
/2)*d)) + (b*Cot[c + d*x])/(3*a*(a - b)*d*(a + b*Cot[c + d*x]^2)^(3/2)) + (
(5*a - 2*b)*b*Cot[c + d*x])/(3*a^2*(a - b)^2*d*Sqrt[a + b*Cot[c + d*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^{5/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^{5/2}}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= \frac{b\cot(c+dx)}{3a(a-b)d\left(a+b\cot^2(c+dx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{3a-2b-2bx^2}{(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \cot(c+\right.}{3a(a-b)d}$$

$$= \frac{b\cot(c+dx)}{3a(a-b)d\left(a+b\cot^2(c+dx)\right)^{3/2}} + \frac{(5a-2b)b\cot(c+dx)}{3a^2(a-b)^2d\sqrt{a+b\cot^2(c+dx)}} -$$

$$= \frac{b\cot(c+dx)}{3a(a-b)d\left(a+b\cot^2(c+dx)\right)^{3/2}} + \frac{(5a-2b)b\cot(c+dx)}{3a^2(a-b)^2d\sqrt{a+b\cot^2(c+dx)}} -$$

$$= \frac{b\cot(c+dx)}{3a(a-b)d\left(a+b\cot^2(c+dx)\right)^{3/2}} + \frac{(5a-2b)b\cot(c+dx)}{3a^2(a-b)^2d\sqrt{a+b\cot^2(c+dx)}} -$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{(a-b)^{5/2}d} + \frac{b\cot(c+dx)}{3a(a-b)d\left(a+b\cot^2(c+dx)\right)^{3/2}} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 8.00, size = 367, normalized size = 2.72

$$-\frac{\cot^5(c+dx)\left(24(a-b)^3\cos^2(c+dx)\,{}_3F_2\left(2,2,2;1,\frac{9}{2};\frac{(a-b)\cos^2(c+dx)}{a}\right)\left(b+a\tan^2(c+dx)\right)^2+24(a-b)^3\cos^2(c+dx)\,{}_2F_1\left(2,2;\frac{9}{2};\frac{(a-b)\cos^2(c+dx)}{a}\right)\left(3b^2+7ab\tan^2(c+dx)+4a^2\tan^4(c+dx)\right)-\frac{35a\left(8b^2+20ab\tan^2(c+dx)+15a^2\tan^4(c+dx)\right)\left(-3\text{ArcSin}\left(\sqrt{\frac{(a-b)\cos^2(c+dx)}{a}}\right)\left(b+a\tan^2(c+dx)\right)^2+a\sec^2(c+dx)\sqrt{\frac{(a-b)\cos^4(c+dx)\left(b+a\tan^2(c+dx)\right)}{a^2}}\left(4b+a\left(-1+3\tan^2(c+dx)\right)\right)\right)}{\sqrt{\frac{(a-b)\cos^4(c+dx)\left(b+a\tan^2(c+dx)\right)}{a^2}}}\right)}{315a^5(a-b)^2d\left(1+\cot^2(c+dx)\right)\sqrt{a+b\cot^2(c+dx)}\left(1+\frac{b\cot^2(c+dx)}{a}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cot[c + d*x]^2)^(-5/2),x]
```

```
[Out] -1/315*(Cot[c + d*x]^5*(24*(a - b)^3*Cos[c + d*x]^2*HypergeometricPFQ[{2, 2
, 2}, {1, 9/2}, ((a - b)*Cos[c + d*x]^2)/a]*(b + a*Tan[c + d*x]^2)^2 + 24*(
a - b)^3*Cos[c + d*x]^2*Hypergeometric2F1[2, 2, 9/2, ((a - b)*Cos[c + d*x]^
2)/a]*(3*b^2 + 7*a*b*Tan[c + d*x]^2 + 4*a^2*Tan[c + d*x]^4) - (35*a*(8*b^2
+ 20*a*b*Tan[c + d*x]^2 + 15*a^2*Tan[c + d*x]^4)*(-3*ArcSin[Sqrt[((a - b)*C
os[c + d*x]^2)/a]]*(b + a*Tan[c + d*x]^2)^2 + a*Sec[c + d*x]^2*Sqrt[((a - b
)*Cos[c + d*x]^4*(b + a*Tan[c + d*x]^2))/a^2]*(4*b + a*(-1 + 3*Tan[c + d*x]
```

```
^2))))/Sqrt[((a - b)*Cos[c + d*x]^4*(b + a*Tan[c + d*x]^2))/a^2]))/(a^5*(a
- b)^2*d*(1 + Cot[c + d*x]^2)*Sqrt[a + b*Cot[c + d*x]^2]*(1 + (b*Cot[c + d*
x]^2)/a))
```

Maple [A]
time = 0.22, size = 162, normalized size = 1.20

method	result
derivativedivides	$\frac{-\frac{\sqrt{b^4(a-b)}\arctan\left(\frac{b^2(a-b)\cot(dx+c)}{\sqrt{b^4(a-b)}\sqrt{a+b\left(\cot^2(dx+c)\right)}}\right)}{(a-b)^3b^2}+\frac{b\left(\frac{\cot(dx+c)}{3a\left(a+b\left(\cot^2(dx+c)\right)\right)^{\frac{3}{2}}}+\frac{}{3a^2\sqrt{a+}}\right.}{a-b}}{d}$
default	$\frac{-\frac{\sqrt{b^4(a-b)}\arctan\left(\frac{b^2(a-b)\cot(dx+c)}{\sqrt{b^4(a-b)}\sqrt{a+b\left(\cot^2(dx+c)\right)}}\right)}{(a-b)^3b^2}+\frac{b\left(\frac{\cot(dx+c)}{3a\left(a+b\left(\cot^2(dx+c)\right)\right)^{\frac{3}{2}}}+\frac{}{3a^2\sqrt{a+}}\right.}{a-b}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cot(d*x+c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/(a-b)^3*(b^4*(a-b))^(1/2)/b^2*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a
+b*cot(d*x+c)^2)^(1/2)*cot(d*x+c))+1/(a-b)*b*(1/3*cot(d*x+c)/a/(a+b*cot(d*x
+c)^2)^(3/2)+2/3/a^2*cot(d*x+c)/(a+b*cot(d*x+c)^2)^(1/2))+1/(a-b)^2*b*cot(d
*x+c)/a/(a+b*cot(d*x+c)^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 423 vs. 2(121) = 242.
time = 3.34, size = 898, normalized size = 6.65

$$\left[-\frac{3\left(a^4+2a^3b+a^2b^2+\left(a^4-2a^3b+a^2b^2\right)\cos(2dx+2c)^2-2\left(a^4-a^2b^2\right)\cos(2dx+2c)\right)\sqrt{-a+b}\log\left(-2\left(a^2-2ab+b^2\right)\cos(2dx+2c)^2-2\left((a-b)\cos(2dx+2c)-b\right)\sqrt{-a+b}\sqrt{\frac{(a-b)\cos(2dx+2c)-a-b}{\cos(2dx+2c)-1}}\sin(2dx+2c)+a^2-2b^2+4\left(ab-b^2\right)\cos(2dx+2c)\right)-8\left(3a^3b-2a^2b^2-2ab^3+b^4-\left(3a^3b-7a^2b^2+5ab^3-b^4\right)\cos(2dx+2c)\right)\sqrt{\frac{(a-b)\cos(2dx+2c)-a-b}{\cos(2dx+2c)-1}}\sin(2dx+2c)}{12\left(\left(a^7-5a^6b+10a^5b^2-10a^4b^3+5a^3b^4-a^2b^5\right)d\cos(2dx+2c)^2-2\left(a^7-3a^6b+2a^5b^2+2a^4b^3-3a^3b^4+a^2b^5\right)d\cos(2dx+2c)+\left(a^7-a^6b-2a^5b^2+2a^4b^3+a^3b^4-a^2b^5\right)d\right)}, -\frac{3\left(a^4+2a^3b+a^2b^2+\left(a^4-2a^3b+a^2b^2\right)\cos(2dx+2c)^2-2\left(a^4-a^2b^2\right)\cos(2dx+2c)\right)\sqrt{a-b}\arctan\left(-\frac{\sqrt{a-b}\sqrt{\frac{(a-b)\cos(2dx+2c)-a-b}{\cos(2dx+2c)-1}}\sin(2dx+2c)}{(a-b)\cos(2dx+2c)-b}\right)-4\left(3a^3b-2a^2b^2-2ab^3+b^4-\left(3a^3b-7a^2b^2+5ab^3-b^4\right)\cos(2dx+2c)\right)\sqrt{\frac{(a-b)\cos(2dx+2c)-a-b}{\cos(2dx+2c)-1}}\sin(2dx+2c)}{6\left(\left(a^7-5a^6b+10a^5b^2-10a^4b^3+5a^3b^4-a^2b^5\right)d\cos(2dx+2c)^2-2\left(a^7-3a^6b+2a^5b^2+2a^4b^3-3a^3b^4+a^2b^5\right)d\cos(2dx+2c)+\left(a^7-a^6b-2a^5b^2+2a^4b^3+a^3b^4-a^2b^5\right)d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(a^4 + 2*a^3*b + a^2*b^2 + (a^4 - 2*a^3*b + a^2*b^2)*cos(2*d*x +
2*c)^2 - 2*(a^4 - a^2*b^2)*cos(2*d*x + 2*c))*sqrt(-a + b)*log(-2*(a^2 - 2*a
*b + b^2)*cos(2*d*x + 2*c)^2 - 2*((a - b)*cos(2*d*x + 2*c) - b)*sqrt(-a + b
)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x
 + 2*c) + a^2 - 2*b^2 + 4*(a*b - b^2)*cos(2*d*x + 2*c)) - 8*(3*a^3*b - 2*a^
2*b^2 - 2*a*b^3 + b^4 - (3*a^3*b - 7*a^2*b^2 + 5*a*b^3 - b^4)*cos(2*d*x + 2
*c))*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*sin(2*
d*x + 2*c))/((a^7 - 5*a^6*b + 10*a^5*b^2 - 10*a^4*b^3 + 5*a^3*b^4 - a^2*b^5
)*d*cos(2*d*x + 2*c)^2 - 2*(a^7 - 3*a^6*b + 2*a^5*b^2 + 2*a^4*b^3 - 3*a^3*b
^4 + a^2*b^5)*d*cos(2*d*x + 2*c) + (a^7 - a^6*b - 2*a^5*b^2 + 2*a^4*b^3 + a
^3*b^4 - a^2*b^5)*d), -1/6*(3*(a^4 + 2*a^3*b + a^2*b^2 + (a^4 - 2*a^3*b + a
^2*b^2)*cos(2*d*x + 2*c)^2 - 2*(a^4 - a^2*b^2)*cos(2*d*x + 2*c))*sqrt(a - b
)*arctan(-sqrt(a - b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x +
2*c) - 1))*sin(2*d*x + 2*c)/((a - b)*cos(2*d*x + 2*c) - b)) - 4*(3*a^3*b -
2*a^2*b^2 - 2*a*b^3 + b^4 - (3*a^3*b - 7*a^2*b^2 + 5*a*b^3 - b^4)*cos(2*d*x
 + 2*c))*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d*x + 2*c) - 1))*si
n(2*d*x + 2*c))/((a^7 - 5*a^6*b + 10*a^5*b^2 - 10*a^4*b^3 + 5*a^3*b^4 - a^2
*b^5)*d*cos(2*d*x + 2*c)^2 - 2*(a^7 - 3*a^6*b + 2*a^5*b^2 + 2*a^4*b^3 - 3*a
^3*b^4 + a^2*b^5)*d*cos(2*d*x + 2*c) + (a^7 - a^6*b - 2*a^5*b^2 + 2*a^4*b^3
 + a^3*b^4 - a^2*b^5)*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\cot^{2}\left(c + dx\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)**2)**(5/2),x)
```

```
[Out] Integral((a + b*cot(c + d*x)**2)**(-5/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1160 vs. 2(121) = 242.
time = 1.49, size = 1160, normalized size = 8.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(((((5*a^9*b^2*sgn(sin(d*x + c)) - 42*a^8*b^3*sgn(sin(d*x + c)) + 156*
a^7*b^4*sgn(sin(d*x + c)) - 336*a^6*b^5*sgn(sin(d*x + c)) + 462*a^5*b^6*sgn
(sin(d*x + c)) - 420*a^4*b^7*sgn(sin(d*x + c)) + 252*a^3*b^8*sgn(sin(d*x +
```

```
c)) - 96*a^2*b^9*sgn(sin(d*x + c)) + 21*a*b^10*sgn(sin(d*x + c)) - 2*b^11*s
gn(sin(d*x + c)))*tan(1/2*d*x + 1/2*c)^2/(a^12 - 10*a^11*b + 45*a^10*b^2 -
120*a^9*b^3 + 210*a^8*b^4 - 252*a^7*b^5 + 210*a^6*b^6 - 120*a^5*b^7 + 45*a^
4*b^8 - 10*a^3*b^9 + a^2*b^10) + 3*(8*a^10*b*sgn(sin(d*x + c)) - 73*a^9*b^2
*sgn(sin(d*x + c)) + 298*a^8*b^3*sgn(sin(d*x + c)) - 716*a^7*b^4*sgn(sin(d*
x + c)) + 1120*a^6*b^5*sgn(sin(d*x + c)) - 1190*a^5*b^6*sgn(sin(d*x + c)) +
 868*a^4*b^7*sgn(sin(d*x + c)) - 428*a^3*b^8*sgn(sin(d*x + c)) + 136*a^2*b^
9*sgn(sin(d*x + c)) - 25*a*b^10*sgn(sin(d*x + c)) + 2*b^11*sgn(sin(d*x + c)
))/(a^12 - 10*a^11*b + 45*a^10*b^2 - 120*a^9*b^3 + 210*a^8*b^4 - 252*a^7*b^
5 + 210*a^6*b^6 - 120*a^5*b^7 + 45*a^4*b^8 - 10*a^3*b^9 + a^2*b^10))*tan(1/
2*d*x + 1/2*c)^2 - 3*(8*a^10*b*sgn(sin(d*x + c)) - 73*a^9*b^2*sgn(sin(d*x +
 c)) + 298*a^8*b^3*sgn(sin(d*x + c)) - 716*a^7*b^4*sgn(sin(d*x + c)) + 1120
*a^6*b^5*sgn(sin(d*x + c)) - 1190*a^5*b^6*sgn(sin(d*x + c)) + 868*a^4*b^7*s
gn(sin(d*x + c)) - 428*a^3*b^8*sgn(sin(d*x + c)) + 136*a^2*b^9*sgn(sin(d*x
+ c)) - 25*a*b^10*sgn(sin(d*x + c)) + 2*b^11*sgn(sin(d*x + c)))/(a^12 - 10*
a^11*b + 45*a^10*b^2 - 120*a^9*b^3 + 210*a^8*b^4 - 252*a^7*b^5 + 210*a^6*b^
6 - 120*a^5*b^7 + 45*a^4*b^8 - 10*a^3*b^9 + a^2*b^10))*tan(1/2*d*x + 1/2*c)
^2 - (5*a^9*b^2*sgn(sin(d*x + c)) - 42*a^8*b^3*sgn(sin(d*x + c)) + 156*a^7*
b^4*sgn(sin(d*x + c)) - 336*a^6*b^5*sgn(sin(d*x + c)) + 462*a^5*b^6*sgn(sin
(d*x + c)) - 420*a^4*b^7*sgn(sin(d*x + c)) + 252*a^3*b^8*sgn(sin(d*x + c))
- 96*a^2*b^9*sgn(sin(d*x + c)) + 21*a*b^10*sgn(sin(d*x + c)) - 2*b^11*sgn(s
in(d*x + c)))/(a^12 - 10*a^11*b + 45*a^10*b^2 - 120*a^9*b^3 + 210*a^8*b^4 -
 252*a^7*b^5 + 210*a^6*b^6 - 120*a^5*b^7 + 45*a^4*b^8 - 10*a^3*b^9 + a^2*b^
10))/(b*tan(1/2*d*x + 1/2*c)^4 + 4*a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d
*x + 1/2*c)^2 + b)^(3/2) - 6*arctan(-1/2*(sqrt(b)*tan(1/2*d*x + 1/2*c)^2 -
sqrt(b*tan(1/2*d*x + 1/2*c)^4 + 4*a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*
x + 1/2*c)^2 + b) + sqrt(b))/sqrt(a - b))/((a^2*sgn(sin(d*x + c)) - 2*a*b*s
gn(sin(d*x + c)) + b^2*sgn(sin(d*x + c)))*sqrt(a - b)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\cot\left(c+d\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cot(c + d*x)^2)^(5/2),x)
```

```
[Out] int(1/(a + b*cot(c + d*x)^2)^(5/2), x)
```

3.37 $\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=190

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{(a-b)^{7/2}d}+\frac{b\cot(c+dx)}{5a(a-b)d\left(a+b\cot^2(c+dx)\right)^{5/2}}+\frac{(9a-4b)b\cot(c+dx)}{15a^2(a-b)^2d\left(a+b\cot^2(c+dx)\right)}$$

```
[Out] -arctan(cot(d*x+c)*(a-b)^(1/2)/(a+b*cot(d*x+c)^2)^(1/2))/(a-b)^(7/2)/d+1/5*
b*cot(d*x+c)/a/(a-b)/d/(a+b*cot(d*x+c)^2)^(5/2)+1/15*(9*a-4*b)*b*cot(d*x+c)
/a^2/(a-b)^2/d/(a+b*cot(d*x+c)^2)^(3/2)+1/15*b*(33*a^2-26*a*b+8*b^2)*cot(d*
x+c)/a^3/(a-b)^3/d/(a+b*cot(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3742, 425, 541, 12, 385, 209}

$$\frac{b(9a-4b)\cot(c+dx)}{15a^2d(a-b)^2\left(a+b\cot^2(c+dx)\right)^{3/2}}+\frac{b(33a^2-26ab+8b^2)\cot(c+dx)}{15a^3d(a-b)^3\sqrt{a+b\cot^2(c+dx)}}-\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{d(a-b)^{7/2}}+\frac{b\cot(c+dx)}{5ad(a-b)\left(a+b\cot^2(c+dx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[c + d*x]^2)^(-7/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]^2]]/((a - b)^(7
/2)*d)) + (b*Cot[c + d*x])/(5*a*(a - b)*d*(a + b*Cot[c + d*x]^2)^(5/2)) + (
(9*a - 4*b)*b*Cot[c + d*x])/(15*a^2*(a - b)^2*d*(a + b*Cot[c + d*x]^2)^(3/2
)) + (b*(33*a^2 - 26*a*b + 8*b^2)*Cot[c + d*x])/(15*a^3*(a - b)^3*d*Sqrt[a
+ b*Cot[c + d*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\left(a+b\cot^2(c+dx)\right)^{7/2}}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^{7/2}}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= \frac{b\cot(c+dx)}{5a(a-b)d\left(a+b\cot^2(c+dx)\right)^{5/2}} - \frac{\operatorname{Subst}\left(\int \frac{5a-4b-4bx^2}{(1+x^2)(a+bx^2)^{5/2}}\,dx, x, \cot(c+\right.}{5a(a-b)d}$$

$$= \frac{b\cot(c+dx)}{5a(a-b)d\left(a+b\cot^2(c+dx)\right)^{5/2}} + \frac{(9a-4b)b\cot(c+dx)}{15a^2(a-b)^2d\left(a+b\cot^2(c+dx)\right)^{3/2}} -$$

$$= \frac{b\cot(c+dx)}{5a(a-b)d\left(a+b\cot^2(c+dx)\right)^{5/2}} + \frac{(9a-4b)b\cot(c+dx)}{15a^2(a-b)^2d\left(a+b\cot^2(c+dx)\right)^{3/2}} +$$

$$= \frac{b\cot(c+dx)}{5a(a-b)d\left(a+b\cot^2(c+dx)\right)^{5/2}} + \frac{(9a-4b)b\cot(c+dx)}{15a^2(a-b)^2d\left(a+b\cot^2(c+dx)\right)^{3/2}} +$$

$$= \frac{b\cot(c+dx)}{5a(a-b)d\left(a+b\cot^2(c+dx)\right)^{5/2}} + \frac{(9a-4b)b\cot(c+dx)}{15a^2(a-b)^2d\left(a+b\cot^2(c+dx)\right)^{3/2}} +$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\cot(c+dx)}{\sqrt{a+b\cot^2(c+dx)}}\right)}{(a-b)^{7/2}d} + \frac{b\cot(c+dx)}{5a(a-b)d\left(a+b\cot^2(c+dx)\right)^{5/2}} + \frac{}{1}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 14.78, size = 2553, normalized size = 13.44

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cot[c + d*x]^2)^(-7/2),x]
```

```
[Out] -1/4725*(Cot[c + d*x]*(-33075*ArcSin[Sqrt[((a - b)*Cos[c + d*x]^2)/a]] + (9
9225*(a - b)*ArcSin[Sqrt[((a - b)*Cos[c + d*x]^2)/a]]*Cos[c + d*x]^2)/a - (
99225*(a - b)^2*ArcSin[Sqrt[((a - b)*Cos[c + d*x]^2)/a]]*Cos[c + d*x]^4)/a^
2 + (33075*(a - b)^3*ArcSin[Sqrt[((a - b)*Cos[c + d*x]^2)/a]]*Cos[c + d*x]^
6)/a^3 - (66150*b*ArcSin[Sqrt[((a - b)*Cos[c + d*x]^2)/a]]*Cot[c + d*x]^2)/
```

a + (198450*(a - b)*b*ArcSin[Sqrt[((a - b)*Cos[c + d*x]^2)/a]]*Cos[c + d*x]
^2*Cot[c + d*x]^2)/a^2 + (66150*(a - b)^3*b*ArcSin[Sqrt[((a - b)*Cos[c + d*
x]^2)/a]]*Cos[c + d*x]^6*Cot[c + d*x]^2)/a^4 - (52920*b^2*ArcSin[Sqrt[((a -
 b)*Cos[c + d*x]^2)/a]]*Cot[c + d*x]^4)/a^2 + (158760*(a - b)*b^2*ArcSin[Sq
rt[((a - b)*Cos[c + d*x]^2)/a]]*Cos[c + d*x]^2*Cot[c + d*x]^4)/a^3 - (15876
0*(a - b)^2*b^2*ArcSin[Sqrt[((a - b)*Cos[c + d*x]^2)/a]]*Cos[c + d*x]^4*Cot
[c + d*x]^4)/a^4 + (52920*(a - b)^3*b^2*ArcSin[Sqrt[((a - b)*Cos[c + d*x]^2
)/a]]*Cos[c + d*x]^6*Cot[c + d*x]^4)/a^5 - (15120*b^3*ArcSin[Sqrt[((a - b)*
Cos[c + d*x]^2)/a]]*Cot[c + d*x]^6)/a^3 + (45360*(a - b)*b^3*ArcSin[Sqrt[((
a - b)*Cos[c + d*x]^2)/a]]*Cos[c + d*x]^2*Cot[c + d*x]^6)/a^4 - (45360*(a -
 b)^2*b^3*ArcSin[Sqrt[((a - b)*Cos[c + d*x]^2)/a]]*Cos[c + d*x]^4*Cot[c + d
x]^6)/a^5 + (15120(a - b)^3*b^3*ArcSin[Sqrt[((a - b)*Cos[c + d*x]^2)/a]]*
Cos[c + d*x]^6*Cot[c + d*x]^6)/a^6 - 77175*(((a - b)*Cos[c + d*x]^2)/a)^(3/
2)*Sqrt[(Cos[c + d*x]^2*(b + a*Tan[c + d*x]^2))/a] + 50715*(((a - b)*Cos[c
 + d*x]^2)/a)^(5/2)*Sqrt[(Cos[c + d*x]^2*(b + a*Tan[c + d*x]^2))/a] - (15435
0*b*(((a - b)*Cos[c + d*x]^2)/a)^(3/2)*Cot[c + d*x]^2*Sqrt[(Cos[c + d*x]^2*
(b + a*Tan[c + d*x]^2))/a])/a + (101430*b*(((a - b)*Cos[c + d*x]^2)/a)^(5/2
)*Cot[c + d*x]^2*Sqrt[(Cos[c + d*x]^2*(b + a*Tan[c + d*x]^2))/a])/a - (1234
80*b^2*(((a - b)*Cos[c + d*x]^2)/a)^(3/2)*Cot[c + d*x]^4*Sqrt[(Cos[c + d*x]
^2*(b + a*Tan[c + d*x]^2))/a])/a^2 + (81144*b^2*(((a - b)*Cos[c + d*x]^2)/a
)^(5/2)*Cot[c + d*x]^4*Sqrt[(Cos[c + d*x]^2*(b + a*Tan[c + d*x]^2))/a])/a^2
 - (35280*b^3*(((a - b)*Cos[c + d*x]^2)/a)^(3/2)*Cot[c + d*x]^6*Sqrt[(Cos[c
 + d*x]^2*(b + a*Tan[c + d*x]^2))/a])/a^3 + (23184*b^3*(((a - b)*Cos[c + d*
x]^2)/a)^(5/2)*Cot[c + d*x]^6*Sqrt[(Cos[c + d*x]^2*(b + a*Tan[c + d*x]^2))/
a])/a^3 + 1420*(((a - b)*Cos[c + d*x]^2)/a)^(9/2)*Hypergeometric2F1[2, 2, 1
1/2, ((a - b)*Cos[c + d*x]^2)/a]*Sqrt[(Cos[c + d*x]^2*(b + a*Tan[c + d*x]^2
))/a] + (3540*b*(((a - b)*Cos[c + d*x]^2)/a)^(9/2)*Cot[c + d*x]^2*Hypergeom
etric2F1[2, 2, 11/2, ((a - b)*Cos[c + d*x]^2)/a]*Sqrt[(Cos[c + d*x]^2*(b +
a*Tan[c + d*x]^2))/a])/a + (3000*b^2*(((a - b)*Cos[c + d*x]^2)/a)^(9/2)*Cot
[c + d*x]^4*Hypergeometric2F1[2, 2, 11/2, ((a - b)*Cos[c + d*x]^2)/a]*Sqrt[
(Cos[c + d*x]^2*(b + a*Tan[c + d*x]^2))/a])/a^2 + (880*b^3*(((a - b)*Cos[c
 + d*x]^2)/a)^(9/2)*Cot[c + d*x]^6*Hypergeometric2F1[2, 2, 11/2, ((a - b)*Co
s[c + d*x]^2)/a]*Sqrt[(Cos[c + d*x]^2*(b + a*Tan[c + d*x]^2))/a])/a^3 + 600
*(((a - b)*Cos[c + d*x]^2)/a)^(9/2)*HypergeometricPFQ[{2, 2, 2}, {1, 11/2},
 ((a - b)*Cos[c + d*x]^2)/a]*Sqrt[(Cos[c + d*x]^2*(b + a*Tan[c + d*x]^2))/a
] + (1680*b*(((a - b)*Cos[c + d*x]^2)/a)^(9/2)*Cot[c + d*x]^2*Hypergeometri
cPFQ[{2, 2, 2}, {1, 11/2}, ((a - b)*Cos[c + d*x]^2)/a]*Sqrt[(Cos[c + d*x]^2
*(b + a*Tan[c + d*x]^2))/a])/a + (1560*b^2*(((a - b)*Cos[c + d*x]^2)/a)^(9/
2)*Cot[c + d*x]^4*HypergeometricPFQ[{2, 2, 2}, {1, 11/2}, ((a - b)*Cos[c +
d*x]^2)/a]*Sqrt[(Cos[c + d*x]^2*(b + a*Tan[c + d*x]^2))/a])/a^2 + (480*b^3*
(((a - b)*Cos[c + d*x]^2)/a)^(9/2)*Cot[c + d*x]^6*HypergeometricPFQ[{2, 2,
2}, {1, 11/2}, ((a - b)*Cos[c + d*x]^2)/a]*Sqrt[(Cos[c + d*x]^2*(b + a*Tan[
c + d*x]^2))/a])/a^3 + 80*(((a - b)*Cos[c + d*x]^2)/a)^(9/2)*Hypergeometric
PFQ[{2, 2, 2, 2}, {1, 1, 11/2}, ((a - b)*Cos[c + d*x]^2)/a]*Sqrt[(Cos[c + d
x]^2(b + a*Tan[c + d*x]^2))/a] + (240*b*(((a - b)*Cos[c + d*x]^2)/a)^(9/2

```
)*Cot[c + d*x]^2*HypergeometricPFQ[{2, 2, 2, 2}, {1, 1, 11/2}, ((a - b)*Cos
[c + d*x]^2)/a]*Sqrt[(Cos[c + d*x]^2*(b + a*Tan[c + d*x]^2))/a])/a + (240*b
^2*(((a - b)*Cos[c + d*x]^2)/a)^(9/2)*Cot[c + d*x]^4*HypergeometricPFQ[{2,
2, 2, 2}, {1, 1, 11/2}, ((a - b)*Cos[c + d*x]^2)/a]*Sqrt[(Cos[c + d*x]^2*(b
 + a*Tan[c + d*x]^2))/a])/a^2 + (80*b^3*(((a - b)*Cos[c + d*x]^2)/a)^(9/2)*
Cot[c + d*x]^6*HypergeometricPFQ[{2, 2, 2, 2}, {1, 1, 11/2}, ((a - b)*Cos[c
 + d*x]^2)/a]*Sqrt[(Cos[c + d*x]^2*(b + a*Tan[c + d*x]^2))/a])/a^3 + 33075*
Sqrt[((a - b)*Cos[c + d*x]^4*(b + a*Tan[c + d*x]^2))/a^2] + (66150*b*Cot[c
+ d*x]^2*Sqrt[((a - b)*Cos[c + d*x]^4*(b + a*Tan[c + d*x]^2))/a^2])/a + (52
920*b^2*Cot[c + d*x]^4*Sqrt[((a - b)*Cos[c + d*x]^4*(b + a*Tan[c + d*x]^2))
/a^2])/a^2 + (15120*b^3*Cot[c + d*x]^6*Sqrt[((a - b)*Cos[c + d*x]^4*(b + a*
Tan[c + d*x]^2))/a^2])/a^3 - (198450*(a - b)^2*b*ArcSin[Sqrt[((a - b)*Cos[c
 + d*x]^2)/a]])/(a^3*(Tan[c + d*x] + Tan[c + d*x]^3)^2)))/(a^3*d*(((a - b)*
Cos[c + d*x]^2)/a)^(7/2)*(1 + Cot[c + d*x]^2)*Sqrt[a + b*Cot[c + d*x]^2]*(1
 + (b*Cot[c + d*x]^2)/a)^2*Sqrt[(Cos[c + d*x]^2*(b + a*Tan[c + d*x]^2))/a])
```

Maple [A]
time = 0.23, size = 253, normalized size = 1.33

method	result
derivativedivides	$\frac{\frac{b\cot(dx+c)}{(a-b)^3 a\sqrt{a+b(\cot^2(dx+c))}}+\frac{b\left(\frac{\cot(dx+c)}{3a\left(a+b\left(\cot^2(dx+c)\right)\right)^{\frac{3}{2}}}+\frac{2\cot(dx+c)}{3a^2\sqrt{a+b(\cot^2(dx+c))}}\right)}{(a-b)^2}+\frac{b\left(\frac{}{5a(a+}\right.}{}$
default	$\frac{\frac{b\cot(dx+c)}{(a-b)^3 a\sqrt{a+b(\cot^2(dx+c))}}+\frac{b\left(\frac{\cot(dx+c)}{3a\left(a+b\left(\cot^2(dx+c)\right)\right)^{\frac{3}{2}}}+\frac{2\cot(dx+c)}{3a^2\sqrt{a+b(\cot^2(dx+c))}}\right)}{(a-b)^2}+\frac{b\left(\frac{}{5a(a+}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cot(d*x+c)^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b/(a-b)^3*cot(d*x+c)/a/(a+b*cot(d*x+c)^2)^(1/2)+1/(a-b)^2*b*(1/3*cot(d
*x+c)/a/(a+b*cot(d*x+c)^2)^(3/2)+2/3/a^2*cot(d*x+c)/(a+b*cot(d*x+c)^2)^(1/2
))+1/(a-b)*b*(1/5*cot(d*x+c)/a/(a+b*cot(d*x+c)^2)^(5/2)+4/5/a*(1/3*cot(d*x+
c)/a/(a+b*cot(d*x+c)^2)^(3/2)+2/3/a^2*cot(d*x+c)/(a+b*cot(d*x+c)^2)^(1/2)))
-1/(a-b)^4*(b^4*(a-b))^(1/2)/b^2*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*co
t(d*x+c)^2)^(1/2)*cot(d*x+c)))
```

Maple [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 700 vs. $2(172) = 344$.

time = 3.19, size = 1452, normalized size = 7.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/60*(15*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3 - (a^6 - 3*a^5*b + 3*a^4*b^
2 - a^3*b^3)*cos(2*d*x + 2*c)^3 + 3*(a^6 - a^5*b - a^4*b^2 + a^3*b^3)*cos(2
*d*x + 2*c)^2 - 3*(a^6 + a^5*b - a^4*b^2 - a^3*b^3)*cos(2*d*x + 2*c))*sqrt(
-a + b)*log(-2*(a^2 - 2*a*b + b^2)*cos(2*d*x + 2*c)^2 + 2*((a - b)*cos(2*d*
x + 2*c) - b)*sqrt(-a + b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(cos(2*d
*x + 2*c) - 1))*sin(2*d*x + 2*c) + a^2 - 2*b^2 + 4*(a*b - b^2)*cos(2*d*x +
2*c)) + 4*(45*a^5*b - 15*a^4*b^2 - 47*a^3*b^3 + 11*a^2*b^4 + 14*a*b^5 - 8*b
^6 + (45*a^5*b - 165*a^4*b^2 + 233*a^3*b^3 - 159*a^2*b^4 + 54*a*b^5 - 8*b^6
)*cos(2*d*x + 2*c)^2 - 2*(45*a^5*b - 90*a^4*b^2 + 27*a^3*b^3 + 44*a^2*b^4 -
 34*a*b^5 + 8*b^6)*cos(2*d*x + 2*c))*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b
)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c))/((a^10 - 7*a^9*b + 21*a^8*b^2 -
 35*a^7*b^3 + 35*a^6*b^4 - 21*a^5*b^5 + 7*a^4*b^6 - a^3*b^7)*d*cos(2*d*x +
2*c)^3 - 3*(a^10 - 5*a^9*b + 9*a^8*b^2 - 5*a^7*b^3 - 5*a^6*b^4 + 9*a^5*b^5
- 5*a^4*b^6 + a^3*b^7)*d*cos(2*d*x + 2*c)^2 + 3*(a^10 - 3*a^9*b + a^8*b^2 +
 5*a^7*b^3 - 5*a^6*b^4 - a^5*b^5 + 3*a^4*b^6 - a^3*b^7)*d*cos(2*d*x + 2*c)
- (a^10 - a^9*b - 3*a^8*b^2 + 3*a^7*b^3 + 3*a^6*b^4 - 3*a^5*b^5 - a^4*b^6 +
 a^3*b^7)*d), 1/30*(15*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3 - (a^6 - 3*a^5*
b + 3*a^4*b^2 - a^3*b^3)*cos(2*d*x + 2*c)^3 + 3*(a^6 - a^5*b - a^4*b^2 + a^
3*b^3)*cos(2*d*x + 2*c)^2 - 3*(a^6 + a^5*b - a^4*b^2 - a^3*b^3)*cos(2*d*x +
 2*c))*sqrt(a - b)*arctan(-sqrt(a - b)*sqrt(((a - b)*cos(2*d*x + 2*c) - a -
 b)/(cos(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c)/((a - b)*cos(2*d*x + 2*c) - b)
) - 2*(45*a^5*b - 15*a^4*b^2 - 47*a^3*b^3 + 11*a^2*b^4 + 14*a*b^5 - 8*b^6 +
 (45*a^5*b - 165*a^4*b^2 + 233*a^3*b^3 - 159*a^2*b^4 + 54*a*b^5 - 8*b^6)*co
s(2*d*x + 2*c)^2 - 2*(45*a^5*b - 90*a^4*b^2 + 27*a^3*b^3 + 44*a^2*b^4 - 34*
a*b^5 + 8*b^6)*cos(2*d*x + 2*c))*sqrt(((a - b)*cos(2*d*x + 2*c) - a - b)/(c
os(2*d*x + 2*c) - 1))*sin(2*d*x + 2*c))/((a^10 - 7*a^9*b + 21*a^8*b^2 - 35*
a^7*b^3 + 35*a^6*b^4 - 21*a^5*b^5 + 7*a^4*b^6 - a^3*b^7)*d*cos(2*d*x + 2*c)
^3 - 3*(a^10 - 5*a^9*b + 9*a^8*b^2 - 5*a^7*b^3 - 5*a^6*b^4 + 9*a^5*b^5 - 5*
```

```
a^4*b^6 + a^3*b^7)*d*cos(2*d*x + 2*c)^2 + 3*(a^10 - 3*a^9*b + a^8*b^2 + 5*a
^7*b^3 - 5*a^6*b^4 - a^5*b^5 + 3*a^4*b^6 - a^3*b^7)*d*cos(2*d*x + 2*c) - (a
^10 - a^9*b - 3*a^8*b^2 + 3*a^7*b^3 + 3*a^6*b^4 - 3*a^5*b^5 - a^4*b^6 + a^3
*b^7)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\cot^2\left(c + dx\right)\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)**2)**(7/2),x)
```

```
[Out] Integral((a + b*cot(c + d*x)**2)**(-7/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 3249 vs. $2(172) = 344$.
time = 2.69, size = 3249, normalized size = 17.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(d*x+c)^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*(30*arctan(-1/2*(sqrt(b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(b*tan(1/2*d*x +
 1/2*c)^4 + 4*a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c)^2 + b) +
sqrt(b))/sqrt(a - b))/((a^3*sgn(sin(d*x + c)) - 3*a^2*b*sgn(sin(d*x + c)) +
 3*a*b^2*sgn(sin(d*x + c)) - b^3*sgn(sin(d*x + c)))*sqrt(a - b)) - ((((((33
*a^20*b^3*sgn(sin(d*x + c)) - 620*a^19*b^4*sgn(sin(d*x + c)) + 5525*a^18*b^
5*sgn(sin(d*x + c)) - 31050*a^17*b^6*sgn(sin(d*x + c)) + 123420*a^16*b^7*sg
n(sin(d*x + c)) - 368832*a^15*b^8*sgn(sin(d*x + c)) + 859860*a^14*b^9*sgn(s
in(d*x + c)) - 1601400*a^13*b^10*sgn(sin(d*x + c)) + 2419950*a^12*b^11*sgn(
sin(d*x + c)) - 2996760*a^11*b^12*sgn(sin(d*x + c)) + 3058198*a^10*b^13*sgn
(sin(d*x + c)) - 2576860*a^9*b^14*sgn(sin(d*x + c)) + 1790100*a^8*b^15*sgn(
sin(d*x + c)) - 1020000*a^7*b^16*sgn(sin(d*x + c)) + 472260*a^6*b^17*sgn(si
n(d*x + c)) - 175032*a^5*b^18*sgn(sin(d*x + c)) + 50745*a^4*b^19*sgn(sin(d*
x + c)) - 11100*a^3*b^20*sgn(sin(d*x + c)) + 1725*a^2*b^21*sgn(sin(d*x + c)
) - 170*a*b^22*sgn(sin(d*x + c)) + 8*b^23*sgn(sin(d*x + c)))*tan(1/2*d*x +
1/2*c)^2/(a^24 - 21*a^23*b + 210*a^22*b^2 - 1330*a^21*b^3 + 5985*a^20*b^4 -
 20349*a^19*b^5 + 54264*a^18*b^6 - 116280*a^17*b^7 + 203490*a^16*b^8 - 2939
30*a^15*b^9 + 352716*a^14*b^10 - 352716*a^13*b^11 + 293930*a^12*b^12 - 2034
90*a^11*b^13 + 116280*a^10*b^14 - 54264*a^9*b^15 + 20349*a^8*b^16 - 5985*a^
7*b^17 + 1330*a^6*b^18 - 210*a^5*b^19 + 21*a^4*b^20 - a^3*b^21) + 5*(60*a^2
1*b^2*sgn(sin(d*x + c)) - 1165*a^20*b^3*sgn(sin(d*x + c)) + 10752*a^19*b^4*
sgn(sin(d*x + c)) - 62729*a^18*b^5*sgn(sin(d*x + c)) + 259530*a^17*b^6*sgn(
```

sin(d*x + c)) - 809676*a^16*b^7*sgn(sin(d*x + c)) + 1977168*a^15*b^8*sgn(si
n(d*x + c)) - 3871716*a^14*b^9*sgn(sin(d*x + c)) + 6178752*a^13*b^10*sgn(si
n(d*x + c)) - 8121750*a^12*b^11*sgn(sin(d*x + c)) + 8850608*a^11*b^12*sgn(s
in(d*x + c)) - 8020974*a^10*b^13*sgn(sin(d*x + c)) + 6045676*a^9*b^14*sgn(s
in(d*x + c)) - 3778692*a^8*b^15*sgn(sin(d*x + c)) + 1946160*a^7*b^16*sgn(si
n(d*x + c)) - 817428*a^6*b^17*sgn(sin(d*x + c)) + 275604*a^5*b^18*sgn(sin(d
*x + c)) - 72837*a^4*b^19*sgn(sin(d*x + c)) + 14544*a^3*b^20*sgn(sin(d*x +
c)) - 2065*a^2*b^21*sgn(sin(d*x + c)) + 186*a*b^22*sgn(sin(d*x + c)) - 8*b^
23*sgn(sin(d*x + c)))/(a^24 - 21*a^23*b + 210*a^22*b^2 - 1330*a^21*b^3 + 59
85*a^20*b^4 - 20349*a^19*b^5 + 54264*a^18*b^6 - 116280*a^17*b^7 + 203490*a^
16*b^8 - 293930*a^15*b^9 + 352716*a^14*b^10 - 352716*a^13*b^11 + 293930*a^1
2*b^12 - 203490*a^11*b^13 + 116280*a^10*b^14 - 54264*a^9*b^15 + 20349*a^8*b
^16 - 5985*a^7*b^17 + 1330*a^6*b^18 - 210*a^5*b^19 + 21*a^4*b^20 - a^3*b^21
))*tan(1/2*d*x + 1/2*c)^2 + 10*(72*a^22*b*sgn(sin(d*x + c)) - 1458*a^21*b^2
*sgn(sin(d*x + c)) + 14067*a^20*b^3*sgn(sin(d*x + c)) - 86018*a^19*b^4*sgn(
sin(d*x + c)) + 374075*a^18*b^5*sgn(sin(d*x + c)) - 1230570*a^17*b^6*sgn(si
n(d*x + c)) + 3179748*a^16*b^7*sgn(sin(d*x + c)) - 6614904*a^15*b^8*sgn(sin
(d*x + c)) + 11265084*a^14*b^9*sgn(sin(d*x + c)) - 15882420*a^13*b^10*sgn(s
in(d*x + c)) + 18674058*a^12*b^11*sgn(sin(d*x + c)) - 18386316*a^11*b^12*sg
n(sin(d*x + c)) + 15180490*a^10*b^13*sgn(sin(d*x + c)) - 10497364*a^9*b^14*
sgn(sin(d*x + c)) + 6055740*a^8*b^15*sgn(sin(d*x + c)) - 2893944*a^7*b^16*s
gn(sin(d*x + c)) + 1133220*a^6*b^17*sgn(sin(d*x + c)) - 357786*a^5*b^18*sgn
(sin(d*x + c)) + 88923*a^4*b^19*sgn(sin(d*x + c)) - 16770*a^3*b^20*sgn(sin(
d*x + c)) + 2259*a^2*b^21*sgn(sin(d*x + c)) - 194*a*b^22*sgn(sin(d*x + c))
+ 8*b^23*sgn(sin(d*x + c)))/(a^24 - 21*a^23*b + 210*a^22*b^2 - 1330*a^21*b^
3 + 5985*a^20*b^4 - 20349*a^19*b^5 + 54264*a^18*b^6 - 116280*a^17*b^7 + 203
490*a^16*b^8 - 293930*a^15*b^9 + 352716*a^14*b^10 - 352716*a^13*b^11 + 2939
30*a^12*b^12 - 203490*a^11*b^13 + 116280*a^10*b^14 - 54264*a^9*b^15 + 20349
*a^8*b^16 - 5985*a^7*b^17 + 1330*a^6*b^18 - 210*a^5*b^19 + 21*a^4*b^20 - a^
3*b^21))*tan(1/2*d*x + 1/2*c)^2 - 10*(72*a^22*b*sgn(sin(d*x + c)) - 1458*a^
21*b^2*sgn(sin(d*x + c)) + 14067*a^20*b^3*sgn(sin(d*x + c)) - 86018*a^19*b^
4*sgn(sin(d*x + c)) + 374075*a^18*b^5*sgn(sin(d*x + c)) - 1230570*a^17*b^6*
sgn(sin(d*x + c)) + 3179748*a^16*b^7*sgn(sin(d*x + c)) - 6614904*a^15*b^8*s
gn(sin(d*x + c)) + 11265084*a^14*b^9*sgn(sin(d*x + c)) - 15882420*a^13*b^10
*sgn(sin(d*x + c)) + 18674058*a^12*b^11*sgn(sin(d*x + c)) - 18386316*a^11*b
^12*sgn(sin(d*x + c)) + 15180490*a^10*b^13*sgn(sin(d*x + c)) - 10497364*a^9
*b^14*sgn(sin(d*x + c)) + 6055740*a^8*b^15*sgn(sin(d*x + c)) - 2893944*a^7*
b^16*sgn(sin(d*x + c)) + 1133220*a^6*b^17*sgn(sin(d*x + c)) - 357786*a^5*b^
18*sgn(sin(d*x + c)) + 88923*a^4*b^19*sgn(sin(d*x + c)) - 16770*a^3*b^20*sg
n(sin(d*x + c)) + 2259*a^2*b^21*sgn(sin(d*x + c)) - 194*a*b^22*sgn(sin(d*x
+ c)) + 8*b^23*sgn(sin(d*x + c)))/(a^24 - 21*a^23*b + 210*a^22*b^2 - 1330*a
^21*b^3 + 5985*a^20*b^4 - 20349*a^19*b^5 + 54264*a^18*b^6 - 116280*a^17*b^7
+ 203490*a^16*b^8 - 293930*a^15*b^9 + 352716*a^14*b^10 - 352716*a^13*b^11
+ 293930*a^12*b^12 - 203490*a^11*b^13 + 116280*a^10*b^14 - 54264*a^9*b^15 +
20349*a^8*b^16 - 5985*a^7*b^17 + 1330*a^6*b^18 - 210*a^5*b^19 + 21*a^4*b^2

```
0 - a^3*b^21))*tan(1/2*d*x + 1/2*c)^2 - 5*(60*a...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\cot\left(c+d\,x\right)^2+a\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cot(c + d*x)^2)^(7/2),x)
[Out] int(1/(a + b*cot(c + d*x)^2)^(7/2), x)
```

3.38 $\int \left(1-\cot^2(x)\right)^{3/2} dx$

Optimal. Leaf size=54

$$\frac{5}{2}\text{ArcSin}(\cot(x))-2\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\cot(x)}{\sqrt{1-\cot^2(x)}}\right)+\frac{1}{2}\cot(x)\sqrt{1-\cot^2(x)}$$

```
[Out] 5/2*arcsin(cot(x))-2*arctan(cot(x)*2^(1/2)/(1-cot(x)^2)^(1/2))*2^(1/2)+1/2*
cot(x)*(1-cot(x)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3742, 427, 537, 222, 385, 209}

$$\frac{5}{2}\text{ArcSin}(\cot(x))-2\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\cot(x)}{\sqrt{1-\cot^2(x)}}\right)+\frac{1}{2}\cot(x)\sqrt{1-\cot^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cot[x]^2)^(3/2),x]
```

```
[Out] (5*ArcSin[Cot[x]])/2 - 2*Sqrt[2]*ArcTan[(Sqrt[2]*Cot[x])/Sqrt[1 - Cot[x]^2]
] + (Cot[x]*Sqrt[1 - Cot[x]^2])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
```

```
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$
\begin{aligned}
\int \left(1-\cot^2(x)\right)^{3/2}\,dx &= -\text{Subst}\left(\int \frac{\left(1-x^2\right)^{3/2}}{1+x^2}\,dx, x, \cot(x)\right)\\
&= \frac{1}{2}\cot(x)\sqrt{1-\cot^2(x)}-\frac{1}{2}\text{Subst}\left(\int \frac{3-5x^2}{\sqrt{1-x^2}\,\left(1+x^2\right)}\,dx, x, \cot(x)\right)\\
&= \frac{1}{2}\cot(x)\sqrt{1-\cot^2(x)}+\frac{5}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \cot(x)\right)-4\text{Subst}\left(\int \frac{}{\sqrt{1-}}\right.\\
&= \frac{5}{2}\sin^{-1}(\cot(x))+\frac{1}{2}\cot(x)\sqrt{1-\cot^2(x)}-4\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{\cot(x)}{\sqrt{1-\cot^2}}\right.\\
&= \frac{5}{2}\sin^{-1}(\cot(x))-2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{1-\cot^2(x)}}\right)+\frac{1}{2}\cot(x)\sqrt{1-\cot^2(x)}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 123 vs. 2(54) = 108.

time = 0.44, size = 123, normalized size = 2.28

$$
\frac{1}{2}\left(1-\cot^2(x)\right)^{3/2}\sec^2(2x)\left(\text{ArcTan}\left(\frac{\cos(x)}{\sqrt{-\cos(2x)}}\right)\sqrt{-\cos(2x)}\,\sin^3(x)+4\tanh^{-1}\left(\frac{\cos(x)}{\sqrt{\cos(2x)}}\right)\sqrt{\cos(2x)}\,\sin^3(x)-4\sqrt{2}\,\sqrt{\cos(2x)}\,\log\left(\sqrt{2}\,\cos(x)+\sqrt{\cos(2x)}\right)\sin^3(x)-\frac{1}{4}\sin(4x)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cot[x]^2)^(3/2),x]
```

```
[Out] ((1 - Cot[x]^2)^(3/2)*Sec[2*x]^2*(ArcTan[Cos[x]/Sqrt[-Cos[2*x]]]*Sqrt[-Cos[
2*x]]*Sin[x]^3 + 4*ArcTanh[Cos[x]/Sqrt[Cos[2*x]]]*Sqrt[Cos[2*x]]*Sin[x]^3 -
 4*Sqrt[2]*Sqrt[Cos[2*x]]*Log[Sqrt[2]*Cos[x] + Sqrt[Cos[2*x]]]*Sin[x]^3 - S
in[4*x]/4))/2
```

Maple [**A**]
time = 0.20, size = 51, normalized size = 0.94

method	result	siz
derivativedivides	$\frac{\cot(x)\sqrt{1-(\cot^2(x))}}{2}+\frac{5\arcsin(\cot(x))}{2}+2\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\ \sqrt{1-(\cot^2(x))}\ \cot(x)}{-1+\cot^2(x)}\right)$	51
default	$\frac{\cot(x)\sqrt{1-(\cot^2(x))}}{2}+\frac{5\arcsin(\cot(x))}{2}+2\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\ \sqrt{1-(\cot^2(x))}\ \cot(x)}{-1+\cot^2(x)}\right)$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cot(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*cot(x)*(1-cot(x)^2)^(1/2)+5/2*arcsin(cot(x))+2*2^(1/2)*arctan(2^(1/2)*(
1-cot(x)^2)^(1/2)/(-1+cot(x)^2)*cot(x))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cot(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-cot(x)^2 + 1)^(3/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(42) = 84.
time = 2.40, size = 110, normalized size = 2.04

$$\frac{4\sqrt{2}\ \arctan\left(\frac{\sqrt{\frac{\cos(2x)}{\cos(2x)-1}}\ \sin(2x)}{\cos(2x)+1}\right)\sin(2x)+\sqrt{2}\sqrt{\frac{\cos(2x)}{\cos(2x)-1}}\ (\cos(2x)+1)-5\ \arctan\left(\frac{\sqrt{2}\sqrt{\frac{\cos(2x)}{\cos(2x)-1}}\ \sin(2x)}{\cos(2x)+1}\right)\sin(2x)}{2\ \sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cot(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*sqrt(2)*arctan(sqrt(cos(2*x)/(cos(2*x) - 1))*sin(2*x)/(cos(2*x) + 1)
)*sin(2*x) + sqrt(2)*sqrt(cos(2*x)/(cos(2*x) - 1))*(cos(2*x) + 1) - 5*arcta
n(sqrt(2)*sqrt(cos(2*x)/(cos(2*x) - 1))*sin(2*x)/(cos(2*x) + 1))*sin(2*x))/
sin(2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(1-\cot^{2}\left(x\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cot(x)**2)**(3/2),x)
```

```
[Out] Integral((1 - cot(x)**2)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(42) = 84.
time = 0.48, size = 257, normalized size = 4.76

$$\frac{1}{4}\left(5\,\pi\mathrm{sgn}\left(\cos\left(x\right)\right)-4\,\sqrt{2}\left(\pi\mathrm{sgn}\left(\cos\left(x\right)\right)+2\,\arctan\left(-\frac{\left(\frac{\left(\sqrt{2}\,\sqrt{-2\,\cos\left(x\right)^{2}+1}-\sqrt{2}\right)^{2}}{\cos\left(x\right)^{2}}-4\right)\cos\left(x\right)}{4\left(\sqrt{2}\,\sqrt{-2\,\cos\left(x\right)^{2}+1}-\sqrt{2}\right)}\right)\right)+\frac{4\,\sqrt{2}\left(\frac{\sqrt{2}\,\sqrt{-2\,\cos\left(x\right)^{2}+1}-\sqrt{2}}{\cos\left(x\right)}-\frac{4\,\cos\left(x\right)}{\sqrt{2}\,\sqrt{-2\,\cos\left(x\right)^{2}+1}-\sqrt{2}}\right)}{\left(\frac{\sqrt{2}\,\sqrt{-2\,\cos\left(x\right)^{2}+1}-\sqrt{2}}{\cos\left(x\right)}-\frac{4\,\cos\left(x\right)}{\sqrt{2}\,\sqrt{-2\,\cos\left(x\right)^{2}+1}-\sqrt{2}}\right)^{2}+8}+10\,\arctan\left(-\frac{\sqrt{2}\left(\frac{\left(\sqrt{2}\,\sqrt{-2\,\cos\left(x\right)^{2}+1}-\sqrt{2}\right)^{2}}{\cos\left(x\right)^{2}}-4\right)\cos\left(x\right)}{4\left(\sqrt{2}\,\sqrt{-2\,\cos\left(x\right)^{2}+1}-\sqrt{2}\right)}\right)\right)\mathrm{sgn}\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cot(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(5*pi*sgn(cos(x)) - 4*sqrt(2)*(pi*sgn(cos(x)) + 2*arctan(-1/4*((sqrt(2)
*sqrt(-2*cos(x)^2 + 1) - sqrt(2))^2/cos(x)^2 - 4)*cos(x)/(sqrt(2)*sqrt(-2*c
os(x)^2 + 1) - sqrt(2)))) + 4*sqrt(2)*((sqrt(2)*sqrt(-2*cos(x)^2 + 1) - sqr
t(2))/cos(x) - 4*cos(x)/(sqrt(2)*sqrt(-2*cos(x)^2 + 1) - sqrt(2)))/(((sqrt(
2)*sqrt(-2*cos(x)^2 + 1) - sqrt(2))/cos(x) - 4*cos(x)/(sqrt(2)*sqrt(-2*cos(
x)^2 + 1) - sqrt(2)))^2 + 8) + 10*arctan(-1/4*sqrt(2)*((sqrt(2)*sqrt(-2*cos
(x)^2 + 1) - sqrt(2))^2/cos(x)^2 - 4)*cos(x)/(sqrt(2)*sqrt(-2*cos(x)^2 + 1)
 - sqrt(2))))*sgn(sin(x))
```

Mupad [B]
time = 0.84, size = 104, normalized size = 1.93

$$\frac{5\,\mathrm{asin}\left(\mathrm{cot}\left(x\right)\right)}{2}+\frac{\mathrm{cot}\left(x\right)\,\sqrt{1-{\mathrm{cot}\left(x\right)}^2}}{2}-\sqrt{2}\,\ln\left(\frac{\frac{\sqrt{2}\,\left(-1+\mathrm{cot}\left(x\right)\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{2}-\sqrt{1-{\mathrm{cot}\left(x\right)}^2}\,1{}\mathrm{i}}{\mathrm{cot}\left(x\right)-\mathrm{i}}\right)\,1{}\mathrm{i}+\sqrt{2}\,\ln\left(\frac{\frac{\sqrt{2}\,\left(1+\mathrm{cot}\left(x\right)\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{2}+\sqrt{1-{\mathrm{cot}\left(x\right)}^2}\,1{}\mathrm{i}}{\mathrm{cot}\left(x\right)+1{}\mathrm{i}}\right)\,1{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - cot(x)^2)^(3/2),x)
```

```
[Out] (5*asin(cot(x)))/2 + (cot(x)*(1 - cot(x)^2)^(1/2))/2 - 2^(1/2)*log(((2^(1/2
)*(cot(x)*1i - 1)*1i)/2 - (1 - cot(x)^2)^(1/2)*1i)/(cot(x) - 1i))*1i + 2^(1
/2)*log(((2^(1/2)*(cot(x)*1i + 1)*1i)/2 + (1 - cot(x)^2)^(1/2)*1i)/(cot(x)
+ 1i))*1i
```

3.39 $\int \sqrt{1-\cot^2(x)}\,dx$

Optimal. Leaf size=32

$$\text{ArcSin}(\cot(x))-\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\cot(x)}{\sqrt{1-\cot^2(x)}}\right)$$

```
[Out] arcsin(cot(x))-arctan(cot(x)*2^(1/2)/(1-cot(x)^2)^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3742, 399, 222, 385, 209}

$$\text{ArcSin}(\cot(x))-\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\cot(x)}{\sqrt{1-\cot^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Cot[x]^2],x]
```

```
[Out] ArcSin[Cot[x]] - Sqrt[2]*ArcTan[(Sqrt[2]*Cot[x])/Sqrt[1 - Cot[x]^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
```

```
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{1-\cot^2(x)}\,dx &= -\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{1+x^2}\,dx,x,\cot(x)\right)\\
&= -\left(2\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\,(1+x^2)}\,dx,x,\cot(x)\right)\right)+\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx,x,\cot(x)\right)\\
&= \sin^{-1}(\cot(x))-2\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\frac{\cot(x)}{\sqrt{1-\cot^2(x)}}\right)\\
&= \sin^{-1}(\cot(x))-\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{2}\,\cot(x)}{\sqrt{1-\cot^2(x)}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 62, normalized size = 1.94

$$
\frac{\sqrt{1-\cot^2(x)}\left(-\tanh^{-1}\left(\frac{\cos(x)}{\sqrt{\cos(2x)}}\right)+\sqrt{2}\,\log\left(\sqrt{2}\,\cos(x)+\sqrt{\cos(2x)}\right)\right)\sin(x)}{\sqrt{\cos(2x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Cot[x]^2],x]
```

```
[Out] (Sqrt[1 - Cot[x]^2]*(-ArcTanh[Cos[x]/Sqrt[Cos[2*x]]] + Sqrt[2]*Log[Sqrt[2]*
Cos[x] + Sqrt[Cos[2*x]]])*Sin[x])/Sqrt[Cos[2*x]]
```

Maple [A]
time = 0.21, size = 34, normalized size = 1.06

method	result	size

derivativedivides	$\arcsin(\cot(x)) + \sqrt{2}\, \arctan\left(\frac{\sqrt{2}\, \sqrt{1-(\cot^2(x))}\, \cot(x)}{-1+\cot^2(x)}\right)$	34
default	$\arcsin(\cot(x)) + \sqrt{2}\, \arctan\left(\frac{\sqrt{2}\, \sqrt{1-(\cot^2(x))}\, \cot(x)}{-1+\cot^2(x)}\right)$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arcsin(cot(x))+2^(1/2)*arctan(2^(1/2)*(1-cot(x)^2)^(1/2)/(-1+cot(x)^2)*cot(
x))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(26) = 52$.
time = 2.58, size = 68, normalized size = 2.12

$$\sqrt{2}\, \arctan\left(\frac{\sqrt{\frac{\cos(2x)}{\cos(2x)-1}}\, \sin(2x)}{\cos(2x)+1}\right) - \arctan\left(\frac{\sqrt{2}\, \sqrt{\frac{\cos(2x)}{\cos(2x)-1}}\, \sin(2x)}{\cos(2x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*arctan(sqrt(cos(2*x)/(cos(2*x) - 1))*sin(2*x)/(cos(2*x) + 1)) - arc
tan(sqrt(2)*sqrt(cos(2*x)/(cos(2*x) - 1))*sin(2*x)/(cos(2*x) + 1))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{1-\cot^2(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cot(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(1 - cot(x)**2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.50, size = 170, normalized size = 5.31

$$-\frac{1}{2}\left(\pi-\sqrt{2}\,\pi-2\sqrt{2}\arctan\left(-\frac{1}{2}i\sqrt{2}\right)+2\arctan\left(-i\right)\right)\mathrm{sgn}\left(\sin\left(x\right)\right)+\frac{1}{2}\left(\pi\mathrm{sgn}\left(\cos\left(x\right)\right)-\sqrt{2}\left(\pi\mathrm{sgn}\left(\cos\left(x\right)\right)+2\arctan\left(-\frac{\left(\frac{\left(\sqrt{2}\sqrt{-2\cos\left(x\right)^{2}+1}-\sqrt{2}\right)^{2}}{\cos\left(x\right)^{2}}-4\right)\cos\left(x\right)}{4\left(\sqrt{2}\sqrt{-2\cos\left(x\right)^{2}+1}-\sqrt{2}\right)}\right)\right)+2\arctan\left(-\frac{\sqrt{2}\left(\frac{\left(\sqrt{2}\sqrt{-2\cos\left(x\right)^{2}+1}-\sqrt{2}\right)^{2}}{\cos\left(x\right)^{2}}-4\right)\cos\left(x\right)}{4\left(\sqrt{2}\sqrt{-2\cos\left(x\right)^{2}+1}-\sqrt{2}\right)}\right)\right)\mathrm{sgn}\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(pi - sqrt(2)*pi - 2*sqrt(2)*arctan(-1/2*I*sqrt(2)) + 2*arctan(-I))*sg
n(sin(x)) + 1/2*(pi*sgn(cos(x)) - sqrt(2)*(pi*sgn(cos(x)) + 2*arctan(-1/4*(
(sqrt(2)*sqrt(-2*cos(x)^2 + 1) - sqrt(2))^2/cos(x)^2 - 4)*cos(x)/(sqrt(2)*s
qrt(-2*cos(x)^2 + 1) - sqrt(2)))) + 2*arctan(-1/4*sqrt(2)*((sqrt(2)*sqrt(-2
*cos(x)^2 + 1) - sqrt(2))^2/cos(x)^2 - 4)*cos(x)/(sqrt(2)*sqrt(-2*cos(x)^2
+ 1) - sqrt(2))))*sgn(sin(x))
```

Mupad [B]
time = 0.96, size = 88, normalized size = 2.75

$$\mathrm{asin}\left(\cot\left(x\right)\right)-\frac{\sqrt{2}\,\ln\left(\frac{\frac{\sqrt{2}\,\left(-1+\cot\left(x\right)\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{2}-\sqrt{1-{\cot\left(x\right)}^2}\,1{}\mathrm{i}}{\cot\left(x\right)-\mathrm{i}}\right)\,1{}\mathrm{i}}{2}+\frac{\sqrt{2}\,\ln\left(\frac{\frac{\sqrt{2}\,\left(1+\cot\left(x\right)\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{2}+\sqrt{1-{\cot\left(x\right)}^2}\,1{}\mathrm{i}}{\cot\left(x\right)+1{}\mathrm{i}}\right)\,1{}\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - cot(x)^2)^(1/2),x)
```

```
[Out] asin(cot(x)) - (2^(1/2)*log(((2^(1/2)*(cot(x)*1i - 1)*1i)/2 - (1 - cot(x)^2
)^(1/2)*1i)/(cot(x) - 1i))*1i)/2 + (2^(1/2)*log(((2^(1/2)*(cot(x)*1i + 1)*1
i)/2 + (1 - cot(x)^2)^(1/2)*1i)/(cot(x) + 1i))*1i)/2
```

3.40 $\int \frac{1}{\sqrt{1-\cot^2(x)}}\,dx$

Optimal. Leaf size=28

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{1-\cot^2(x)}}\right)}{\sqrt{2}}$$

```
[Out] -1/2*arctan(cot(x)*2^(1/2)/(1-cot(x)^2)^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3742, 385, 209}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{1-\cot^2(x)}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 - Cot[x]^2],x]
```

```
[Out] -(ArcTan[(Sqrt[2]*Cot[x])/Sqrt[1 - Cot[x]^2]]/Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{1-\cot^2(x)}}\,dx &= -\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\,(1+x^2)}\,dx, x, \cot(x)\right)\\
&= -\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{\cot(x)}{\sqrt{1-\cot^2(x)}}\right)\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{1-\cot^2(x)}}\right)}{\sqrt{2}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.07, size = 42, normalized size = 1.50

$$
-\frac{\sqrt{\cos(2x)}\csc(x)\log\left(\sqrt{2}\cos(x)+\sqrt{\cos(2x)}\right)}{\sqrt{2-2\cot^2(x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 - Cot[x]^2],x]
```

```
[Out] -((Sqrt[Cos[2*x]]*Csc[x]*Log[Sqrt[2]*Cos[x] + Sqrt[Cos[2*x]]])/Sqrt[2 - 2*C
ot[x]^2])
```

Maple [**A**]
time = 0.26, size = 31, normalized size = 1.11

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{1-(\cot^2(x))}\,\cot(x)}{-1+\cot^2(x)}\right)}{2}$	31
default	$\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{1-(\cot^2(x))}\,\cot(x)}{-1+\cot^2(x)}\right)}{2}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*2^(1/2)*arctan(2^(1/2)*(1-cot(x)^2)^(1/2)/(-1+cot(x)^2)*cot(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(22) = 44$.
time = 0.55, size = 90, normalized size = 3.21

$$\frac{1}{4}\sqrt{2}\arctan\left(\left(\cos\left(4x\right)^{2}+\sin\left(4x\right)^{2}+2\cos\left(4x\right)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\arctan\left(\sin\left(4x\right),\cos\left(4x\right)+1\right)\right)+\sin\left(2x\right),\left(\cos\left(4x\right)^{2}+\sin\left(4x\right)^{2}+2\cos\left(4x\right)+1\right)^{\frac{1}{4}}\cos\left(\frac{1}{2}\arctan\left(\sin\left(4x\right),\cos\left(4x\right)+1\right)\right)+\cos\left(2x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan2((cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*sin(1/
2*arctan2(sin(4*x), cos(4*x) + 1)) + sin(2*x), (cos(4*x)^2 + sin(4*x)^2 + 2
*cos(4*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + cos(2*x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(22) = 44$.
time = 3.13, size = 56, normalized size = 2.00

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{2}\cos\left(2x\right)+\sqrt{2}\right)\sqrt{\frac{\cos\left(2x\right)}{\cos\left(2x\right)-1}}\sin\left(2x\right)}{4\left(\cos\left(2x\right)^{2}+\cos\left(2x\right)\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(1/4*sqrt(2)*(2*sqrt(2)*cos(2*x) + sqrt(2))*sqrt(cos(2*x)
/(cos(2*x) - 1))*sin(2*x)/(cos(2*x)^2 + cos(2*x)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{1-\cot^{2}\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cot(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(1 - cot(x)**2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.45, size = 34, normalized size = 1.21

$$-\frac{1}{2}i\sqrt{2}\log\left(i\sqrt{2}+i\right)\operatorname{sgn}(\sin\left(x\right))-\frac{\sqrt{2}\arcsin\left(\sqrt{2}\cos\left(x\right)\right)}{2\operatorname{sgn}\left(\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*I*sqrt(2)*log(I*sqrt(2) + I)*sgn(sin(x)) - 1/2*sqrt(2)*arcsin(sqrt(2)*
cos(x))/sgn(sin(x))
```

Mupad [B]
time = 0.63, size = 85, normalized size = 3.04

$$-\frac{\sqrt{2}\ \ln\left(\frac{\frac{\sqrt{2}\,(-1+\cot(x)\,1\mathrm{i})\,1\mathrm{i}}{2}-\sqrt{1-\cot\left(x\right)^2}\ 1\mathrm{i}}{\cot(x)-\mathrm{i}}\right)\ 1\mathrm{i}}{4}+\frac{\sqrt{2}\ \ln\left(\frac{\frac{\sqrt{2}\,(1+\cot(x)\,1\mathrm{i})\,1\mathrm{i}}{2}+\sqrt{1-\cot\left(x\right)^2}\ 1\mathrm{i}}{\cot(x)+1\mathrm{i}}\right)\ 1\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - cot(x)^2)^(1/2),x)
```

```
[Out] (2^(1/2)*log(((2^(1/2)*(cot(x)*1i + 1)*1i)/2 + (1 - cot(x)^2)^(1/2)*1i)/(co
t(x) + 1i))*1i)/4 - (2^(1/2)*log(((2^(1/2)*(cot(x)*1i - 1)*1i)/2 - (1 - cot
(x)^2)^(1/2)*1i)/(cot(x) - 1i))*1i)/4
```

3.41 $\int \left(-1+\cot^2(x)\right)^{3/2} dx$

Optimal. Leaf size=61

$$\frac{5}{2}\tanh^{-1}\left(\frac{\cot(x)}{\sqrt{-1+\cot^2(x)}}\right)-2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{-1+\cot^2(x)}}\right)-\frac{1}{2}\cot(x)\sqrt{-1+\cot^2(x)}$$

```
[Out] 5/2*arctanh(cot(x)/(-1+cot(x)^2)^(1/2))-2*arctanh(cot(x)*2^(1/2)/(-1+cot(x)
^2)^(1/2))*2^(1/2)-1/2*cot(x)*(-1+cot(x)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3742, 427, 537, 223, 212, 385}

$$-\frac{1}{2}\cot(x)\sqrt{\cot^2(x)-1}+\frac{5}{2}\tanh^{-1}\left(\frac{\cot(x)}{\sqrt{\cot^2(x)-1}}\right)-2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{\cot^2(x)-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Cot[x]^2)^(3/2),x]
```

```
[Out] (5*ArcTanh[Cot[x]/Sqrt[-1 + Cot[x]^2]])/2 - 2*Sqrt[2]*ArcTanh[(Sqrt[2]*Cot[
x])/Sqrt[-1 + Cot[x]^2]] - (Cot[x]*Sqrt[-1 + Cot[x]^2])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
```

```
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}
\int \left(-1+\cot^2(x)\right)^{3/2}\,dx &= -\mathrm{Subst}\left(\int \frac{\left(-1+x^2\right)^{3/2}}{1+x^2}\,dx,x,\cot(x)\right)\\
&= -\frac{1}{2}\cot(x)\sqrt{-1+\cot^2(x)}-\frac{1}{2}\mathrm{Subst}\left(\int \frac{3-5x^2}{\sqrt{-1+x^2}\,\left(1+x^2\right)}\,dx,x,\cot(x)\right)\\
&= -\frac{1}{2}\cot(x)\sqrt{-1+\cot^2(x)}+\frac{5}{2}\mathrm{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}}\,dx,x,\cot(x)\right)-4\mathrm{Subst}\Bigg(\\
&= -\frac{1}{2}\cot(x)\sqrt{-1+\cot^2(x)}+\frac{5}{2}\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\cot(x)}{\sqrt{-1+\cot^2(x)}}\right)-4\mathrm{S}\\
&= \frac{5}{2}\tanh^{-1}\left(\frac{\cot(x)}{\sqrt{-1+\cot^2(x)}}\right)-2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\cot(x)}{\sqrt{-1+\cot^2(x)}}\right)-\frac{1}{2}\cot(x)
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 121, normalized size = 1.98

$$\frac{1}{2}\left(-1+\cot^2(x)\right)^{3/2}\sec^2(2x)\left(\mathrm{ArcTan}\left(\frac{\cos(x)}{\sqrt{-\cos(2x)}}\right)\sqrt{-\cos(2x)}\,\sin^3(x)+4\tanh^{-1}\left(\frac{\cos(x)}{\sqrt{\cos(2x)}}\right)\sqrt{\cos(2x)}\,\sin^3(x)-4\sqrt{2}\,\sqrt{\cos(2x)}\,\log\left(\sqrt{2}\,\cos(x)+\sqrt{\cos(2x)}\right)\sin^3(x)-\frac{1}{4}\sin(4x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Cot[x]^2)^(3/2),x]
```

```
[Out] ((-1 + Cot[x]^2)^(3/2)*Sec[2*x]^2*(ArcTan[Cos[x]/Sqrt[-Cos[2*x]]]*Sqrt[-Cos
[2*x]]*Sin[x]^3 + 4*ArcTanh[Cos[x]/Sqrt[Cos[2*x]]]*Sqrt[Cos[2*x]]*Sin[x]^3
- 4*Sqrt[2]*Sqrt[Cos[2*x]]*Log[Sqrt[2]*Cos[x] + Sqrt[Cos[2*x]]]*Sin[x]^3 -
Sin[4*x]/4))/2
```

<u>Maple</u> [A]
time = 0.18, size = 48, normalized size = 0.79

method	result
derivativedivides	$-\frac{\cot(x)\sqrt{-1+\cot^2(x)}}{2}+\frac{5\ln\left(\cot(x)+\sqrt{-1+\cot^2(x)}\right)}{2}-2\,\mathrm{arctanh}\left(\frac{\cot(x)\sqrt{2}}{\sqrt{-1+\cot^2(x)}}\right)$
default	$-\frac{\cot(x)\sqrt{-1+\cot^2(x)}}{2}+\frac{5\ln\left(\cot(x)+\sqrt{-1+\cot^2(x)}\right)}{2}-2\,\mathrm{arctanh}\left(\frac{\cot(x)\sqrt{2}}{\sqrt{-1+\cot^2(x)}}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+cot(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*cot(x)*(-1+cot(x)^2)^(1/2)+5/2*ln(cot(x)+(-1+cot(x)^2)^(1/2))-2*arctan
h(cot(x)*2^(1/2)/(-1+cot(x)^2)^(1/2))*2^(1/2)
```

<u>Maxima</u> [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cot(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((cot(x)^2 - 1)^(3/2), x)
```

<u>Fricas</u> [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(47) = 94$.
time = 2.64, size = 170, normalized size = 2.79

$$\frac{4\sqrt{2}\log\left(2\sqrt{-\frac{\cos(2x)}{\cos(2x)-1}}\sin(2x)-2\cos(2x)-1\right)\sin(2x)-2\sqrt{2}\sqrt{-\frac{\cos(2x)}{\cos(2x)-1}}(\cos(2x)+1)+5\log\left(\frac{\sqrt{2}\sqrt{-\frac{\cos(2x)}{\cos(2x)-1}}\sin(2x)+\cos(2x)+1}{\cos(2x)+1}\right)\sin(2x)-5\log\left(\frac{\sqrt{2}\sqrt{-\frac{\cos(2x)}{\cos(2x)-1}}\sin(2x)-\cos(2x)-1}{\cos(2x)+1}\right)\sin(2x)}{4\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cot(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*log(2*sqrt(-cos(2*x)/(cos(2*x) - 1))*sin(2*x) - 2*cos(2*x) -
 1)*sin(2*x) - 2*sqrt(2)*sqrt(-cos(2*x)/(cos(2*x) - 1))*(cos(2*x) + 1) + 5*
log((sqrt(2)*sqrt(-cos(2*x)/(cos(2*x) - 1))*sin(2*x) + cos(2*x) + 1)/(cos(2
*x) + 1))*sin(2*x) - 5*log((sqrt(2)*sqrt(-cos(2*x)/(cos(2*x) - 1))*sin(2*x)
 - cos(2*x) - 1)/(cos(2*x) + 1))*sin(2*x))/sin(2*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\cot^{2}\left(x\right) - 1\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cot(x)**2)**(3/2),x)
```

```
[Out] Integral((cot(x)**2 - 1)**(3/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(47) = 94.
time = 0.74, size = 179, normalized size = 2.93

$$\frac{1}{4} \left(4 \sqrt{2} \log\left(\left(\sqrt{2} \cos\left(x\right) - \sqrt{2 \cos\left(x\right)^{2} - 1}\right)^{2}\right) - \frac{4 \sqrt{2} \left(3 \left(\sqrt{2} \cos\left(x\right) - \sqrt{2 \cos\left(x\right)^{2} - 1}\right)^{2} - 1\right)}{\left(\sqrt{2} \cos\left(x\right) - \sqrt{2 \cos\left(x\right)^{2} - 1}\right)^{4} - 6 \left(\sqrt{2} \cos\left(x\right) - \sqrt{2 \cos\left(x\right)^{2} - 1}\right)^{2} + 1} + 5 \log\left(\frac{\left|2 \left(\sqrt{2} \cos\left(x\right) - \sqrt{2 \cos\left(x\right)^{2} - 1}\right)^{2} - 4 \sqrt{2} - 6\right|}{\left|2 \left(\sqrt{2} \cos\left(x\right) - \sqrt{2 \cos\left(x\right)^{2} - 1}\right)^{2} + 4 \sqrt{2} - 6\right|}\right)\right) \mathrm{sgn}\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cot(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(4*sqrt(2)*log((sqrt(2)*cos(x) - sqrt(2*cos(x)^2 - 1))^2) - 4*sqrt(2)*(
3*(sqrt(2)*cos(x) - sqrt(2*cos(x)^2 - 1))^2 - 1)/((sqrt(2)*cos(x) - sqrt(2*
cos(x)^2 - 1))^4 - 6*(sqrt(2)*cos(x) - sqrt(2*cos(x)^2 - 1))^2 + 1) + 5*log
(abs(2*(sqrt(2)*cos(x) - sqrt(2*cos(x)^2 - 1))^2 - 4*sqrt(2) - 6)/abs(2*(sq
rt(2)*cos(x) - sqrt(2*cos(x)^2 - 1))^2 + 4*sqrt(2) - 6)))*sgn(sin(x))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left({\mathrm{cot}\left(x\right)}^2-1\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(x)^2 - 1)^(3/2),x)
```

```
[Out] int((cot(x)^2 - 1)^(3/2), x)
```

3.42 $\int \sqrt{-1+\cot^2(x)}\,dx$

Optimal. Leaf size=42

$$-\tanh^{-1}\left(\frac{\cot(x)}{\sqrt{-1+\cot^2(x)}}\right)+\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{-1+\cot^2(x)}}\right)$$

```
[Out] -arctanh(cot(x)/(-1+cot(x)^2)^(1/2))+arctanh(cot(x)*2^(1/2)/(-1+cot(x)^2)^(
1/2))*2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.500$, Rules used = {3742, 399, 223, 212, 385}

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{\cot^2(x)-1}}\right)-\tanh^{-1}\left(\frac{\cot(x)}{\sqrt{\cot^2(x)-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + Cot[x]^2],x]
```

```
[Out] -ArcTanh[Cot[x]/Sqrt[-1 + Cot[x]^2]] + Sqrt[2]*ArcTanh[(Sqrt[2]*Cot[x])/Sqr
t[-1 + Cot[x]^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
```

```
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{-1+\cot^2(x)}\,dx &= -\text{Subst}\left(\int \frac{\sqrt{-1+x^2}}{1+x^2}\,dx, x, \cot(x)\right)\\
&= 2\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}\,(1+x^2)}\,dx, x, \cot(x)\right) - \text{Subst}\Bigg(\int \frac{1}{\sqrt{-1+x^2}}\,dx, x, \cot(\\
&= 2\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{\cot(x)}{\sqrt{-1+\cot^2(x)}}\right) - \text{Subst}\Bigg(\int \frac{1}{1-x^2}\,dx, x, \frac{\cot(}{\sqrt{-1+\mathrm{c}}}\\
&= -\tanh^{-1}\left(\frac{\cot(x)}{\sqrt{-1+\cot^2(x)}}\right) + \sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{-1+\cot^2(x)}}\right)
\end{aligned}
$$

Mathematica [**A**]
time = 0.05, size = 60, normalized size = 1.43

$$
\frac{\sqrt{-1+\cot^2(x)}\left(-\tanh^{-1}\left(\frac{\cos(x)}{\sqrt{\cos(2x)}}\right)+\sqrt{2}\log\left(\sqrt{2}\cos(x)+\sqrt{\cos(2x)}\right)\right)\sin(x)}{\sqrt{\cos(2x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + Cot[x]^2],x]
```

```
[Out] (Sqrt[-1 + Cot[x]^2]*(-ArcTanh[Cos[x]/Sqrt[Cos[2*x]]] + Sqrt[2]*Log[Sqrt[2]
*Cos[x] + Sqrt[Cos[2*x]]])*Sin[x])/Sqrt[Cos[2*x]]
```

Maple [**A**]
time = 0.17, size = 35, normalized size = 0.83

method	result	size
derivativedivides	$-\ln\left(\cot(x)+\sqrt{-1+\cot^2(x)}\right)+\operatorname{arctanh}\left(\frac{\cot(x)\sqrt{2}}{\sqrt{-1+\cot^2(x)}}\right)\sqrt{2}$	35
default	$-\ln\left(\cot(x)+\sqrt{-1+\cot^2(x)}\right)+\operatorname{arctanh}\left(\frac{\cot(x)\sqrt{2}}{\sqrt{-1+\cot^2(x)}}\right)\sqrt{2}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(cot(x)+(-1+cot(x)^2)^(1/2))+arctanh(cot(x)*2^(1/2)/(-1+cot(x)^2)^(1/2))
*2^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(34) = 68$.
time = 2.61, size = 123, normalized size = 2.93

$$\frac{1}{2}\sqrt{2}\log\left(-2\sqrt{-\frac{\cos(2x)}{\cos(2x)-1}}\sin(2x)-2\cos(2x)-1\right)-\frac{1}{2}\log\left(\frac{\sqrt{2}\sqrt{-\frac{\cos(2x)}{\cos(2x)-1}}\sin(2x)+\cos(2x)+1}{\cos(2x)+1}\right)+\frac{1}{2}\log\left(\frac{\sqrt{2}\sqrt{-\frac{\cos(2x)}{\cos(2x)-1}}\sin(2x)-\cos(2x)-1}{\cos(2x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(-2*sqrt(-cos(2*x)/(cos(2*x) - 1))*sin(2*x) - 2*cos(2*x) - 1
) - 1/2*log((sqrt(2)*sqrt(-cos(2*x)/(cos(2*x) - 1))*sin(2*x) + cos(2*x) + 1
)/(cos(2*x) + 1)) + 1/2*log((sqrt(2)*sqrt(-cos(2*x)/(cos(2*x) - 1))*sin(2*x
) - cos(2*x) - 1)/(cos(2*x) + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\cot^2(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cot(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(cot(x)**2 - 1), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 0.43, size = 34, normalized size = 0.81

$$\sqrt{2}\ \mathrm{atanh}\left(\frac{\sqrt{2}\ \cot(x)}{\sqrt{\cot(x)^2-1}}\right)-\ln\left(\cot(x)+\sqrt{\cot(x)^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(x)^2 - 1)^(1/2),x)
```

```
[Out] 2^(1/2)*atanh((2^(1/2)*cot(x))/(cot(x)^2 - 1)^(1/2)) - log(cot(x) + (cot(x)
^2 - 1)^(1/2))
```

3.43 $\int \frac{1}{\sqrt{-1+\cot^2(x)}}\,dx$

Optimal. Leaf size=26

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{-1+\cot^2(x)}}\right)}{\sqrt{2}}$$

[Out] -1/2*arctanh(cot(x)*2^(1/2)/(-1+cot(x)^2)^(1/2))*2^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3742, 385, 212}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{\cot^2(x)-1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-1 + Cot[x]^2],x]

[Out] -(ArcTanh[(Sqrt[2]*Cot[x])/Sqrt[-1 + Cot[x]^2]]/Sqrt[2])

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+\cot^2(x)}}\,dx = -\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}\,(1+x^2)}\,dx, x, \cot(x)\right)$$
$$= -\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{\cot(x)}{\sqrt{-1+\cot^2(x)}}\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{2}\cot(x)}{\sqrt{-1+\cot^2(x)}}\right)}{\sqrt{2}}$$

Mathematica [A]
time = 0.03, size = 45, normalized size = 1.73

$$-\frac{\sqrt{\cos(2x)}\csc(x)\log\left(\sqrt{2}\cos(x)+\sqrt{\cos(2x)}\right)}{\sqrt{2}\sqrt{-1+\cot^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 + Cot[x]^2],x]
[Out] -((Sqrt[Cos[2*x]]*Csc[x]*Log[Sqrt[2]*Cos[x] + Sqrt[Cos[2*x]]])/(Sqrt[2]*Sqr
t[-1 + Cot[x]^2]))
```

Maple [A]
time = 0.18, size = 21, normalized size = 0.81

method	result	size
derivativedivides	$-\frac{\operatorname{arctanh}\left(\frac{\cot(x)\sqrt{2}}{\sqrt{-1+\cot^2(x)}}\right)\sqrt{2}}{2}$	21
default	$-\frac{\operatorname{arctanh}\left(\frac{\cot(x)\sqrt{2}}{\sqrt{-1+\cot^2(x)}}\right)\sqrt{2}}{2}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*arctanh(cot(x)*2^(1/2)/(-1+cot(x)^2)^(1/2))*2^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(20) = 40$.
time = 0.56, size = 143, normalized size = 5.50

$$-\frac{1}{8}\sqrt{2}\left(2\operatorname{arsinh}(1)+\log\left(\cos(2x)^2+\sin(2x)^2+\sqrt{\cos(4x)^2+\sin(4x)^2+2\cos(4x)+1}\left(\cos\left(\frac{1}{2}\arctan(\sin(4x),\cos(4x)+1)\right)^2+\sin\left(\frac{1}{2}\arctan(\sin(4x),\cos(4x)+1)\right)^2\right)+2\left(\cos(4x)^2+\sin(4x)^2+2\cos(4x)+1\right)^{\frac{1}{4}}\left(\cos(2x)\cos\left(\frac{1}{2}\arctan(\sin(4x),\cos(4x)+1)\right)+\sin(2x)\sin\left(\frac{1}{2}\arctan(\sin(4x),\cos(4x)+1)\right)\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*(2*arcsinh(1) + log(cos(2*x)^2 + sin(2*x)^2 + sqrt(cos(4*x)^2
+ sin(4*x)^2 + 2*cos(4*x) + 1)*(cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2
+ sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2) + 2*(cos(4*x)^2 + sin(4*x)^2
+ 2*cos(4*x) + 1)^(1/4)*(cos(2*x)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))
+ sin(2*x)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1)))))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(20) = 40$.
time = 3.39, size = 60, normalized size = 2.31

$$\frac{1}{8}\sqrt{2}\log\left(2\sqrt{2}\left(2\sqrt{2}\cos(2x)+\sqrt{2}\right)\sqrt{-\frac{\cos(2x)}{\cos(2x)-1}}\sin(2x)-8\cos(2x)^2-8\cos(2x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log(2*sqrt(2)*(2*sqrt(2)*cos(2*x) + sqrt(2))*sqrt(-cos(2*x)/(co
s(2*x) - 1))*sin(2*x) - 8*cos(2*x)^2 - 8*cos(2*x) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{\cot^2(x)-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+cot(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(cot(x)**2 - 1), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(20) = 40$.
time = 0.43, size = 45, normalized size = 1.73

$$-\frac{1}{2}\sqrt{2}\log\left(\sqrt{2}-1\right)\operatorname{sgn}(\sin(x))+\frac{\sqrt{2}\log\left(\left|-\sqrt{2}\cos(x)+\sqrt{2\cos(x)^2-1}\right|\right)}{2\operatorname{sgn}(\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(sqrt(2) - 1)*sgn(sin(x)) + 1/2*sqrt(2)*log(abs(-sqrt(2)*co
s(x) + sqrt(2*cos(x)^2 - 1)))/sgn(sin(x))
```

Mupad [B]
time = 0.50, size = 20, normalized size = 0.77

$$-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\cot(x)}{\sqrt{\cot\left(x\right)^2-1}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)^2 - 1)^(1/2),x)
```

```
[Out] -(2^(1/2)*atanh((2^(1/2)*cot(x))/(cot(x)^2 - 1)^(1/2)))/2
```

3.44 $\int \frac{\cot^3(x)}{\sqrt{a+b\cot^2(x)}}\,dx$

Optimal. Leaf size=52

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{\sqrt{a+b\cot^2(x)}}{b}$$

```
[Out] -arctanh((a+b*cot(x)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)-(a+b*cot(x)^2)^(1/2)
/b
```

Rubi [A]
time = 0.07, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3751, 457, 81, 65, 214}

$$-\frac{\sqrt{a+b\cot^2(x)}}{b}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^3/Sqrt[a + b*Cot[x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]]/Sqrt[a - b]) - Sqrt[a + b*Cot[x
]^2]/b
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^3(x)}{\sqrt{a+b\cot^2(x)}}\,dx &= -\text{Subst}\left(\int \frac{x^3}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \cot(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{x}{(1+x)\sqrt{a+bx}}\,dx, x, \cot^2(x)\right)\right)\\
&= -\frac{\sqrt{a+b\cot^2(x)}}{b} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \cot^2(x)\right)\\
&= -\frac{\sqrt{a+b\cot^2(x)}}{b} + \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cot^2(x)}\right)}{b}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} - \frac{\sqrt{a+b\cot^2(x)}}{b}
\end{aligned}
$$

Mathematica [A]

time = 0.16, size = 52, normalized size = 1.00

$$-\frac{\frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\cot^{2}(x)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}+\sqrt{a+b\cot^{2}(x)}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3/Sqrt[a + b*Cot[x]^2],x]
```

```
[Out] -(((b*ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]])/Sqrt[a - b] + Sqrt[a + b*C
ot[x]^2])/b)
```

Maple [A]
time = 0.13, size = 44, normalized size = 0.85

method	result	size
derivativedivides	$-\frac{\sqrt{a+b\left(\cot^{2}\left(x\right)\right)}}{b}+\frac{\arctan\left(\frac{\sqrt{a+b\left(\cot^{2}\left(x\right)\right)}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$	44
default	$-\frac{\sqrt{a+b\left(\cot^{2}\left(x\right)\right)}}{b}+\frac{\arctan\left(\frac{\sqrt{a+b\left(\cot^{2}\left(x\right)\right)}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3/(a+b*cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(a+b*cot(x)^2)^(1/2)/b+1/(-a+b)^(1/2)*arctan((a+b*cot(x)^2)^(1/2)/(-a+b)^(
1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+b*cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(44) = 88$.
time = 3.66, size = 284, normalized size = 5.46

$$\left[\frac{\sqrt{a-b}\, b \log\left(-2\,(a^2-2\,ab+b^2)\cos(2\,x)^2 - 2\,a^2 + b^2 + 2\,((a-b)\cos(2\,x)^2 - (2\,a-b)\cos(2\,x) + a)\sqrt{a-b}\sqrt{\frac{(a-b)\cos(2\,x)-a-b}{\cos(2\,x)-1}} + 4\,(a^2-ab)\cos(2\,x)\right) - 4\,(a-b)\sqrt{\frac{(a-b)\cos(2\,x)-a-b}{\cos(2\,x)-1}}}{4\,(ab-b^2)}, -\frac{\sqrt{-a+b}\, b \arctan\left(-\frac{\sqrt{-a+b}\sqrt{\frac{(a-b)\cos(2\,x)-a-b}{\cos(2\,x)-1}}\,(\cos(2\,x)-1)}{(a-b)\cos(2\,x)-a}\right) + 2\,(a-b)\sqrt{\frac{(a-b)\cos(2\,x)-a-b}{\cos(2\,x)-1}}}{2\,(ab-b^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+b*cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a - b)*b*log(-2*(a^2 - 2*a*b + b^2)*cos(2*x)^2 - 2*a^2 + b^2 + 2
*((a - b)*cos(2*x)^2 - (2*a - b)*cos(2*x) + a)*sqrt(a - b)*sqrt(((a - b)*co
s(2*x) - a - b)/(cos(2*x) - 1)) + 4*(a^2 - a*b)*cos(2*x)) - 4*(a - b)*sqrt(
((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)))/(a*b - b^2), -1/2*(sqrt(-a + b)
*b*arctan(-sqrt(-a + b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*(co
s(2*x) - 1)/((a - b)*cos(2*x) - a)) + 2*(a - b)*sqrt(((a - b)*cos(2*x) - a
- b)/(cos(2*x) - 1)))/(a*b - b^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(x)}{\sqrt{a + b\cot^2(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**3/(a+b*cot(x)**2)**(1/2),x)
```

```
[Out] Integral(cot(x)**3/sqrt(a + b*cot(x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(44) = 88$.
time = 0.48, size = 96, normalized size = 1.85

$$\frac{\frac{\log\left(\left(\sqrt{a-b}\sin(x) - \sqrt{a\sin(x)^2 - b\sin(x)^2 + b}\right)^2\right)}{\sqrt{a-b}} + \frac{4\sqrt{a-b}}{\left(\sqrt{a-b}\sin(x) - \sqrt{a\sin(x)^2 - b\sin(x)^2 + b}\right)^2 - b}}{2\,\mathrm{sgn}(\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+b*cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(log((sqrt(a - b)*sin(x) - sqrt(a*sin(x)^2 - b*sin(x)^2 + b))^2)/sqrt(a
 - b) + 4*sqrt(a - b)/((sqrt(a - b)*sin(x) - sqrt(a*sin(x)^2 - b*sin(x)^2 +
 b))^2 - b))/sgn(sin(x))
```

Mupad [B]
time = 1.21, size = 44, normalized size = 0.85

$$-\frac{\sqrt{b\cot\left(x\right)^2+a}}{b}-\frac{\mathrm{atanh}\left(\frac{\sqrt{b\cot\left(x\right)^2+a}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3/(a + b*cot(x)^2)^(1/2),x)
[Out] - (a + b*cot(x)^2)^(1/2)/b - atanh((a + b*cot(x)^2)^(1/2)/(a - b)^(1/2))/(a
 - b)^(1/2)
```

3.45 $\int \frac{\cot^2(x)}{\sqrt{a+b\cot^2(x)}}\,dx$

Optimal. Leaf size=64

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{\sqrt{a-b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{\sqrt{b}}$$

```
[Out] arctan(cot(x)*(a-b)^(1/2)/(a+b*cot(x)^2)^(1/2))/(a-b)^(1/2)-arctanh(cot(x)*
b^(1/2)/(a+b*cot(x)^2)^(1/2))/b^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3751, 494, 223, 212, 385, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{\sqrt{a-b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2/Sqrt[a + b*Cot[x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Cot[x])/Sqrt[a + b*Cot[x]^2]]/Sqrt[a - b] - ArcTanh[(Sq
rt[b]*Cot[x])/Sqrt[a + b*Cot[x]^2]]/Sqrt[b]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^2(x)}{\sqrt{a+b\cot^2(x)}}\,dx &= -\text{Subst}\left(\int \frac{x^2}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \cot(x)\right)\\ &= -\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \cot(x)\right) + \text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \cot(x)\right.\\ &= -\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\cot(x)}{\sqrt{a+b\cot^2(x)}}\right) + \text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, -\right.\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{\sqrt{a-b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 158 vs. 2(64) = 128.

time = 0.29, size = 158, normalized size = 2.47

$$\frac{\left(-\sqrt{-b}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a-b}\,\cos(x)}{\sqrt{-a-b+(a-b)\cos(2x)}}\right)+\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{-b}\,\cos(x)}{\sqrt{-a-b+(a-b)\cos(2x)}}\right)\right)\sqrt{(a+b+(-a+b)\cos(2x))\csc^2(x)}\,\sin(x)}{\sqrt{a-b}\,\sqrt{-b}\,\sqrt{-a-b+(a-b)\cos(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2/Sqrt[a + b*Cot[x]^2],x]
```

```
[Out] ((-(Sqrt[-b]*ArcTanh[(Sqrt[2]*Sqrt[a - b]*Cos[x])/Sqrt[-a - b + (a - b)*Cos
[2*x]]]) + Sqrt[a - b]*ArcTanh[(Sqrt[2]*Sqrt[-b]*Cos[x])/Sqrt[-a - b + (a -
 b)*Cos[2*x]]])*Sqrt[(a + b + (-a + b)*Cos[2*x])*Csc[x]^2]*Sin[x])/(Sqrt[a
- b]*Sqrt[-b]*Sqrt[-a - b + (a - b)*Cos[2*x]])
```

Maple [A]
time = 0.17, size = 80, normalized size = 1.25

method	result
derivativedivides	$-\frac{\ln\left(\sqrt{b}\,\cot(x)+\sqrt{a+b\left(\cot^2(x)\right)}\right)}{\sqrt{b}}+\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\cot(x)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\left(\cot^2(x\right.}}\right.}{b^2(a-b)}$
default	$-\frac{\ln\left(\sqrt{b}\,\cot(x)+\sqrt{a+b\left(\cot^2(x)\right)}\right)}{\sqrt{b}}+\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\cot(x)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\left(\cot^2(x\right.}}\right.}{b^2(a-b)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/(a+b*cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(b^(1/2)*cot(x)+(a+b*cot(x)^2)^(1/2))/b^(1/2)+(b^4*(a-b))^(1/2)/b^2/(a-b
)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*cot(x)^2)^(1/2)*cot(x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+b*cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(x)^2/sqrt(b*cot(x)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(52) = 104$.
time = 2.98, size = 588, normalized size = 9.19

$$\left[-\frac{\sqrt{-a+b}\,b\log\left(-(a-b)\cos(2x)+\sqrt{-a+b}\,\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\,\sin(2x)+b\right)-(a-b)\sqrt{b}\,\log\left(\frac{(a-2b)\cos(2x)+2\sqrt{b}\,\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\,\sin(2x)-a-2b}{\cos(2x)-1}\right)}{2(ab-b^2)},\ \frac{2(a-b)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\,\sin(2x)}{b\cos(2x)+b}\right)-\sqrt{-a+b}\,b\log\left(-(a-b)\cos(2x)+\sqrt{-a+b}\,\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\,\sin(2x)+b\right)}{2(ab-b^2)},\ \frac{2\sqrt{a-b}\,b\arctan\left(-\frac{\sqrt{a-b}\,\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\,\sin(2x)}{(a-b)\cos(2x)+a-b}\right)+(a-b)\sqrt{b}\,\log\left(\frac{(a-2b)\cos(2x)+2\sqrt{b}\,\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\,\sin(2x)-a-2b}{\cos(2x)-1}\right)}{2(ab-b^2)},\ \frac{\sqrt{a-b}\,b\arctan\left(-\frac{\sqrt{a-b}\,\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\,\sin(2x)}{(a-b)\cos(2x)+a-b}\right)+(a-b)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\,\sin(2x)}{b\cos(2x)+b}\right)}{ab-b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+b*cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a + b)*b*log(-(a - b)*cos(2*x) + sqrt(-a + b)*sqrt(((a - b)*co
s(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x) + b) - (a - b)*sqrt(b)*log(((a - 2
*b)*cos(2*x) + 2*sqrt(b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*si
n(2*x) - a - 2*b)/(cos(2*x) - 1)))/(a*b - b^2), 1/2*(2*(a - b)*sqrt(-b)*arc
tan(sqrt(-b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x)/(b*co
s(2*x) + b)) - sqrt(-a + b)*b*log(-(a - b)*cos(2*x) + sqrt(-a + b)*sqrt(((a
 - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x) + b))/(a*b - b^2), 1/2*(2*
sqrt(a - b)*b*arctan(-sqrt(a - b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x)
 - 1))*sin(2*x)/((a - b)*cos(2*x) + a - b)) + (a - b)*sqrt(b)*log(((a - 2*b
)*cos(2*x) + 2*sqrt(b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(
2*x) - a - 2*b)/(cos(2*x) - 1)))/(a*b - b^2), (sqrt(a - b)*b*arctan(-sqrt(a
 - b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x)/((a - b)*cos
(2*x) + a - b)) + (a - b)*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(2*x) -
 a - b)/(cos(2*x) - 1))*sin(2*x)/(b*cos(2*x) + b)))/(a*b - b^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(x)}{\sqrt{a+b\cot^2(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2/(a+b*cot(x)**2)**(1/2),x)
```

```
[Out] Integral(cot(x)**2/sqrt(a + b*cot(x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(52) = 104.
time = 1.92, size = 229, normalized size = 3.58

$$\frac{\left(2\,a\arctan\left(\frac{\sqrt{-a+b}\,\sqrt{b}}{\sqrt{ab-b^2}}\right)-2\,b\arctan\left(\frac{\sqrt{-a+b}\,\sqrt{b}}{\sqrt{ab-b^2}}\right)+\sqrt{ab-b^2}\,\log\left(-a-2\,\sqrt{-a+b}\,\sqrt{b}+2\,b\right)\right)\mathrm{sgn}(\sin(x))}{2\,\sqrt{ab-b^2}\,\sqrt{-a+b}}-\frac{\frac{2\,\sqrt{-a+b}\arctan\left(\frac{\left(\sqrt{-a+b}\cos(x)-\sqrt{-a\cos(x)^2+b\cos(x)^2+a}\right)^2+a-2\,b}{2\,\sqrt{ab-b^2}}\right)}{\sqrt{ab-b^2}}+\frac{\log\left(\left(\sqrt{-a+b}\cos(x)-\sqrt{-a\cos(x)^2+b\cos(x)^2+a}\right)^2\right)}{\sqrt{-a+b}}}{2\,\mathrm{sgn}(\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+b*cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*a*arctan(sqrt(-a + b)*sqrt(b)/sqrt(a*b - b^2)) - 2*b*arctan(sqrt(-a
+ b)*sqrt(b)/sqrt(a*b - b^2)) + sqrt(a*b - b^2)*log(-a - 2*sqrt(-a + b)*sqr
t(b) + 2*b))*sgn(sin(x))/(sqrt(a*b - b^2)*sqrt(-a + b)) - 1/2*(2*sqrt(-a +
b)*arctan(1/2*((sqrt(-a + b)*cos(x) - sqrt(-a*cos(x)^2 + b*cos(x)^2 + a))^2
 + a - 2*b)/sqrt(a*b - b^2))/sqrt(a*b - b^2) + log((sqrt(-a + b)*cos(x) - s
qrt(-a*cos(x)^2 + b*cos(x)^2 + a))^2)/sqrt(-a + b))/sgn(sin(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cot(x)^2}{\sqrt{b\cot(x)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/(a + b*cot(x)^2)^(1/2),x)
```

```
[Out] int(cot(x)^2/(a + b*cot(x)^2)^(1/2), x)
```

3.46 $\int \frac{\cot(x)}{\sqrt{a+b\cot^2(x)}}\,dx$

Optimal. Leaf size=33

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

[Out] arctanh((a+b*cot(x)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)

Rubi [A]
time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3751, 455, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

[In] Int[Cot[x]/Sqrt[a + b*Cot[x]^2],x]

[Out] ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]]/Sqrt[a - b]

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(x)}{\sqrt{a+b\cot^2(x)}}\,dx &= -\mathrm{Subst}\left(\int \frac{x}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \cot(x)\right)\\ &= -\left(\frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \cot^2(x)\right)\right)\\ &= -\frac{\mathrm{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cot^2(x)}\right)}{b}\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/Sqrt[a + b*Cot[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]]/Sqrt[a - b]
```

Maple [A]
time = 0.21, size = 29, normalized size = 0.88

method	result	size

derivativedivides	$-\frac{\arctan\left(\frac{\sqrt{a+b\left(\cot^{2}\left(x\right)\right)}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$	29
default	$-\frac{\arctan\left(\frac{\sqrt{a+b\left(\cot^{2}\left(x\right)\right)}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a+b*cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(-a+b)^(1/2)*arctan((a+b*cot(x)^2)^(1/2)/(-a+b)^(1/2))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(27) = 54$.
time = 3.16, size = 127, normalized size = 3.85

$$\left[\frac{\log\left(-\sqrt{a-b}\sqrt{\frac{(a-b)\cos\left(2\,x\right)-a-b}{\cos\left(2\,x\right)-1}}\left(\cos\left(2\,x\right)-1\right)-(a-b)\cos\left(2\,x\right)+a\right)}{2\sqrt{a-b}},\ \frac{\sqrt{-a+b}\arctan\left(-\frac{\sqrt{-a+b}\sqrt{\frac{(a-b)\cos\left(2\,x\right)-a-b}{\cos\left(2\,x\right)-1}}}{a-b}\right)}{a-b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-sqrt(a - b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*(cos(
2*x) - 1) - (a - b)*cos(2*x) + a)/sqrt(a - b), sqrt(-a + b)*arctan(-sqrt(-a
 + b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))/(a - b))/(a - b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\sqrt{a + b\cot^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)**2)**(1/2),x)
```

```
[Out] Integral(cot(x)/sqrt(a + b*cot(x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(27) = 54.
time = 0.46, size = 61, normalized size = 1.85

$$\frac{\log(|b|)\operatorname{sgn}(\sin(x))}{2\sqrt{a-b}} - \frac{\log\left(\left|-\sqrt{a-b}\sin(x) + \sqrt{a\sin(x)^2 - b\sin(x)^2 + b}\right|\right)}{\sqrt{a-b}\operatorname{sgn}(\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b))*sgn(sin(x))/sqrt(a - b) - log(abs(-sqrt(a - b)*sin(x) + sqr
t(a*sin(x)^2 - b*sin(x)^2 + b)))/(sqrt(a - b)*sgn(sin(x)))
```

Mupad [B]
time = 0.96, size = 27, normalized size = 0.82

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b\cot(x)^2 + a}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a + b*cot(x)^2)^(1/2),x)
```

```
[Out] atanh((a + b*cot(x)^2)^(1/2)/(a - b)^(1/2))/(a - b)^(1/2)
```

3.47 $\int \frac{\tan(x)}{\sqrt{a+b\cot^2(x)}}\,dx$

Optimal. Leaf size=60

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

```
[Out] arctanh((a+b*cot(x)^2)^(1/2)/a^(1/2))/a^(1/2)-arctanh((a+b*cot(x)^2)^(1/2)/
(a-b)^(1/2))/(a-b)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3751, 457, 88, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/Sqrt[a + b*Cot[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a]]/Sqrt[a] - ArcTanh[Sqrt[a + b*Cot[x]^2
]/Sqrt[a - b]]/Sqrt[a - b]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan(x)}{\sqrt{a+b\cot^2(x)}}\,dx &= -\mathrm{Subst}\left(\int \frac{1}{x\left(1+x^2\right)\sqrt{a+bx^2}}\,dx, x, \cot(x)\right)\\
&= -\left(\frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{x(1+x)\sqrt{a+bx}}\,dx, x, \cot^2(x)\right)\right)\\
&= -\left(\frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \cot^2(x)\right)\right)+\frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x\right.\\
&= -\frac{\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cot^2(x)}\right)}{b}+\frac{\mathrm{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cot^2\right.}{b}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}
\end{aligned}
$$

Mathematica [A]

time = 0.05, size = 60, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/Sqrt[a + b*Cot[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a]]/Sqrt[a] - ArcTanh[Sqrt[a + b*Cot[x]^2
]/Sqrt[a - b]]/Sqrt[a - b]
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.63, size = 376, normalized size = 6.27

method	result
default	$-\frac{2\sqrt{2}\,\sqrt{\frac{\cos(x)\sqrt{a}\,\sqrt{a-b}-a\cos(x)+b\cos(x)-\sqrt{a}\,\sqrt{a-b}+a}{(\cos(x)+1)b}}\,\sqrt{-\frac{2\left(\cos(x)\sqrt{a}\,\sqrt{a-b}+a\cos(x)-b\cos(x)-\right.}{(\cos(x)+1)b}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a+b*cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*2^(1/2)*((cos(x)*a^(1/2)*(a-b)^(1/2)-a*cos(x)+b*cos(x)-a^(1/2)*(a-b)^(1/
2)+a)/(cos(x)+1)/b)^(1/2)*(-2*(cos(x)*a^(1/2)*(a-b)^(1/2)+a*cos(x)-b*cos(x)
-a^(1/2)*(a-b)^(1/2)-a)/(cos(x)+1)/b)^(1/2)*(EllipticPi((-1+cos(x))*((2*a^(
1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/sin(x),-1/(2*a^(1/2)*(a-b)^(1/2)-2*a+b)*b,
(-(2*a^(1/2)*(a-b)^(1/2)+2*a-b)/b)^(1/2)/((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^
(1/2))-EllipticPi((-1+cos(x))*((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/sin(x
),1/(2*a^(1/2)*(a-b)^(1/2)-2*a+b)*b,(-(2*a^(1/2)*(a-b)^(1/2)+2*a-b)/b)^(1/2
)/((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)))*sin(x)/((cos(x)^2*a-b*cos(x)^2-
a)/(cos(x)^2-1))^(1/2)/(-1+cos(x))/((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)/sqrt(b*cot(x)^2 + a), x)
```

Fricas [A]
time = 2.49, size = 419, normalized size = 6.98

$$\left[\frac{(a-b)\sqrt{a}\log\left(2\,a\tan(x)^2+2\,\sqrt{a}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan(x)^2+b\right)+\sqrt{a-b}\,a\log\left(\frac{(2a-b)\tan(x)^2-2\sqrt{a-b}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan(x)^2+b}{\tan(x)^2+1}\right)}{2\left(a^2-ab\right)},\ -\frac{2\,a\sqrt{-a+b}\arctan\left(-\frac{\sqrt{-a+b}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}}{a-b}\right)-(a-b)\sqrt{a}\log\left(2\,a\tan(x)^2+2\,\sqrt{a}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan(x)^2+b\right)}{2\left(a^2-ab\right)},\ -\frac{2\sqrt{-a}\,(a-b)\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}}{a}\right)-\sqrt{a-b}\,a\log\left(\frac{(2a-b)\tan(x)^2-2\sqrt{a-b}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan(x)^2+b}{\tan(x)^2+1}\right)}{2\left(a^2-ab\right)},\ -\frac{\sqrt{-a}\,(a-b)\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}}{a}\right)+a\sqrt{-a+b}\arctan\left(-\frac{\sqrt{-a+b}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}}{a-b}\right)}{a^2-ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a - b)*sqrt(a)*log(2*a*tan(x)^2 + 2*sqrt(a)*sqrt((a*tan(x)^2 + b)/ta
n(x)^2)*tan(x)^2 + b) + sqrt(a - b)*a*log(((2*a - b)*tan(x)^2 - 2*sqrt(a -
b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2 + b)/(tan(x)^2 + 1)))/(a^2 - a*
b), -1/2*(2*a*sqrt(-a + b)*arctan(-sqrt(-a + b)*sqrt((a*tan(x)^2 + b)/tan(x
)^2)/(a - b)) - (a - b)*sqrt(a)*log(2*a*tan(x)^2 + 2*sqrt(a)*sqrt((a*tan(x)
^2 + b)/tan(x)^2)*tan(x)^2 + b))/(a^2 - a*b), -1/2*(2*sqrt(-a)*(a - b)*arct
an(sqrt(-a)*sqrt((a*tan(x)^2 + b)/tan(x)^2)/a) - sqrt(a - b)*a*log(((2*a -
b)*tan(x)^2 - 2*sqrt(a - b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2 + b)/(
tan(x)^2 + 1)))/(a^2 - a*b), -(sqrt(-a)*(a - b)*arctan(sqrt(-a)*sqrt((a*tan
(x)^2 + b)/tan(x)^2)/a) + a*sqrt(-a + b)*arctan(-sqrt(-a + b)*sqrt((a*tan(x
)^2 + b)/tan(x)^2)/(a - b)))/(a^2 - a*b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{\sqrt{a+b\cot^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cot(x)**2)**(1/2),x)
```

```
[Out] Integral(tan(x)/sqrt(a + b*cot(x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(48) = 96.
time = 0.49, size = 236, normalized size = 3.93

$$-\frac{\left(2\,\sqrt{a-b}\,a\arctan\left(-\frac{a-b}{\sqrt{-a^2+ab}}\right)-2\,\sqrt{a-b}\,b\arctan\left(-\frac{a-b}{\sqrt{-a^2+ab}}\right)+\sqrt{-a^2+ab}\,\sqrt{a-b}\,\log(b)\right)\operatorname{sgn}(\sin(x))}{2\left(\sqrt{-a^2+ab}\,a-\sqrt{-a^2+ab}\,b\right)}+\frac{\frac{2\sqrt{a-b}\arctan\left(\frac{\left(\sqrt{a-b}\sin(x)-\sqrt{a\sin(x)^2-b\sin(x)^2+b}\right)^2-2a+b}{2\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}}+\frac{\log\left(\left(\sqrt{a-b}\sin(x)-\sqrt{a\sin(x)^2-b\sin(x)^2+b}\right)^2\right)}{\sqrt{a-b}}}{2\operatorname{sgn}(\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(2*sqrt(a - b)*a*arctan(-(a - b)/sqrt(-a^2 + a*b)) - 2*sqrt(a - b)*b*a
rctan(-(a - b)/sqrt(-a^2 + a*b)) + sqrt(-a^2 + a*b)*sqrt(a - b)*log(b))*sgn
```

```
(sin(x))/(sqrt(-a^2 + a*b)*a - sqrt(-a^2 + a*b)*b) + 1/2*(2*sqrt(a - b)*arc
tan(1/2*((sqrt(a - b)*sin(x) - sqrt(a*sin(x)^2 - b*sin(x)^2 + b))^2 - 2*a +
 b)/sqrt(-a^2 + a*b))/sqrt(-a^2 + a*b) + log((sqrt(a - b)*sin(x) - sqrt(a*s
in(x)^2 - b*sin(x)^2 + b))^2)/sqrt(a - b))/sgn(sin(x))
```

Mupad [B]
time = 0.51, size = 93, normalized size = 1.55

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\tan\left(x\right)^2}}}{\sqrt{a-b}}+\frac{2\,\sqrt{a-b}\,\sqrt{a+\frac{b}{\tan\left(x\right)^2}}}{b}-\frac{2\,a\,\sqrt{a+\frac{b}{\tan\left(x\right)^2}}}{b\,\sqrt{a-b}}\right)}{\sqrt{a-b}}+\frac{\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\tan\left(x\right)^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a + b*cot(x)^2)^(1/2),x)
```

```
[Out] atanh((a + b/tan(x)^2)^(1/2)/(a - b)^(1/2) + (2*(a - b)^(1/2)*(a + b/tan(x)
^2)^(1/2))/b - (2*a*(a + b/tan(x)^2)^(1/2))/(b*(a - b)^(1/2)))/(a - b)^(1/2
) + atanh((a + b/tan(x)^2)^(1/2)/a^(1/2))/a^(1/2)
```

3.48 $\int \frac{\tan^2(x)}{\sqrt{a+b\cot^2(x)}}\,dx$

Optimal. Leaf size=54

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{\sqrt{a-b}}+\frac{\sqrt{a+b\cot^2(x)}\tan(x)}{a}$$

```
[Out] arctan(cot(x)*(a-b)^(1/2)/(a+b*cot(x)^2)^(1/2))/(a-b)^(1/2)+(a+b*cot(x)^2)^
(1/2)*tan(x)/a
```

Rubi [A]
time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3751, 491, 12, 385, 209}

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{\sqrt{a-b}}+\frac{\tan(x)\sqrt{a+b\cot^2(x)}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^2/Sqrt[a + b*Cot[x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Cot[x])/Sqrt[a + b*Cot[x]^2]]/Sqrt[a - b] + (Sqrt[a + b
*Cot[x]^2]*Tan[x])/a
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^2(x)}{\sqrt{a+b\cot^2(x)}}\,dx &= -\text{Subst}\left(\int \frac{1}{x^2\left(1+x^2\right)\sqrt{a+bx^2}}\,dx,x,\cot(x)\right)\\
&= \frac{\sqrt{a+b\cot^2(x)}\,\tan(x)}{a}+\frac{\text{Subst}\left(\int \frac{a}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx,x,\cot(x)\right)}{a}\\
&= \frac{\sqrt{a+b\cot^2(x)}\,\tan(x)}{a}+\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx,x,\cot(x)\right)\\
&= \frac{\sqrt{a+b\cot^2(x)}\,\tan(x)}{a}+\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx,x,\frac{\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{\sqrt{a-b}}+\frac{\sqrt{a+b\cot^2(x)}\,\tan(x)}{a}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 1.67, size = 134, normalized size = 2.48

$$\frac{\left(1+\frac{b\cot^2(x)}{a}\right)\sin^2(x)\left(4(a-b)\cos^2(x)\left(a+b\cot^2(x)\right)\,{}_2F_1\left(2,2;\frac{5}{2};\frac{(a-b)\cos^2(x)}{a}\right)+\frac{3a\mathrm{ArcSin}\left(\sqrt{\frac{(a-b)\cos^2(x)}{a}}\right)(a+2b\cot^2(x))}{\sqrt{\frac{(a-b)\cos^2(x)\left(a+b\cot^2(x)\right)\sin^2(x)}{a^2}}}\right)\tan(x)}{3a^2\sqrt{a+b\cot^2(x)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[x]^2/Sqrt[a + b*Cot[x]^2],x]
```

```
[Out] ((1 + (b*Cot[x]^2)/a)*Sin[x]^2*(4*(a - b)*Cos[x]^2*(a + b*Cot[x]^2)*Hyperge
ometric2F1[2, 2, 5/2, ((a - b)*Cos[x]^2)/a] + (3*a*ArcSin[Sqrt[((a - b)*Cos
[x]^2)/a]]*(a + 2*b*Cot[x]^2))/Sqrt[((a - b)*Cos[x]^2*(a + b*Cot[x]^2)*Sin[
x]^2)/a^2])*Tan[x])/(3*a^2*Sqrt[a + b*Cot[x]^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(46) = 92$.
time = 0.68, size = 328, normalized size = 6.07

method	result
default	$-\frac{\sin(x)\left(\left(\cos^2(x)\right)\sqrt{-\frac{\left(\cos^2(x)\right)a-b\left(\cos^2(x)\right)-a}{(\cos(x)+1)^2}}\ \ln\left(4\cos(x)\sqrt{-a+b}\ \sqrt{-\frac{\left(\cos^2(x)\right)a-b\left(\cos^2(x)\right)-a}{(\cos(x)+1)^2}}-4a\cos(x)+4b\cos\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(a+b*cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -sin(x)*(cos(x)^2*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*ln(4*cos(
x)*(-a+b)^(1/2)*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)-4*a*cos(x)+
4*b*cos(x)+4*(-a+b)^(1/2)*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2))*
a-cos(x)^2*(-a+b)^(1/2)*a+cos(x)^2*(-a+b)^(1/2)*b+cos(x)*(-(cos(x)^2*a-b*co
s(x)^2-a)/(cos(x)+1)^2)^(1/2)*ln(4*cos(x)*(-a+b)^(1/2)*(-(cos(x)^2*a-b*cos(
x)^2-a)/(cos(x)+1)^2)^(1/2)-4*a*cos(x)+4*b*cos(x)+4*(-a+b)^(1/2)*(-(cos(x)^
2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2))*a+(-a+b)^(1/2)*a)/cos(x)/((cos(x)^2*
a-b*cos(x)^2-a)/(cos(x)^2-1))^(1/2)/(cos(x)^2-1)/(-a+b)^(1/2)/a
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+b*cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)^2/sqrt(b*cot(x)^2 + a), x)
```

Fricas [A]
time = 3.50, size = 229, normalized size = 4.24

$$\left[-\frac{a\sqrt{-a+b}\log\left(-\frac{a^2\tan(x)^4-2\left(3a^2-4ab\right)\tan(x)^2+a^2-8ab+8b^2+4\left(a\tan(x)^3-(a-2b)\tan(x)\right)\sqrt{-a+b}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}}{\tan(x)^4+2\tan(x)^2+1}\right)-4(a-b)\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan(x)}{4\left(a^2-ab\right)}, \frac{\sqrt{a-b}\,a\arctan\left(\frac{2\sqrt{a-b}\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan(x)}{a\tan(x)^2-a+2b}\right)+2(a-b)\sqrt{\frac{a\tan(x)^2+b}{\tan(x)^2}}\tan(x)}{2\left(a^2-ab\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+b*cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(a*sqrt(-a + b)*log(-(a^2*tan(x)^4 - 2*(3*a^2 - 4*a*b)*tan(x)^2 + a^2
 - 8*a*b + 8*b^2 + 4*(a*tan(x)^3 - (a - 2*b)*tan(x))*sqrt(-a + b)*sqrt((a*t
an(x)^2 + b)/tan(x)^2))/(tan(x)^4 + 2*tan(x)^2 + 1)) - 4*(a - b)*sqrt((a*ta
n(x)^2 + b)/tan(x)^2)*tan(x))/(a^2 - a*b), 1/2*(sqrt(a - b)*a*arctan(2*sqrt
(a - b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)/(a*tan(x)^2 - a + 2*b)) + 2*
(a - b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x))/(a^2 - a*b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(x)}{\sqrt{a+b\cot^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**2/(a+b*cot(x)**2)**(1/2),x)
```

```
[Out] Integral(tan(x)**2/sqrt(a + b*cot(x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(46) = 92.
time = 0.47, size = 216, normalized size = 4.00

$$\frac{\left(a\log\left(-a-2\sqrt{-a+b}\sqrt{b}+2b\right)+\sqrt{-a+b}\sqrt{b}\log\left(-a-2\sqrt{-a+b}\sqrt{b}+2b\right)-b\log\left(-a-2\sqrt{-a+b}\sqrt{b}+2b\right)+2a-2b\right)\mathrm{sgn}(\sin(x))}{2\left(a\sqrt{-a+b}-a\sqrt{b}-\sqrt{-a+b}\,b+b^{\frac{3}{2}}\right)}-\frac{\frac{\log\left(\left(\sqrt{-a+b}\cos(x)-\sqrt{-a\cos(x)^2+b\cos(x)^2+a}\right)^2\right)}{\sqrt{-a+b}}+\frac{4\sqrt{-a+b}}{\left(\sqrt{-a+b}\cos(x)-\sqrt{-a\cos(x)^2+b\cos(x)^2+a}\right)^2-a}}{2\,\mathrm{sgn}(\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+b*cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(a*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b) + sqrt(-a + b)*sqrt(b)*log(-a
 - 2*sqrt(-a + b)*sqrt(b) + 2*b) - b*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b)
 + 2*a - 2*b)*sgn(sin(x))/(a*sqrt(-a + b) - a*sqrt(b) - sqrt(-a + b)*b + b^
(3/2)) - 1/2*(log((sqrt(-a + b)*cos(x) - sqrt(-a*cos(x)^2 + b*cos(x)^2 + a)
```

```
)^2)/sqrt(-a + b) + 4*sqrt(-a + b)/((sqrt(-a + b)*cos(x) - sqrt(-a*cos(x)^2
 + b*cos(x)^2 + a))^2 - a))/sgn(sin(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tan(x)^2}{\sqrt{b\cot(x)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(a + b*cot(x)^2)^(1/2),x)
```

```
[Out] int(tan(x)^2/(a + b*cot(x)^2)^(1/2), x)
```

3.49 $\int \frac{\cot^3(x)}{(a+b\cot^2(x))^{3/2}}\,dx$

Optimal. Leaf size=59

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}+\frac{a}{(a-b)b\sqrt{a+b\cot^2(x)}}$$

```
[Out] -arctanh((a+b*cot(x)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)+a/(a-b)/b/(a+b*cot(x
)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3751, 457, 79, 65, 214}

$$\frac{a}{b(a-b)\sqrt{a+b\cot^2(x)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^3/(a + b*Cot[x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Cot[x]^2/Sqrt[a - b]]/(a - b)^(3/2)) + a/((a - b)*b*S
qrt[a + b*Cot[x]^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
```

```
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^3(x)}{\left(a+b\cot^2(x)\right)^{3/2}}\,dx &= -\text{Subst}\left(\int \frac{x^3}{\left(1+x^2\right)\left(a+bx^2\right)^{3/2}}\,dx, x, \cot(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{x}{(1+x)(a+bx)^{3/2}}\,dx, x, \cot^2(x)\right)\right)\\
&= \frac{a}{(a-b)b\sqrt{a+b\cot^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \cot^2(x)\right)}{2(a-b)}\\
&= \frac{a}{(a-b)b\sqrt{a+b\cot^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cot^2(x)}\right)}{(a-b)b}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}} + \frac{a}{(a-b)b\sqrt{a+b\cot^2(x)}}
\end{aligned}
$$

Mathematica [A]
time = 0.24, size = 59, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}+\frac{a}{(a-b)b\sqrt{a+b\cot^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3/(a + b*Cot[x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]]/(a - b)^(3/2)) + a/((a - b)*b*S
qrt[a + b*Cot[x]^2])
```

Maple [A]
time = 0.13, size = 68, normalized size = 1.15

method	result	size
derivativedivides	$\frac{1}{b\sqrt{a+b\left(\cot^2\left(x\right)\right)}}+\frac{\arctan\left(\frac{\sqrt{a+b\left(\cot^2\left(x\right)\right)}}{\sqrt{-a+b}}\right)}{(a-b)\sqrt{-a+b}}+\frac{1}{(a-b)\sqrt{a+b\left(\cot^2\left(x\right)\right)}}$	68
default	$\frac{1}{b\sqrt{a+b\left(\cot^2\left(x\right)\right)}}+\frac{\arctan\left(\frac{\sqrt{a+b\left(\cot^2\left(x\right)\right)}}{\sqrt{-a+b}}\right)}{(a-b)\sqrt{-a+b}}+\frac{1}{(a-b)\sqrt{a+b\left(\cot^2\left(x\right)\right)}}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3/(a+b*cot(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(a+b*cot(x)^2)^(1/2)+1/(a-b)/(-a+b)^(1/2)*arctan((a+b*cot(x)^2)^(1/2)/(
-a+b)^(1/2))+1/(a-b)/(a+b*cot(x)^2)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+b*cot(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(51) = 102$.
time = 2.49, size = 385, normalized size = 6.53

$$\left[-\frac{\left(ab+b^2-\left(ab-b^2\right)\cos\left(2x\right)\right)\sqrt{a-b}\log\left(-\sqrt{a-b}\sqrt{\frac{(a-b)\cos\left(2x\right)-a-b}{\cos\left(2x\right)-1}}\left(\cos\left(2x\right)-1\right)-(a-b)\cos\left(2x\right)+a\right)-2\left(a^2-ab-\left(a^2-ab\right)\cos\left(2x\right)\right)\sqrt{\frac{(a-b)\cos\left(2x\right)-a-b}{\cos\left(2x\right)-1}}}{2\left(a^3b-a^2b^2-ab^3+b^4-\left(a^3b-3a^2b^2+3ab^3-b^4\right)\cos\left(2x\right)\right)},-\frac{\left(ab+b^2-\left(ab-b^2\right)\cos\left(2x\right)\right)\sqrt{-a+b}\arctan\left(-\frac{\sqrt{-a+b}\sqrt{\frac{(a-b)\cos\left(2x\right)-a-b}{\cos\left(2x\right)-1}}}{a-b}\right)-\left(a^2-ab-\left(a^2-ab\right)\cos\left(2x\right)\right)\sqrt{\frac{(a-b)\cos\left(2x\right)-a-b}{\cos\left(2x\right)-1}}}{a^3b-a^2b^2-ab^3+b^4-\left(a^3b-3a^2b^2+3ab^3-b^4\right)\cos\left(2x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+b*cot(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((a*b + b^2 - (a*b - b^2)*cos(2*x))*sqrt(a - b)*log(-sqrt(a - b)*sqrt
(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*(cos(2*x) - 1) - (a - b)*cos(2*
x) + a) - 2*(a^2 - a*b - (a^2 - a*b)*cos(2*x))*sqrt(((a - b)*cos(2*x) - a -
 b)/(cos(2*x) - 1)))/(a^3*b - a^2*b^2 - a*b^3 + b^4 - (a^3*b - 3*a^2*b^2 +
3*a*b^3 - b^4)*cos(2*x)), -((a*b + b^2 - (a*b - b^2)*cos(2*x))*sqrt(-a + b)
*arctan(-sqrt(-a + b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))/(a -
b)) - (a^2 - a*b - (a^2 - a*b)*cos(2*x))*sqrt(((a - b)*cos(2*x) - a - b)/(c
os(2*x) - 1)))/(a^3*b - a^2*b^2 - a*b^3 + b^4 - (a^3*b - 3*a^2*b^2 + 3*a*b^
3 - b^4)*cos(2*x))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3\left(x\right)}{\left(a+b\cot^2\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**3/(a+b*cot(x)**2)**(3/2),x)
```

```
[Out] Integral(cot(x)**3/(a + b*cot(x)**2)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(51) = 102$.
time = 0.47, size = 109, normalized size = 1.85

$$-\frac{\log\left(|b|\right)\mathrm{sgn}\left(\sin\left(x\right)\right)}{2\left(\sqrt{a-b}\,a-\sqrt{a-b}\,b\right)}+\frac{\frac{a\sin\left(x\right)}{\sqrt{a\sin\left(x\right)^2-b\sin\left(x\right)^2+b}\,(ab-b^2)}+\frac{\log\left(\left|-\sqrt{a-b}\,\sin(x)+\sqrt{a\sin\left(x\right)^2-b\sin\left(x\right)^2+b}\right|\right)}{(a-b)^{\frac{3}{2}}}}{\mathrm{sgn}\left(\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+b*cot(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(b))*sgn(sin(x))/(sqrt(a - b)*a - sqrt(a - b)*b) + (a*sin(x)/(s
qrt(a*sin(x)^2 - b*sin(x)^2 + b)*(a*b - b^2)) + log(abs(-sqrt(a - b)*sin(x)
 + sqrt(a*sin(x)^2 - b*sin(x)^2 + b)))/(a - b)^(3/2))/sgn(sin(x))
```

Mupad [B]
time = 1.92, size = 52, normalized size = 0.88

$$\frac{a}{(a\,b-b^2)\,\sqrt{b\cot(x)^2+a}}-\frac{\operatorname{atanh}\left(\frac{\sqrt{b\cot(x)^2+a}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3/(a + b*cot(x)^2)^(3/2),x)
```

```
[Out] a/((a*b - b^2)*(a + b*cot(x)^2)^(1/2)) - atanh((a + b*cot(x)^2)^(1/2)/(a -
b)^(1/2))/(a - b)^(3/2)
```

3.50 $\int \frac{\cot^2(x)}{(a+b\cot^2(x))^{3/2}}\,dx$

Optimal. Leaf size=59

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{(a-b)^{3/2}}-\frac{\cot(x)}{(a-b)\sqrt{a+b\cot^2(x)}}$$

```
[Out] arctan(cot(x)*(a-b)^(1/2)/(a+b*cot(x)^2)^(1/2))/(a-b)^(3/2)-cot(x)/(a-b)/(a
+b*cot(x)^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3751, 482, 385, 209}

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{(a-b)^{3/2}}-\frac{\cot(x)}{(a-b)\sqrt{a+b\cot^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2/(a + b*Cot[x]^2)^(3/2),x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Cot[x])/Sqrt[a + b*Cot[x]^2]]/(a - b)^(3/2) - Cot[x]/((
a - b)*Sqrt[a + b*Cot[x]^2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
```

```
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^2(x)}{\left(a+b\cot^2(x)\right)^{3/2}}\,dx &= -\text{Subst}\left(\int \frac{x^2}{(1+x^2)\left(a+bx^2\right)^{3/2}}\,dx, x, \cot(x)\right)\\
&= -\frac{\cot(x)}{(a-b)\sqrt{a+b\cot^2(x)}}+\frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \cot(x)\right)}{a-b}\\
&= -\frac{\cot(x)}{(a-b)\sqrt{a+b\cot^2(x)}}+\frac{\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{a-b}\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{(a-b)^{3/2}}-\frac{\cot(x)}{(a-b)\sqrt{a+b\cot^2(x)}}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 137 vs. 2(59) = 118.
time = 0.75, size = 137, normalized size = 2.32

$$\frac{(-a+b)\cot(x)\sqrt{1+\frac{b\cot^2(x)}{a}}+\frac{1}{2}\tanh^{-1}\left(\frac{\sqrt{-\frac{(a-b)\cot^2(x)}{a}}}{\sqrt{1+\frac{b\cot^2(x)}{a}}}\right)(-a-b+(a-b)\cos(2x))\sqrt{-\frac{(a-b)\cot^2(x)}{a}}\csc(x)\sec(x)}{(a-b)^2\sqrt{a+b\cot^2(x)}\sqrt{1+\frac{b\cot^2(x)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2/(a + b*Cot[x]^2)^(3/2),x]
```

```
[Out] ((-a + b)*Cot[x]*Sqrt[1 + (b*Cot[x]^2)/a] + (ArcTanh[Sqrt[-(((a - b)*Cot[x]
^2)/a)]/Sqrt[1 + (b*Cot[x]^2)/a]]*(-a - b + (a - b)*Cos[2*x])*Sqrt[-(((a -
b)*Cot[x]^2)/a)]*Csc[x]*Sec[x])/2)/((a - b)^2*Sqrt[a + b*Cot[x]^2]*Sqrt[1 +
 (b*Cot[x]^2)/a])
```

Maple [A]
time = 0.15, size = 99, normalized size = 1.68

method	result
derivativedivides	$-\frac{\cot(x)}{a\sqrt{a+b\left(\cot^2(x)\right)}}-\frac{b\cot(x)}{(a-b)a\sqrt{a+b\left(\cot^2(x)\right)}}+\frac{\sqrt{b^4(a-b)}\ \arctan\left(\frac{b^2(a-}{\sqrt{b^4(a-b)}\ \sqrt{}}\right.}{(a-b)^2b^2}$
default	$-\frac{\cot(x)}{a\sqrt{a+b\left(\cot^2(x)\right)}}-\frac{b\cot(x)}{(a-b)a\sqrt{a+b\left(\cot^2(x)\right)}}+\frac{\sqrt{b^4(a-b)}\ \arctan\left(\frac{b^2(a-}{\sqrt{b^4(a-b)}\ \sqrt{}}\right.}{(a-b)^2b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/(a+b*cot(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -cot(x)/a/(a+b*cot(x)^2)^(1/2)-1/(a-b)*b*cot(x)/a/(a+b*cot(x)^2)^(1/2)+1/(a
-b)^2*(b^4*(a-b))^(1/2)/b^2*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*cot(x)^
2)^(1/2)*cot(x))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+b*cot(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(51) = 102.
time = 4.06, size = 388, normalized size = 6.58

$$\left[-\frac{((a-b)\cos(2x)-a-b)\sqrt{-a+b}\,\log\left(-2\left(a^2-2ab+b^2\right)\cos(2x)^2-2\left((a-b)\cos(2x)-b\right)\sqrt{-a+b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\sin(2x)+a^2-2b^2+4\left(ab-b^2\right)\cos(2x)\right)+4(a-b)\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\sin(2x)}{4\left(a^3-a^2b-ab^2+b^3-\left(a^3-3a^2b+3ab^2-b^3\right)\cos(2x)\right)},\ -\frac{((a-b)\cos(2x)-a-b)\sqrt{a-b}\,\arctan\left(-\frac{\sqrt{a-b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\sin(2x)}{(a-b)\cos(2x)-b}\right)+2(a-b)\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\sin(2x)}{2\left(a^3-a^2b-ab^2+b^3-\left(a^3-3a^2b+3ab^2-b^3\right)\cos(2x)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+b*cot(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((a - b)*cos(2*x) - a - b)*sqrt(-a + b)*log(-2*(a^2 - 2*a*b + b^2)*c
os(2*x)^2 - 2*((a - b)*cos(2*x) - b)*sqrt(-a + b)*sqrt(((a - b)*cos(2*x) -
a - b)/(cos(2*x) - 1))*sin(2*x) + a^2 - 2*b^2 + 4*(a*b - b^2)*cos(2*x)) + 4
*(a - b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x))/(a^3 - a
^2*b - a*b^2 + b^3 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(2*x)), -1/2*(((a -
 b)*cos(2*x) - a - b)*sqrt(a - b)*arctan(-sqrt(a - b)*sqrt(((a - b)*cos(2*x
) - a - b)/(cos(2*x) - 1))*sin(2*x)/((a - b)*cos(2*x) - b)) + 2*(a - b)*sqr
t(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x))/(a^3 - a^2*b - a*b^2
 + b^3 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(2*x))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(x)}{(a + b\cot^2(x))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2/(a+b*cot(x)**2)**(3/2),x)
```

```
[Out] Integral(cot(x)**2/(a + b*cot(x)**2)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(51) = 102.
time = 0.50, size = 161, normalized size = 2.73

$$-\frac{\left(\sqrt{-a+b}\,\sqrt{b}\,\log\left(\left|-\sqrt{-a+b}+\sqrt{b}\right|\right)-a+b\right)\operatorname{sgn}(\sin(x))}{a^2\sqrt{b}-2\,ab^{\frac{3}{2}}+b^{\frac{5}{2}}}+\frac{\frac{\sqrt{-a+b}\,\log\left(\left|-\sqrt{-a+b}\,\cos(x)+\sqrt{-a\cos(x)^2+b\cos(x)^2+a}\right|\right)}{a^2-2\,ab+b^2}+\frac{\sqrt{-a\cos(x)^2+b\cos(x)^2+a}\,\cos(x)}{\left(a\cos(x)^2-b\cos(x)^2-a\right)(a-b)}}{\operatorname{sgn}(\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+b*cot(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(sqrt(-a + b)*sqrt(b)*log(abs(-sqrt(-a + b) + sqrt(b))) - a + b)*sgn(sin(x
))/(a^2*sqrt(b) - 2*a*b^(3/2) + b^(5/2)) + (sqrt(-a + b)*log(abs(-sqrt(-a +
 b)*cos(x) + sqrt(-a*cos(x)^2 + b*cos(x)^2 + a)))/(a^2 - 2*a*b + b^2) + sqr
t(-a*cos(x)^2 + b*cos(x)^2 + a)*cos(x)/((a*cos(x)^2 - b*cos(x)^2 - a)*(a -
b)))/sgn(sin(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cot(x)^2}{\left(b\cot(x)^2+a\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/(a + b*cot(x)^2)^(3/2),x)
```

```
[Out] int(cot(x)^2/(a + b*cot(x)^2)^(3/2), x)
```

3.51 $\int \frac{\cot(x)}{\left(a+b\cot^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=55

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}-\frac{1}{(a-b)\sqrt{a+b\cot^2(x)}}$$

```
[Out] arctanh((a+b*cot(x)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)-1/(a-b)/(a+b*cot(x)^2
)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3751, 455, 53, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}-\frac{1}{(a-b)\sqrt{a+b\cot^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(a + b*Cot[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]]/(a - b)^(3/2) - 1/((a - b)*Sqrt[a
 + b*Cot[x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(x)}{\left(a+b\cot^2(x)\right)^{3/2}}\,dx &= -\text{Subst}\left(\int \frac{x}{\left(1+x^2\right)\left(a+bx^2\right)^{3/2}}\,dx, x, \cot(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{3/2}}\,dx, x, \cot^2(x)\right)\right)\\
&= -\frac{1}{(a-b)\sqrt{a+b\cot^2(x)}}-\frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \cot^2(x)\right)}{2(a-b)}\\
&= -\frac{1}{(a-b)\sqrt{a+b\cot^2(x)}}-\frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cot^2(x)}\right)}{(a-b)b}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}-\frac{1}{(a-b)\sqrt{a+b\cot^2(x)}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 44, normalized size = 0.80

$$\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\cot^2(x)}{a-b}\right)}{(-a+b)\sqrt{a+b\cot^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(a + b*Cot[x]^2)^(3/2),x]
```

```
[Out] Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Cot[x]^2)/(a - b)]/((-a + b)*Sqrt[a
+ b*Cot[x]^2])
```

Maple [A]
time = 0.13, size = 56, normalized size = 1.02

method	result	size
derivativedivides	$-\frac{\arctan\left(\frac{\sqrt{a+b\left(\cot^2\left(x\right)\right)}}{\sqrt{-a+b}}\right)}{(a-b)\sqrt{-a+b}}-\frac{1}{(a-b)\sqrt{a+b\left(\cot^2\left(x\right)\right)}}$	56
default	$-\frac{\arctan\left(\frac{\sqrt{a+b\left(\cot^2\left(x\right)\right)}}{\sqrt{-a+b}}\right)}{(a-b)\sqrt{-a+b}}-\frac{1}{(a-b)\sqrt{a+b\left(\cot^2\left(x\right)\right)}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a+b*cot(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(a-b)/(-a+b)^(1/2)*arctan((a+b*cot(x)^2)^(1/2)/(-a+b)^(1/2))-1/(a-b)/(a+
b*cot(x)^2)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(47) = 94$.
time = 3.00, size = 344, normalized size = 6.25

$$\left[\frac{\left((a-b)\cos\left(2x\right)-a-b\right)\sqrt{a-b}\log\left(\sqrt{a-b}\sqrt{\frac{(a-b)\cos\left(2x\right)-a-b}{\cos\left(2x\right)-1}}\left(\cos\left(2x\right)-1\right)-(a-b)\cos\left(2x\right)+a\right)+2\left((a-b)\cos\left(2x\right)-a+b\right)\sqrt{\frac{(a-b)\cos\left(2x\right)-a-b}{\cos\left(2x\right)-1}}}{2\left(a^3-a^2b-ab^2+b^3-\left(a^3-3a^2b+3ab^2-b^3\right)\cos\left(2x\right)\right)}, -\frac{\left((a-b)\cos\left(2x\right)-a-b\right)\sqrt{-a+b}\arctan\left(-\frac{\sqrt{-a+b}\sqrt{\frac{(a-b)\cos\left(2x\right)-a-b}{\cos\left(2x\right)-1}}}{a-b}\right)-\left((a-b)\cos\left(2x\right)-a+b\right)\sqrt{\frac{(a-b)\cos\left(2x\right)-a-b}{\cos\left(2x\right)-1}}}{a^3-a^2b-ab^2+b^3-\left(a^3-3a^2b+3ab^2-b^3\right)\cos\left(2x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((a - b)*cos(2*x) - a - b)*sqrt(a - b)*log(sqrt(a - b)*sqrt(((a - b)*
cos(2*x) - a - b)/(cos(2*x) - 1))*(cos(2*x) - 1) - (a - b)*cos(2*x) + a) +
2*((a - b)*cos(2*x) - a + b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)
))/(a^3 - a^2*b - a*b^2 + b^3 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(2*x)),
-(((a - b)*cos(2*x) - a - b)*sqrt(-a + b)*arctan(-sqrt(-a + b)*sqrt(((a - b
)*cos(2*x) - a - b)/(cos(2*x) - 1))/(a - b)) - ((a - b)*cos(2*x) - a + b)*s
qrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)))/(a^3 - a^2*b - a*b^2 + b^3
- (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(2*x))]
```

Sympy [A]
time = 4.74, size = 48, normalized size = 0.87

$$-\frac{1}{\left(a-b\right)\sqrt{a+b\cot^{2}\left(x\right)}}-\frac{\operatorname{atan}\left(\frac{\sqrt{a+b\cot^{2}\left(x\right)}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}\left(a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)**2)**(3/2),x)
```

```
[Out] -1/((a - b)*sqrt(a + b*cot(x)**2)) - atan(sqrt(a + b*cot(x)**2)/sqrt(-a + b
))/(sqrt(-a + b)*(a - b))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(47) = 94$.
time = 0.47, size = 105, normalized size = 1.91

$$\frac{\log\left(|b|\right)\operatorname{sgn}(\sin\left(x\right))}{2\left(\sqrt{a-b}\,a-\sqrt{a-b}\,b\right)}-\frac{\frac{\log\left(\left|-\sqrt{a-b}\,\sin(x)+\sqrt{a\sin\left(x\right)^{2}-b\sin\left(x\right)^{2}+b}\right|\right)}{(a-b)^{\frac{3}{2}}}+\frac{\sin(x)}{\sqrt{a\sin\left(x\right)^{2}-b\sin\left(x\right)^{2}+b}\,(a-b)}}{\operatorname{sgn}\left(\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b))*sgn(sin(x))/(sqrt(a - b)*a - sqrt(a - b)*b) - (log(abs(-sqr
t(a - b)*sin(x) + sqrt(a*sin(x)^2 - b*sin(x)^2 + b)))/(a - b)^(3/2) + sin(x
)/(sqrt(a*sin(x)^2 - b*sin(x)^2 + b)*(a - b)))/sgn(sin(x))
```

Mupad [B]
time = 1.78, size = 47, normalized size = 0.85

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b\cot\left(x\right)^2+a}}{\sqrt{a-b}}\right)}{\left(a-b\right)^{3/2}}-\frac{1}{\left(a-b\right)\,\sqrt{b\cot\left(x\right)^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a + b*cot(x)^2)^(3/2),x)
```

```
[Out] atanh((a + b*cot(x)^2)^(1/2)/(a - b)^(1/2))/(a - b)^(3/2) - 1/((a - b)*(a +
 b*cot(x)^2)^(1/2))
```

3.52 $\int \frac{\tan(x)}{\left(a+b\cot^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=84

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}+\frac{b}{a(a-b)\sqrt{a+b\cot^2(x)}}$$

```
[Out] arctanh((a+b*cot(x)^2)^(1/2)/a^(1/2))/a^(3/2)-arctanh((a+b*cot(x)^2)^(1/2)/
(a-b)^(1/2))/(a-b)^(3/2)+b/a/(a-b)/(a+b*cot(x)^2)^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3751, 457, 87, 162, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{b}{a(a-b)\sqrt{a+b\cot^2(x)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(a + b*Cot[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a]]/a^(3/2) - ArcTanh[Sqrt[a + b*Cot[x]^2
]/Sqrt[a - b]]/(a - b)^(3/2) + b/(a*(a - b)*Sqrt[a + b*Cot[x]^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[(b*d*e - b*c*f - a*d*f - b*d*f*x
)*((e + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan(x)}{(a+b\cot^2(x))^{3/2}}\,dx = -\text{Subst}\left(\int \frac{1}{x\,(1+x^2)\,(a+bx^2)^{3/2}}\,dx, x, \cot(x)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)(a+bx)^{3/2}}\,dx, x, \cot^2(x)\right)\right)$$
$$= \frac{b}{a(a-b)\sqrt{a+b\cot^2(x)}} - \frac{\text{Subst}\left(\int \frac{a-b-bx}{x(1+x)\sqrt{a+bx}}\,dx, x, \cot^2(x)\right)}{2a(a-b)}$$
$$= \frac{b}{a(a-b)\sqrt{a+b\cot^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \cot^2(x)\right)}{2a} + \frac{\text{Subst}\left(\int \frac{}{(1+x}\right.}{}$$
$$= \frac{b}{a(a-b)\sqrt{a+b\cot^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cot^2(x)}\right)}{ab} + \frac{\text{Subst}\left(\int\right.}{}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}} + \frac{b}{a(a-b)\sqrt{a+b\,c}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.06, size = 75, normalized size = 0.89

$$\frac{a\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\cot^2(x)}{a-b}\right)+(-a+b)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1+\frac{b\cot^2(x)}{a}\right)}{a(a-b)\sqrt{a+b\cot^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(a + b*Cot[x]^2)^(3/2),x]
```

```
[Out] (a*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Cot[x]^2)/(a - b)] + (-a + b)*Hyp
ergeometric2F1[-1/2, 1, 1/2, 1 + (b*Cot[x]^2)/a])/(a*(a - b)*Sqrt[a + b*Cot
[x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.66, size = 962, normalized size = 11.45

method	result

default	$-\frac{\left(\left(\cos^2(x)\right)a-b\left(\cos^2(x)\right)-a\right)\left(\sqrt{2}\,\sqrt{\frac{\cos(x)\sqrt{a}\,\sqrt{a-b}-a\cos(x)+b\cos(x)-\sqrt{a}\,\sqrt{a-b}+a}{(\cos(x)+1)b}}\,\sqrt{-\frac{2\left(\cos(x)\sqrt{a}\,\sqrt{\right.}}{}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a+b*cot(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(cos(x)^2*a-b*cos(x)^2-a)*(2^(1/2)*((cos(x)*a^(1/2)*(a-b)^(1/2)-a*cos(x)+b
*cos(x)-a^(1/2)*(a-b)^(1/2)+a)/(cos(x)+1)/b)^(1/2)*(-2*(cos(x)*a^(1/2)*(a-b
)^(1/2)+a*cos(x)-b*cos(x)-a^(1/2)*(a-b)^(1/2)-a)/(cos(x)+1)/b)^(1/2)*Ellipt
icF((-1+cos(x))*((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/sin(x),((8*a^(3/2)*
(a-b)^(1/2)-4*a^(1/2)*(a-b)^(1/2)*b+8*a^2-8*a*b+b^2)/b^2)^(1/2))*b*sin(x)+2
*2^(1/2)*((cos(x)*a^(1/2)*(a-b)^(1/2)-a*cos(x)+b*cos(x)-a^(1/2)*(a-b)^(1/2)
+a)/(cos(x)+1)/b)^(1/2)*(-2*(cos(x)*a^(1/2)*(a-b)^(1/2)+a*cos(x)-b*cos(x)-a
^(1/2)*(a-b)^(1/2)-a)/(cos(x)+1)/b)^(1/2)*EllipticPi((-1+cos(x))*((2*a^(1/2
)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/sin(x),1/(2*a^(1/2)*(a-b)^(1/2)-2*a+b)*b,(-(2
*a^(1/2)*(a-b)^(1/2)+2*a-b)/b)^(1/2)/((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2
))*a*sin(x)-2*2^(1/2)*((cos(x)*a^(1/2)*(a-b)^(1/2)-a*cos(x)+b*cos(x)-a^(1/2
)*(a-b)^(1/2)+a)/(cos(x)+1)/b)^(1/2)*(-2*(cos(x)*a^(1/2)*(a-b)^(1/2)+a*cos(
x)-b*cos(x)-a^(1/2)*(a-b)^(1/2)-a)/(cos(x)+1)/b)^(1/2)*EllipticPi((-1+cos(x
))*((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/sin(x),1/(2*a^(1/2)*(a-b)^(1/2)-
2*a+b)*b,(-(2*a^(1/2)*(a-b)^(1/2)+2*a-b)/b)^(1/2)/((2*a^(1/2)*(a-b)^(1/2)-2
*a+b)/b)^(1/2))*b*sin(x)-2*2^(1/2)*((cos(x)*a^(1/2)*(a-b)^(1/2)-a*cos(x)+b*
cos(x)-a^(1/2)*(a-b)^(1/2)+a)/(cos(x)+1)/b)^(1/2)*(-2*(cos(x)*a^(1/2)*(a-b)
^(1/2)+a*cos(x)-b*cos(x)-a^(1/2)*(a-b)^(1/2)-a)/(cos(x)+1)/b)^(1/2)*Ellipti
cPi((-1+cos(x))*((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)/sin(x),-1/(2*a^(1/2
)*(a-b)^(1/2)-2*a+b)*b,(-(2*a^(1/2)*(a-b)^(1/2)+2*a-b)/b)^(1/2)/((2*a^(1/2)
*(a-b)^(1/2)-2*a+b)/b)^(1/2))*a*sin(x)+((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1
/2)*b*cos(x)-((2*a^(1/2)*(a-b)^(1/2)-2*a+b)/b)^(1/2)*b)/(-1+cos(x))/sin(x)^
2/((cos(x)^2*a-b*cos(x)^2-a)/(cos(x)^2-1))^(3/2)/((2*a^(1/2)*(a-b)^(1/2)-2*
a+b)/b)^(1/2)/(a-b)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cot(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)/(b*cot(x)^2 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(70) = 140$.
time = 2.89, size = 863, normalized size = 10.27

$$\left[\frac{2\left(a^{2}b-ab^{2}\right)\sqrt{\frac{a\tan\left(x\right)^{2}+b}{\tan\left(x\right)^{2}}}\tan\left(x\right)^{2}+\left(a^{2}b-2ab^{2}+b^{3}+\left(a^{3}-2a^{2}b+ab^{2}\right)\tan\left(x\right)^{2}\right)\sqrt{a}\log\left(2a\tan\left(x\right)^{2}+2\sqrt{a}\sqrt{\frac{a\tan\left(x\right)^{2}+b}{\tan\left(x\right)^{2}}}\tan\left(x\right)^{2}+b\right)-\left(a^{3}\tan\left(x\right)^{2}+a^{2}b\right)\sqrt{a-b}\log\left(\frac{(2a-b)\tan\left(x\right)^{2}+2\sqrt{a-b}\sqrt{\frac{a\tan\left(x\right)^{2}+b}{\tan\left(x\right)^{2}}}\tan\left(x\right)^{2}+b}{\tan\left(x\right)^{2}+1}\right)}{2\left(a^{4}b-2a^{3}b^{2}+a^{2}b^{3}+\left(a^{5}-2a^{4}b+a^{3}b^{2}\right)\tan\left(x\right)^{2}\right)},\ \frac{2\left(a^{2}b-ab^{2}\right)\sqrt{\frac{a\tan\left(x\right)^{2}+b}{\tan\left(x\right)^{2}}}\tan\left(x\right)^{2}-2\left(a^{3}\tan\left(x\right)^{2}+a^{2}b\right)\sqrt{-a+b}\arctan\left(-\frac{\sqrt{-a+b}\sqrt{\frac{a\tan\left(x\right)^{2}+b}{\tan\left(x\right)^{2}}}}{a-b}\right)+\left(a^{2}b-2ab^{2}+b^{3}+\left(a^{3}-2a^{2}b+ab^{2}\right)\tan\left(x\right)^{2}\right)\sqrt{a}\log\left(2a\tan\left(x\right)^{2}+2\sqrt{a}\sqrt{\frac{a\tan\left(x\right)^{2}+b}{\tan\left(x\right)^{2}}}\tan\left(x\right)^{2}+b\right)}{2\left(a^{4}b-2a^{3}b^{2}+a^{2}b^{3}+\left(a^{5}-2a^{4}b+a^{3}b^{2}\right)\tan\left(x\right)^{2}\right)},\ \frac{2\left(a^{2}b-ab^{2}\right)\sqrt{\frac{a\tan\left(x\right)^{2}+b}{\tan\left(x\right)^{2}}}\tan\left(x\right)^{2}-2\left(a^{2}b-2ab^{2}+b^{3}+\left(a^{3}-2a^{2}b+ab^{2}\right)\tan\left(x\right)^{2}\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{a\tan\left(x\right)^{2}+b}{\tan\left(x\right)^{2}}}}{a}\right)-\left(a^{3}\tan\left(x\right)^{2}+a^{2}b\right)\sqrt{a-b}\log\left(\frac{(2a-b)\tan\left(x\right)^{2}+2\sqrt{a-b}\sqrt{\frac{a\tan\left(x\right)^{2}+b}{\tan\left(x\right)^{2}}}\tan\left(x\right)^{2}+b}{\tan\left(x\right)^{2}+1}\right)}{2\left(a^{4}b-2a^{3}b^{2}+a^{2}b^{3}+\left(a^{5}-2a^{4}b+a^{3}b^{2}\right)\tan\left(x\right)^{2}\right)},\ \frac{\left(a^{2}b-ab^{2}\right)\sqrt{\frac{a\tan\left(x\right)^{2}+b}{\tan\left(x\right)^{2}}}\tan\left(x\right)^{2}-\left(a^{2}b-2ab^{2}+b^{3}+\left(a^{3}-2a^{2}b+ab^{2}\right)\tan\left(x\right)^{2}\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{a\tan\left(x\right)^{2}+b}{\tan\left(x\right)^{2}}}}{a}\right)-\left(a^{3}\tan\left(x\right)^{2}+a^{2}b\right)\sqrt{-a+b}\arctan\left(-\frac{\sqrt{-a+b}\sqrt{\frac{a\tan\left(x\right)^{2}+b}{\tan\left(x\right)^{2}}}}{a-b}\right)}{a^{4}b-2a^{3}b^{2}+a^{2}b^{3}+\left(a^{5}-2a^{4}b+a^{3}b^{2}\right)\tan\left(x\right)^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cot(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^2*b - a*b^2)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2 + (a^2*b -
 2*a*b^2 + b^3 + (a^3 - 2*a^2*b + a*b^2)*tan(x)^2)*sqrt(a)*log(2*a*tan(x)^2
 + 2*sqrt(a)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2 + b) - (a^3*tan(x)^2
+ a^2*b)*sqrt(a - b)*log(((2*a - b)*tan(x)^2 + 2*sqrt(a - b)*sqrt((a*tan(x)
^2 + b)/tan(x)^2)*tan(x)^2 + b)/(tan(x)^2 + 1)))/(a^4*b - 2*a^3*b^2 + a^2*b
^3 + (a^5 - 2*a^4*b + a^3*b^2)*tan(x)^2), 1/2*(2*(a^2*b - a*b^2)*sqrt((a*ta
n(x)^2 + b)/tan(x)^2)*tan(x)^2 - 2*(a^3*tan(x)^2 + a^2*b)*sqrt(-a + b)*arct
an(-sqrt(-a + b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)/(a - b)) + (a^2*b - 2*a*b^
2 + b^3 + (a^3 - 2*a^2*b + a*b^2)*tan(x)^2)*sqrt(a)*log(2*a*tan(x)^2 + 2*sq
rt(a)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2 + b))/(a^4*b - 2*a^3*b^2 + a
^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*tan(x)^2), 1/2*(2*(a^2*b - a*b^2)*sqrt((
a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2 - 2*(a^2*b - 2*a*b^2 + b^3 + (a^3 - 2*a^
2*b + a*b^2)*tan(x)^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*tan(x)^2 + b)/tan(x
)^2)/a) - (a^3*tan(x)^2 + a^2*b)*sqrt(a - b)*log(((2*a - b)*tan(x)^2 + 2*sq
rt(a - b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2 + b)/(tan(x)^2 + 1)))/(a
^4*b - 2*a^3*b^2 + a^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*tan(x)^2), ((a^2*b -
 a*b^2)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2 - (a^2*b - 2*a*b^2 + b^3 +
 (a^3 - 2*a^2*b + a*b^2)*tan(x)^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*tan(x)^
2 + b)/tan(x)^2)/a) - (a^3*tan(x)^2 + a^2*b)*sqrt(-a + b)*arctan(-sqrt(-a +
 b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)/(a - b)))/(a^4*b - 2*a^3*b^2 + a^2*b^3
+ (a^5 - 2*a^4*b + a^3*b^2)*tan(x)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(x\right)}{\left(a+b\cot^{2}\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cot(x)**2)**(3/2),x)
```

```
[Out] Integral(tan(x)/(a + b*cot(x)**2)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(70) = 140$.

time = 0.49, size = 340, normalized size = 4.05

$$-\frac{\left(2\sqrt{a-b}\,a^{2}\arctan\left(-\frac{a-b}{\sqrt{-a^{2}+ab}}\right)-4\sqrt{a-b}\,ab\arctan\left(-\frac{a-b}{\sqrt{-a^{2}+ab}}\right)+2\sqrt{a-b}\,b^{2}\arctan\left(-\frac{a-b}{\sqrt{-a^{2}+ab}}\right)+\sqrt{-a^{2}+ab}\,\sqrt{a-b}\,a\log\left(b\right)\right)\mathrm{sgn}\left(\sin\left(x\right)\right)}{2\left(\sqrt{-a^{2}+ab}\,a^{3}-2\sqrt{-a^{2}+ab}\,a^{2}b+\sqrt{-a^{2}+ab}\,ab^{2}\right)}+\frac{\frac{\sqrt{a-b}\log\left(\left(\sqrt{a-b}\sin\left(x\right)-\sqrt{a\sin\left(x\right)^{2}-b\sin\left(x\right)^{2}+b}\right)^{2}\right)}{a^{2}-2ab+b^{2}}+\frac{2b\sin\left(x\right)}{\sqrt{a\sin\left(x\right)^{2}-b\sin\left(x\right)^{2}+b}\,\left(a^{2}-ab\right)}+\frac{2\sqrt{a-b}\arctan\left(\frac{\left(\sqrt{a-b}\sin\left(x\right)-\sqrt{a\sin\left(x\right)^{2}-b\sin\left(x\right)^{2}+b}\right)^{2}-2a+b}{2\sqrt{-a^{2}+ab}}\right)}{\sqrt{-a^{2}+ab}\,a}}{2\,\mathrm{sgn}\left(\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cot(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(2*sqrt(a - b)*a^2*arctan(-(a - b)/sqrt(-a^2 + a*b)) - 4*sqrt(a - b)*a
*b*arctan(-(a - b)/sqrt(-a^2 + a*b)) + 2*sqrt(a - b)*b^2*arctan(-(a - b)/sq
rt(-a^2 + a*b)) + sqrt(-a^2 + a*b)*sqrt(a - b)*a*log(b))*sgn(sin(x))/(sqrt(
-a^2 + a*b)*a^3 - 2*sqrt(-a^2 + a*b)*a^2*b + sqrt(-a^2 + a*b)*a*b^2) + 1/2*
(sqrt(a - b)*log((sqrt(a - b)*sin(x) - sqrt(a*sin(x)^2 - b*sin(x)^2 + b))^2
)/(a^2 - 2*a*b + b^2) + 2*b*sin(x)/(sqrt(a*sin(x)^2 - b*sin(x)^2 + b)*(a^2
- a*b)) + 2*sqrt(a - b)*arctan(1/2*((sqrt(a - b)*sin(x) - sqrt(a*sin(x)^2 -
 b*sin(x)^2 + b))^2 - 2*a + b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a*b)*a))/sgn(
sin(x))
```

Mupad [B]
time = 0.48, size = 1451, normalized size = 17.27

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a + b*cot(x)^2)^(3/2),x)
```

```
[Out] atanh((2*a^2*b^8*(a + b/tan(x)^2)^(1/2))/((a^3)^(1/2)*(2*a*b^8 - 12*a^2*b^7
 + 30*a^3*b^6 - 38*a^4*b^5 + 24*a^5*b^4 - 6*a^6*b^3)) - (12*a^3*b^7*(a + b/
tan(x)^2)^(1/2))/((a^3)^(1/2)*(2*a*b^8 - 12*a^2*b^7 + 30*a^3*b^6 - 38*a^4*b
^5 + 24*a^5*b^4 - 6*a^6*b^3)) + (30*a^4*b^6*(a + b/tan(x)^2)^(1/2))/((a^3)^
(1/2)*(2*a*b^8 - 12*a^2*b^7 + 30*a^3*b^6 - 38*a^4*b^5 + 24*a^5*b^4 - 6*a^6*
b^3)) - (38*a^5*b^5*(a + b/tan(x)^2)^(1/2))/((a^3)^(1/2)*(2*a*b^8 - 12*a^2*
b^7 + 30*a^3*b^6 - 38*a^4*b^5 + 24*a^5*b^4 - 6*a^6*b^3)) + (24*a^6*b^4*(a +
 b/tan(x)^2)^(1/2))/((a^3)^(1/2)*(2*a*b^8 - 12*a^2*b^7 + 30*a^3*b^6 - 38*a^
4*b^5 + 24*a^5*b^4 - 6*a^6*b^3)) - (6*a^7*b^3*(a + b/tan(x)^2)^(1/2))/((a^3
)^(1/2)*(2*a*b^8 - 12*a^2*b^7 + 30*a^3*b^6 - 38*a^4*b^5 + 24*a^5*b^4 - 6*a^
6*b^3)))/(a^3)^(1/2) - (atan(((((a - b)^3)^(1/2)*(((a + b/tan(x)^2)^(1/2)*(
2*a^3*b^7 - 10*a^4*b^6 + 22*a^5*b^5 - 26*a^6*b^4 + 16*a^7*b^3 - 4*a^8*b^2))
/2 + (((a - b)^3)^(1/2)*(12*a^5*b^7 - 2*a^4*b^8 - 28*a^6*b^6 + 32*a^7*b^5 -
 18*a^8*b^4 + 4*a^9*b^3 + ((a + b/tan(x)^2)^(1/2)*((a - b)^3)^(1/2)*(8*a^5*
b^8 - 56*a^6*b^7 + 160*a^7*b^6 - 240*a^8*b^5 + 200*a^9*b^4 - 88*a^10*b^3 +
16*a^11*b^2))/(4*(a - b)^3)))/(2*(a - b)^3))*1i)/(a - b)^3 + (((a - b)^3)^(
1/2)*(((a + b/tan(x)^2)^(1/2)*(2*a^3*b^7 - 10*a^4*b^6 + 22*a^5*b^5 - 26*a^6
```

```
*b^4 + 16*a^7*b^3 - 4*a^8*b^2))/2 + (((a - b)^3)^(1/2)*(2*a^4*b^8 - 12*a^5*
b^7 + 28*a^6*b^6 - 32*a^7*b^5 + 18*a^8*b^4 - 4*a^9*b^3 + ((a + b/tan(x)^2)^
(1/2)*((a - b)^3)^(1/2)*(8*a^5*b^8 - 56*a^6*b^7 + 160*a^7*b^6 - 240*a^8*b^5
 + 200*a^9*b^4 - 88*a^10*b^3 + 16*a^11*b^2))/(4*(a - b)^3)))/(2*(a - b)^3))
*1i)/(a - b)^3)/(2*a^3*b^6 - 6*a^4*b^5 + 6*a^5*b^4 - 2*a^6*b^3 - (((a - b)^
3)^(1/2)*(((a + b/tan(x)^2)^(1/2)*(2*a^3*b^7 - 10*a^4*b^6 + 22*a^5*b^5 - 26
*a^6*b^4 + 16*a^7*b^3 - 4*a^8*b^2))/2 + (((a - b)^3)^(1/2)*(12*a^5*b^7 - 2*
a^4*b^8 - 28*a^6*b^6 + 32*a^7*b^5 - 18*a^8*b^4 + 4*a^9*b^3 + ((a + b/tan(x)
^2)^(1/2)*((a - b)^3)^(1/2)*(8*a^5*b^8 - 56*a^6*b^7 + 160*a^7*b^6 - 240*a^8
*b^5 + 200*a^9*b^4 - 88*a^10*b^3 + 16*a^11*b^2))/(4*(a - b)^3)))/(2*(a - b)
^3)))/(a - b)^3 + (((a - b)^3)^(1/2)*(((a + b/tan(x)^2)^(1/2)*(2*a^3*b^7 -
10*a^4*b^6 + 22*a^5*b^5 - 26*a^6*b^4 + 16*a^7*b^3 - 4*a^8*b^2))/2 + (((a -
b)^3)^(1/2)*(2*a^4*b^8 - 12*a^5*b^7 + 28*a^6*b^6 - 32*a^7*b^5 + 18*a^8*b^4
- 4*a^9*b^3 + ((a + b/tan(x)^2)^(1/2)*((a - b)^3)^(1/2)*(8*a^5*b^8 - 56*a^6
*b^7 + 160*a^7*b^6 - 240*a^8*b^5 + 200*a^9*b^4 - 88*a^10*b^3 + 16*a^11*b^2)
)/(4*(a - b)^3)))/(2*(a - b)^3)))/(a - b)^3))*((a - b)^3)^(1/2)*1i)/(a - b)
^3 - b/((a*b - a^2)*(a + b/tan(x)^2)^(1/2))
```

3.53 $\int \frac{\tan^2(x)}{\left(a+b\cot^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=92

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{(a-b)^{3/2}}+\frac{b\tan(x)}{a(a-b)\sqrt{a+b\cot^2(x)}}+\frac{(a-2b)\sqrt{a+b\cot^2(x)}\tan(x)}{a^2(a-b)}$$

```
[Out] arctan(cot(x)*(a-b)^(1/2)/(a+b*cot(x)^2)^(1/2))/(a-b)^(3/2)+b*tan(x)/a/(a-b
)/(a+b*cot(x)^2)^(1/2)+(a-2*b)*(a+b*cot(x)^2)^(1/2)*tan(x)/a^2/(a-b)
```

Rubi [A]
time = 0.11, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3751, 483, 597, 12, 385, 209}

$$\frac{(a-2b)\tan(x)\sqrt{a+b\cot^2(x)}}{a^2(a-b)}+\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{(a-b)^{3/2}}+\frac{b\tan(x)}{a(a-b)\sqrt{a+b\cot^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^2/(a + b*Cot[x]^2)^(3/2),x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Cot[x])/Sqrt[a + b*Cot[x]^2]]/(a - b)^(3/2) + (b*Tan[x]
)/(a*(a - b)*Sqrt[a + b*Cot[x]^2]) + ((a - 2*b)*Sqrt[a + b*Cot[x]^2]*Tan[x]
)/(a^2*(a - b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^2(x)}{(a+b\cot^2(x))^{3/2}}\,dx = -\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \cot(x)\right)$$

$$= \frac{b\tan(x)}{a(a-b)\sqrt{a+b\cot^2(x)}} - \frac{\text{Subst}\left(\int \frac{a-2b-2bx^2}{x^2(1+x^2)\sqrt{a+bx^2}}\,dx, x, \cot(x)\right)}{a(a-b)}$$

$$= \frac{b\tan(x)}{a(a-b)\sqrt{a+b\cot^2(x)}} + \frac{(a-2b)\sqrt{a+b\cot^2(x)}\,\tan(x)}{a^2(a-b)} + \frac{\text{Subst}\left(\int \frac{a^2}{(1+x^2)\sqrt{a}\right.}{a^2(}$$

$$= \frac{b\tan(x)}{a(a-b)\sqrt{a+b\cot^2(x)}} + \frac{(a-2b)\sqrt{a+b\cot^2(x)}\,\tan(x)}{a^2(a-b)} + \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a}\right.}{a}$$

$$= \frac{b\tan(x)}{a(a-b)\sqrt{a+b\cot^2(x)}} + \frac{(a-2b)\sqrt{a+b\cot^2(x)}\,\tan(x)}{a^2(a-b)} + \frac{\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{(a-b)^{3/2}} + \frac{b\tan(x)}{a(a-b)\sqrt{a+b\cot^2(x)}} + \frac{(a-2b)\sqrt{a+b\cot^2(x)}}{a^2(a-b)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 6.89, size = 674, normalized size = 7.33

$$\frac{\sin^2(x)\left(\frac{12b\csc^2(x)}{a-b} + \frac{8b^2\cot^2(x)\csc^2(x)}{a(a-b)} + \frac{16(a-b)\cos^2(x)\,{}_2F_1\left(2,2;\frac{7}{2};\frac{(a-b)\cos^2(x)}{a}\right)}{15a} + \frac{8(a-b)b\cos^2(x)\cot^2(x)\,{}_2F_1\left(2,2;\frac{7}{2};\frac{(a-b)\cos^2(x)}{a}\right)}{3a^2} + \frac{8(a-b)b^2\cos^2(x)\cot^4(x)\,{}_2F_1\left(2,2;\frac{7}{2};\frac{(a-b)\cos^2(x)}{a}\right)}{5a^3} + \frac{8(a-b)\cos^2(x)\,{}_3F_2\left(2,2,2;1,\frac{7}{2};\frac{(a-b)\cos^2(x)}{a}\right)}{15a} + \frac{16(a-b)b\cos^2(x)\cot^2(x)\,{}_3F_2\left(2,2,2;1,\frac{7}{2};\frac{(a-b)\cos^2(x)}{a}\right)}{15a^2} + \frac{8(a-b)b^2\cos^2(x)\cot^4(x)\,{}_3F_2\left(2,2,2;1,\frac{7}{2};\frac{(a-b)\cos^2(x)}{a}\right)}{15a^3} + \frac{3a\sec^2(x)}{a-b} - \frac{3\text{ArcSin}\left(\sqrt{\frac{(a-b)\cos^2(x)}{a}}\right)}{\left(\frac{(a-b)\cos^2(x)}{a}\right)^{3/2}\sqrt{\frac{(a+b\cot^2(x))\sin^2(x)}{a}}} - \frac{12b\text{ArcSin}\left(\sqrt{\frac{(a-b)\cos^2(x)}{a}}\right)\cot^2(x)}{a\left(\frac{(a-b)\cos^2(x)}{a}\right)^{3/2}\sqrt{\frac{(a+b\cot^2(x))\sin^2(x)}{a}}} - \frac{8b^2\text{ArcSin}\left(\sqrt{\frac{(a-b)\cos^2(x)}{a}}\right)\cot^4(x)}{a^2\left(\frac{(a-b)\cos^2(x)}{a}\right)^{3/2}\sqrt{\frac{(a+b\cot^2(x))\sin^2(x)}{a}}} + \frac{3\text{ArcSin}\left(\sqrt{\frac{(a-b)\cos^2(x)}{a}}\right)}{\sqrt{\frac{(a-b)\cos^2(x)(a+b\cot^2(x))\sin^2(x)}{a^2}}} + \frac{12b\text{ArcSin}\left(\sqrt{\frac{(a-b)\cos^2(x)}{a}}\right)\cot^2(x)}{a\sqrt{\frac{(a-b)\cos^2(x)(a+b\cot^2(x))\sin^2(x)}{a^2}}} + \frac{8b^2\text{ArcSin}\left(\sqrt{\frac{(a-b)\cos^2(x)}{a}}\right)\cot^4(x)}{a^2\sqrt{\frac{(a-b)\cos^2(x)(a+b\cot^2(x))\sin^2(x)}{a^2}}}\right)\tan(x)}{a\sqrt{a+b\cot^2(x)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[x]^2/(a + b*Cot[x]^2)^(3/2),x]
```

```
[Out] (Sin[x]^2*((12*b*Csc[x]^2)/(a - b) + (8*b^2*Cot[x]^2*Csc[x]^2)/(a*(a - b))
+ (16*(a - b)*Cos[x]^2*Hypergeometric2F1[2, 2, 7/2, ((a - b)*Cos[x]^2)/a])/
(15*a) + (8*(a - b)*b*Cos[x]^2*Cot[x]^2*Hypergeometric2F1[2, 2, 7/2, ((a -
b)*Cos[x]^2)/a])/(3*a^2) + (8*(a - b)*b^2*Cos[x]^2*Cot[x]^4*Hypergeometric2
F1[2, 2, 7/2, ((a - b)*Cos[x]^2)/a])/(5*a^3) + (8*(a - b)*Cos[x]^2*Hypergeo
metricPFQ[{2, 2, 2}, {1, 7/2}, ((a - b)*Cos[x]^2)/a])/(15*a) + (16*(a - b)*
b*Cos[x]^2*Cot[x]^2*HypergeometricPFQ[{2, 2, 2}, {1, 7/2}, ((a - b)*Cos[x]^
```

```
2)/a])/(15*a^2) + (8*(a - b)*b^2*Cos[x]^2*Cot[x]^4*HypergeometricPFQ[{2, 2,
 2}, {1, 7/2}, ((a - b)*Cos[x]^2)/a])/(15*a^3) + (3*a*Sec[x]^2)/(a - b) - (
3*ArcSin[Sqrt[((a - b)*Cos[x]^2)/a]])/((((a - b)*Cos[x]^2)/a)^(3/2)*Sqrt[((
a + b*Cot[x]^2)*Sin[x]^2)/a]) - (12*b*ArcSin[Sqrt[((a - b)*Cos[x]^2)/a]]*Co
t[x]^2)/(a*(((a - b)*Cos[x]^2)/a)^(3/2)*Sqrt[((a + b*Cot[x]^2)*Sin[x]^2)/a]
) - (8*b^2*ArcSin[Sqrt[((a - b)*Cos[x]^2)/a]]*Cot[x]^4)/(a^2*(((a - b)*Cos[
x]^2)/a)^(3/2)*Sqrt[((a + b*Cot[x]^2)*Sin[x]^2)/a]) + (3*ArcSin[Sqrt[((a -
b)*Cos[x]^2)/a]])/Sqrt[((a - b)*Cos[x]^2*(a + b*Cot[x]^2)*Sin[x]^2)/a^2] +
(12*b*ArcSin[Sqrt[((a - b)*Cos[x]^2)/a]]*Cot[x]^2)/(a*Sqrt[((a - b)*Cos[x]^
2*(a + b*Cot[x]^2)*Sin[x]^2)/a^2]) + (8*b^2*ArcSin[Sqrt[((a - b)*Cos[x]^2)/
a]]*Cot[x]^4)/(a^2*Sqrt[((a - b)*Cos[x]^2*(a + b*Cot[x]^2)*Sin[x]^2)/a^2]))
*Tan[x])/(a*Sqrt[a + b*Cot[x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(82) = 164$.
time = 1.73, size = 421, normalized size = 4.58

method	result
default	$-\frac{(-1+\cos(x))^2(\cos(x)+1)^2((\cos^2(x))a-b(\cos^2(x))-a)\left((\cos^2(x))\ln\left(4\cos(x)\sqrt{-a+b}\sqrt{-\frac{(\cos^2(x))a-b(\cos^2(x))-a}{(\cos(x)+1)^2}}-4\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(a+b*cot(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-1+cos(x))^2*(cos(x)+1)^2*(cos(x)^2*a-b*cos(x)^2-a)*(cos(x)^2*ln(4*cos(x)
*(-a+b)^(1/2)*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)-4*a*cos(x)+4*
b*cos(x)+4*(-a+b)^(1/2)*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2))*(-
(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*a^2-cos(x)^2*(-a+b)^(1/2)*a^2
+2*cos(x)^2*(-a+b)^(1/2)*a*b-2*cos(x)^2*(-a+b)^(1/2)*b^2+cos(x)*ln(4*cos(x)
*(-a+b)^(1/2)*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)-4*a*cos(x)+4*
b*cos(x)+4*(-a+b)^(1/2)*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2))*(-
(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*a^2+(-a+b)^(1/2)*a^2-(-a+b)^(
1/2)*a*b)*b/cos(x)/((cos(x)^2*a-b*cos(x)^2-a)/(cos(x)^2-1))^(3/2)/sin(x)^7/
(-a+b)^(1/2)/((a*(a-b))^(1/2)-a+b)/((a*(a-b))^(1/2)+a-b)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+b*cot(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)^2/(b*cot(x)^2 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(82) = 164$.
time = 2.85, size = 393, normalized size = 4.27

$$\left[\frac{\left(a^{3}\tan\left(x\right)^{2}+a^{2}b\right)\sqrt{-a+b}\,\log\left(-\frac{a^{2}\tan(x)^{4}-2\,(3\,a^{2}-4\,ab)\tan(x)^{2}+a^{2}-8\,ab+8\,b^{2}-4\left(a\tan(x)^{3}-(a-2\,b)\tan(x)\right)\sqrt{-a+b}\sqrt{\frac{a\tan\left(x\right)^{2}+b}{\tan\left(x\right)^{2}}}}{\tan(x)^{4}+2\tan(x)^{2}+1}\right)+4\left(\left(a^{3}-2\,a^{2}b+ab^{2}\right)\tan\left(x\right)^{3}+\left(a^{2}b-3\,ab^{2}+2\,b^{3}\right)\tan\left(x\right)\right)\sqrt{\frac{a\tan\left(x\right)^{2}+b}{\tan\left(x\right)^{2}}}}{4\left(a^{4}b-2\,a^{3}b^{2}+a^{2}b^{3}+\left(a^{5}-2\,a^{4}b+a^{3}b^{2}\right)\tan\left(x\right)^{2}\right)},\ \frac{\left(a^{3}\tan\left(x\right)^{2}+a^{2}b\right)\sqrt{a-b}\,\arctan\left(\frac{2\sqrt{a-b}\sqrt{\frac{a\tan\left(x\right)^{2}+b}{\tan\left(x\right)^{2}}}\tan(x)}{a\tan(x)^{2}-a+2\,b}\right)+2\left(\left(a^{3}-2\,a^{2}b+ab^{2}\right)\tan\left(x\right)^{3}+\left(a^{2}b-3\,ab^{2}+2\,b^{3}\right)\tan\left(x\right)\right)\sqrt{\frac{a\tan\left(x\right)^{2}+b}{\tan\left(x\right)^{2}}}}{2\left(a^{4}b-2\,a^{3}b^{2}+a^{2}b^{3}+\left(a^{5}-2\,a^{4}b+a^{3}b^{2}\right)\tan\left(x\right)^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+b*cot(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a^3*tan(x)^2 + a^2*b)*sqrt(-a + b)*log(-(a^2*tan(x)^4 - 2*(3*a^2 - 4
*a*b)*tan(x)^2 + a^2 - 8*a*b + 8*b^2 - 4*(a*tan(x)^3 - (a - 2*b)*tan(x))*sq
rt(-a + b)*sqrt((a*tan(x)^2 + b)/tan(x)^2))/(tan(x)^4 + 2*tan(x)^2 + 1)) +
4*((a^3 - 2*a^2*b + a*b^2)*tan(x)^3 + (a^2*b - 3*a*b^2 + 2*b^3)*tan(x))*sqr
t((a*tan(x)^2 + b)/tan(x)^2))/(a^4*b - 2*a^3*b^2 + a^2*b^3 + (a^5 - 2*a^4*b
 + a^3*b^2)*tan(x)^2), 1/2*((a^3*tan(x)^2 + a^2*b)*sqrt(a - b)*arctan(2*sqr
t(a - b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)/(a*tan(x)^2 - a + 2*b)) + 2
*((a^3 - 2*a^2*b + a*b^2)*tan(x)^3 + (a^2*b - 3*a*b^2 + 2*b^3)*tan(x))*sqrt
((a*tan(x)^2 + b)/tan(x)^2))/(a^4*b - 2*a^3*b^2 + a^2*b^3 + (a^5 - 2*a^4*b
+ a^3*b^2)*tan(x)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{2}\left(x\right)}{\left(a+b\cot^{2}\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**2/(a+b*cot(x)**2)**(3/2),x)
```

```
[Out] Integral(tan(x)**2/(a + b*cot(x)**2)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(82) = 164$.
time = 0.46, size = 359, normalized size = 3.90

$$\frac{\left(a^{3}\log\left(-a-2\sqrt{-a+b}\sqrt{b}+2\,b\right)+a^{2}\sqrt{-a+b}\sqrt{b}\log\left(-a-2\sqrt{-a+b}\sqrt{b}+2\,b\right)-a^{2}b\log\left(-a-2\sqrt{-a+b}\sqrt{b}+2\,b\right)+2\,a^{3}-4\,a^{2}b+2\,a\sqrt{-a+b}\,b^{\frac{3}{2}}-2\sqrt{-a+b}\,b^{\frac{5}{2}}+2\,b^{3}\right)\mathrm{sgn}(\sin\left(x\right))}{2\left(a^{4}\sqrt{-a+b}-a^{4}\sqrt{b}-2\,a^{3}\sqrt{-a+b}\,b+2\,a^{3}b^{\frac{3}{2}}+a^{2}\sqrt{-a+b}\,b^{2}-a^{2}b^{\frac{5}{2}}\right)}+\frac{\frac{2\sqrt{-a\cos\left(x\right)^{2}+b\cos\left(x\right)^{2}+a}\,b^{2}\cos(x)}{(a^{3}-a^{2}b)\left(a\cos(x)^{2}-b\cos(x)^{2}-a\right)}-\frac{\log\left(\left(\sqrt{-a+b}\cos(x)-\sqrt{-a\cos\left(x\right)^{2}+b\cos\left(x\right)^{2}+a}\right)^{2}\right)}{(a-b)\sqrt{-a+b}}-\frac{4\sqrt{-a+b}}{\left(\left(\sqrt{-a+b}\cos(x)-\sqrt{-a\cos\left(x\right)^{2}+b\cos\left(x\right)^{2}+a}\right)^{2}-a\right)a}}{2\,\mathrm{sgn}\left(\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+b*cot(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(a^3*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b) + a^2*sqrt(-a + b)*sqrt(b)*
log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b) - a^2*b*log(-a - 2*sqrt(-a + b)*sqrt
(b) + 2*b) + 2*a^3 - 4*a^2*b + 2*a*sqrt(-a + b)*b^(3/2) - 2*sqrt(-a + b)*b^
```

```
(5/2) + 2*b^3)*sgn(sin(x))/(a^4*sqrt(-a + b) - a^4*sqrt(b) - 2*a^3*sqrt(-a
+ b)*b + 2*a^3*b^(3/2) + a^2*sqrt(-a + b)*b^2 - a^2*b^(5/2)) + 1/2*(2*sqrt(
-a*cos(x)^2 + b*cos(x)^2 + a)*b^2*cos(x)/((a^3 - a^2*b)*(a*cos(x)^2 - b*cos
(x)^2 - a)) - log((sqrt(-a + b)*cos(x) - sqrt(-a*cos(x)^2 + b*cos(x)^2 + a)
)^2)/((a - b)*sqrt(-a + b)) - 4*sqrt(-a + b)/(((sqrt(-a + b)*cos(x) - sqrt(
-a*cos(x)^2 + b*cos(x)^2 + a))^2 - a)*a))/sgn(sin(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(x)^2}{\left(b\cot\left(x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(a + b*cot(x)^2)^(3/2),x)
```

```
[Out] int(tan(x)^2/(a + b*cot(x)^2)^(3/2), x)
```

3.54 $\int \frac{\cot^3(x)}{(a+b\cot^2(x))^{5/2}}\,dx$

Optimal. Leaf size=82

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}}+\frac{a}{3(a-b)b\left(a+b\cot^2(x)\right)^{3/2}}+\frac{1}{(a-b)^2\sqrt{a+b\cot^2(x)}}$$

```
[Out] -arctanh((a+b*cot(x)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)+1/3*a/(a-b)/b/(a+b*c
ot(x)^2)^(3/2)+1/(a-b)^2/(a+b*cot(x)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3751, 457, 79, 53, 65, 214}

$$\frac{a}{3b(a-b)\left(a+b\cot^2(x)\right)^{3/2}}+\frac{1}{(a-b)^2\sqrt{a+b\cot^2(x)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^3/(a + b*Cot[x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]]/(a - b)^(5/2)) + a/(3*(a - b)*b
*(a + b*Cot[x]^2)^(3/2)) + 1/((a - b)^2*Sqrt[a + b*Cot[x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^3(x)}{\left(a+b\cot^2(x)\right)^{5/2}}\,dx = -\text{Subst}\left(\int \frac{x^3}{(1+x^2)\left(a+bx^2\right)^{5/2}}\,dx, x, \cot(x)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{x}{(1+x)(a+bx)^{5/2}}\,dx, x, \cot^2(x)\right)\right)$$
$$= \frac{a}{3(a-b)b\left(a+b\cot^2(x)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{3/2}}\,dx, x, \cot^2(x)\right)}{2(a-b)}$$
$$= \frac{a}{3(a-b)b\left(a+b\cot^2(x)\right)^{3/2}} + \frac{1}{(a-b)^2\sqrt{a+b\cot^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\right.}{2(a-b)^2}$$
$$= \frac{a}{3(a-b)b\left(a+b\cot^2(x)\right)^{3/2}} + \frac{1}{(a-b)^2\sqrt{a+b\cot^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x,\right.}{(a-b)^2}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}} + \frac{a}{3(a-b)b\left(a+b\cot^2(x)\right)^{3/2}} + \frac{1}{(a-b)^2\sqrt{a+b\cot^2(x)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.13, size = 69, normalized size = 0.84

$$\frac{a(a-b)+3b\left(a+b\cot^2(x)\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\cot^2(x)}{a-b}\right)}{3(a-b)^2b\left(a+b\cot^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3/(a + b*Cot[x]^2)^(5/2),x]
```

```
[Out] (a*(a - b) + 3*b*(a + b*Cot[x]^2)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Co
t[x]^2)/(a - b)])/(3*(a - b)^2*b*(a + b*Cot[x]^2)^(3/2))
```

Maple [A]
time = 0.16, size = 88, normalized size = 1.07

method	result

derivativedivides	$\frac{1}{3b(a+b(\cot^2(x)))^{\frac{3}{2}}}+\frac{1}{3(a-b)(a+b(\cot^2(x)))^{\frac{3}{2}}}+\frac{\arctan\left(\frac{\sqrt{a+b(\cot^2(x))}}{\sqrt{-a+b}}\right)}{(a-b)^2\sqrt{-a+b}}+\frac{1}{(a-b)^2\sqrt{a+b(\cot^2}}$
default	$\frac{1}{3b(a+b(\cot^2(x)))^{\frac{3}{2}}}+\frac{1}{3(a-b)(a+b(\cot^2(x)))^{\frac{3}{2}}}+\frac{\arctan\left(\frac{\sqrt{a+b(\cot^2(x))}}{\sqrt{-a+b}}\right)}{(a-b)^2\sqrt{-a+b}}+\frac{1}{(a-b)^2\sqrt{a+b(\cot^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3/(a+b*cot(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/b/(a+b*cot(x)^2)^(3/2)+1/3/(a-b)/(a+b*cot(x)^2)^(3/2)+1/(a-b)^2/(-a+b)^
(1/2)*arctan((a+b*cot(x)^2)^(1/2)/(-a+b)^(1/2))+1/(a-b)^2/(a+b*cot(x)^2)^(1
/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+b*cot(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(70) = 140$.
time = 2.88, size = 698, normalized size = 8.51

$$\left[\frac{3\left(a^2b+2ab^2+b^3+\left(a^2b-2ab^2+b^3\right)\cos\left(2x\right)^2-2\left(a^2b-b^3\right)\cos\left(2x\right)\right)\sqrt{a-b}\log\left(\sqrt{a-b}\sqrt{\frac{(a-b)\cos\left(2x\right)-a-b}{\cos\left(2x\right)-1}}\left(\cos\left(2x\right)-1\right)-(a-b)\cos\left(2x\right)+a\right)+2\left(a^3+a^2b+ab^2-3b^3+\left(a^3+a^2b-5ab^2+3b^3\right)\cos\left(2x\right)^2-2\left(a^3+a^2b-2ab^2\right)\cos\left(2x\right)\right)\sqrt{\frac{(a-b)\cos\left(2x\right)-a-b}{\cos\left(2x\right)-1}}}{6\left(a^5b-a^4b^2-2a^3b^3+2a^2b^4+ab^5-b^6+\left(a^5b-5a^4b^2+10a^3b^3-10a^2b^4+5ab^5-b^6\right)\cos\left(2x\right)^2-2\left(a^5b-3a^4b^2+2a^3b^3+2a^2b^4-3ab^5+b^6\right)\cos\left(2x\right)\right)}, -\frac{3\left(a^2b+2ab^2+b^3+\left(a^2b-2ab^2+b^3\right)\cos\left(2x\right)^2-2\left(a^2b-b^3\right)\cos\left(2x\right)\right)\sqrt{-a+b}\arctan\left(-\frac{\sqrt{-a+b}\sqrt{\frac{(a-b)\cos\left(2x\right)-a-b}{\cos\left(2x\right)-1}}}{a-b}\right)-\left(a^3+a^2b+ab^2-3b^3+\left(a^3+a^2b-5ab^2+3b^3\right)\cos\left(2x\right)^2-2\left(a^3+a^2b-2ab^2\right)\cos\left(2x\right)\right)\sqrt{\frac{(a-b)\cos\left(2x\right)-a-b}{\cos\left(2x\right)-1}}}{3\left(a^5b-a^4b^2-2a^3b^3+2a^2b^4+ab^5-b^6+\left(a^5b-5a^4b^2+10a^3b^3-10a^2b^4+5ab^5-b^6\right)\cos\left(2x\right)^2-2\left(a^5b-3a^4b^2+2a^3b^3+2a^2b^4-3ab^5+b^6\right)\cos\left(2x\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+b*cot(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a^2*b + 2*a*b^2 + b^3 + (a^2*b - 2*a*b^2 + b^3)*cos(2*x)^2 - 2*(a^
2*b - b^3)*cos(2*x))*sqrt(a - b)*log(sqrt(a - b)*sqrt(((a - b)*cos(2*x) - a
 - b)/(cos(2*x) - 1))*(cos(2*x) - 1) - (a - b)*cos(2*x) + a) + 2*(a^3 + a^2
*b + a*b^2 - 3*b^3 + (a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cos(2*x)^2 - 2*(a^3 +
a^2*b - 2*a*b^2)*cos(2*x))*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)))
/(a^5*b - a^4*b^2 - 2*a^3*b^3 + 2*a^2*b^4 + a*b^5 - b^6 + (a^5*b - 5*a^4*b^
```

```
2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*cos(2*x)^2 - 2*(a^5*b - 3*a^4*
b^2 + 2*a^3*b^3 + 2*a^2*b^4 - 3*a*b^5 + b^6)*cos(2*x)), -1/3*(3*(a^2*b + 2*
a*b^2 + b^3 + (a^2*b - 2*a*b^2 + b^3)*cos(2*x)^2 - 2*(a^2*b - b^3)*cos(2*x)
)*sqrt(-a + b)*arctan(-sqrt(-a + b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*
x) - 1))/(a - b)) - (a^3 + a^2*b + a*b^2 - 3*b^3 + (a^3 + a^2*b - 5*a*b^2 +
 3*b^3)*cos(2*x)^2 - 2*(a^3 + a^2*b - 2*a*b^2)*cos(2*x))*sqrt(((a - b)*cos(
2*x) - a - b)/(cos(2*x) - 1)))/(a^5*b - a^4*b^2 - 2*a^3*b^3 + 2*a^2*b^4 + a
*b^5 - b^6 + (a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*
cos(2*x)^2 - 2*(a^5*b - 3*a^4*b^2 + 2*a^3*b^3 + 2*a^2*b^4 - 3*a*b^5 + b^6)*
cos(2*x))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(x)}{(a+b\cot^2(x))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**3/(a+b*cot(x)**2)**(5/2),x)
```

```
[Out] Integral(cot(x)**3/(a + b*cot(x)**2)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(70) = 140.
time = 0.49, size = 219, normalized size = 2.67

$$-\frac{\log(|b|)\,\mathrm{sgn}(\sin(x))}{2\left(\sqrt{a-b}\,a^2-2\sqrt{a-b}\,ab+\sqrt{a-b}\,b^2\right)}+\frac{\frac{\left(\frac{(a^3+a^2b-5\,ab^2+3\,b^3)\sin(x)^2}{a^3b-3\,a^2b^2+3\,ab^3-b^4}+\frac{3\,(ab^2-b^3)}{a^3b-3\,a^2b^2+3\,ab^3-b^4}\right)\sin(x)}{(a\sin(x)^2-b\sin(x)^2+b)^{\frac{3}{2}}}+\frac{3\log\left(\left|-\sqrt{a-b}\sin(x)+\sqrt{a\sin(x)^2-b\sin(x)^2+b}\right|\right)}{(a^2-2\,ab+b^2)\sqrt{a-b}}}{3\,\mathrm{sgn}(\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+b*cot(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(b))*sgn(sin(x))/(sqrt(a - b)*a^2 - 2*sqrt(a - b)*a*b + sqrt(a
- b)*b^2) + 1/3*(((a^3 + a^2*b - 5*a*b^2 + 3*b^3)*sin(x)^2/(a^3*b - 3*a^2*b
^2 + 3*a*b^3 - b^4) + 3*(a*b^2 - b^3)/(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4))*
sin(x)/(a*sin(x)^2 - b*sin(x)^2 + b)^(3/2) + 3*log(abs(-sqrt(a - b)*sin(x)
+ sqrt(a*sin(x)^2 - b*sin(x)^2 + b)))/((a^2 - 2*a*b + b^2)*sqrt(a - b)))/sg
n(sin(x))
```

Mupad [B]
time = 4.24, size = 88, normalized size = 1.07

$$\frac{\frac{a}{3\,(a-b)}+\frac{b\,\left(b\cot(x)^2+a\right)}{(a-b)^2}}{b\,\left(b\cot(x)^2+a\right)^{3/2}}-\frac{\mathrm{atanh}\left(\frac{\sqrt{b\cot(x)^2+a}\,\left(2\,a^2-4\,a\,b+2\,b^2\right)}{2\,(a-b)^{5/2}}\right)}{(a-b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3/(a + b*cot(x)^2)^(5/2),x)
```

```
[Out] (a/(3*(a - b)) + (b*(a + b*cot(x)^2))/(a - b)^2)/(b*(a + b*cot(x)^2)^(3/2))
 - atanh(((a + b*cot(x)^2)^(1/2)*(2*a^2 - 4*a*b + 2*b^2))/(2*(a - b)^(5/2))
)/(a - b)^(5/2)
```

3.55 $\int \frac{\cot^2(x)}{(a+b\cot^2(x))^{5/2}}\, dx$

Optimal. Leaf size=94

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{(a-b)^{5/2}}-\frac{\cot(x)}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}}-\frac{(2a+b)\cot(x)}{3a(a-b)^2\sqrt{a+b\cot^2(x)}}$$

```
[Out] arctan(cot(x)*(a-b)^(1/2)/(a+b*cot(x)^2)^(1/2))/(a-b)^(5/2)-1/3*cot(x)/(a-b
)/(a+b*cot(x)^2)^(3/2)-1/3*(2*a+b)*cot(x)/a/(a-b)^2/(a+b*cot(x)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3751, 482, 541, 12, 385, 209}

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{(a-b)^{5/2}}-\frac{(2a+b)\cot(x)}{3a(a-b)^2\sqrt{a+b\cot^2(x)}}-\frac{\cot(x)}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2/(a + b*Cot[x]^2)^(5/2),x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Cot[x])/Sqrt[a + b*Cot[x]^2]]/(a - b)^(5/2) - Cot[x]/(3
*(a - b)*(a + b*Cot[x]^2)^(3/2)) - ((2*a + b)*Cot[x])/(3*a*(a - b)^2*Sqrt[a
 + b*Cot[x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^2(x)}{\left(a+b\cot^2(x)\right)^{5/2}}\,dx &= -\mathrm{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)\left(a+bx^2\right)^{5/2}}\,dx, x, \cot(x)\right)\\
&= -\frac{\cot(x)}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}} + \frac{\mathrm{Subst}\left(\int \frac{1-2x^2}{(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \cot(x)\right)}{3(a-b)}\\
&= -\frac{\cot(x)}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}} - \frac{(2a+b)\cot(x)}{3a(a-b)^2\sqrt{a+b\cot^2(x)}} + \frac{\mathrm{Subst}\left(\int \frac{3a}{(1+x^2)\sqrt{a+}}\right.}{3a(a-}\\
&= -\frac{\cot(x)}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}} - \frac{(2a+b)\cot(x)}{3a(a-b)^2\sqrt{a+b\cot^2(x)}} + \frac{\mathrm{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+}}\right.}{(a-}\\
&= -\frac{\cot(x)}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}} - \frac{(2a+b)\cot(x)}{3a(a-b)^2\sqrt{a+b\cot^2(x)}} + \frac{\mathrm{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\right.}{(a}\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{(a-b)^{5/2}} - \frac{\cot(x)}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}} - \frac{(2a+b)\cot(x)}{3a(a-b)^2\sqrt{a+b\cot^2}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 6.86, size = 200, normalized size = 2.13

$$
\frac{\left(-12(a-b)^3\cos^4(x)\cot^2(x)\left(a+b\cot^2(x)\right)\, {}_2F_1\left(2,2;\frac{9}{2};\frac{(a-b)\cos^2(x)}{a}\right) - \frac{35a\left(5a+2b\cot^2(x)\right)\sin^2(x)\left(3\mathrm{ArcSin}\left(\sqrt{\frac{(a-b)\cos^2(x)}{a}}\right)\left(a+b\cot^2(x)\right)^2+a\left(-3a+(a-4b)\cot^2(x)\right)\csc^2(x)\sqrt{\frac{(a-b)\cos^2(x)\left(a+b\cot^2(x)\right)\sin^2(x)}{a^2}}\right)}{\sqrt{\frac{(a-b)\cos^2(x)\left(a+b\cot^2(x)\right)\sin^2(x)}{a^2}}}\right)\tan(x)}{315a^3(a-b)^2\left(a+b\cot^2(x)\right)^{3/2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[x]^2/(a + b*Cot[x]^2)^(5/2),x]
```

```
[Out] ((-12*(a - b)^3*Cos[x]^4*Cot[x]^2*(a + b*Cot[x]^2)*Hypergeometric2F1[2, 2,
9/2, ((a - b)*Cos[x]^2)/a] - (35*a*(5*a + 2*b*Cot[x]^2)*Sin[x]^2*(3*ArcSin[
Sqrt[((a - b)*Cos[x]^2)/a]]*(a + b*Cot[x]^2)^2 + a*(-3*a + (a - 4*b)*Cot[x]
^2)*Csc[x]^2*Sqrt[((a - b)*Cos[x]^2*(a + b*Cot[x]^2)*Sin[x]^2)/a^2]))/Sqrt[
((a - b)*Cos[x]^2*(a + b*Cot[x]^2)*Sin[x]^2)/a^2])*Tan[x])/(315*a^3*(a - b)
^2*(a + b*Cot[x]^2)^(3/2))
```

Maple [A]
time = 0.14, size = 161, normalized size = 1.71

method	result
derivativedivides	$-\frac{\cot(x)}{3a(a+b(\cot^2(x)))^{\frac{3}{2}}}-\frac{2\cot(x)}{3a^2\sqrt{a+b(\cot^2(x))}}-\frac{b\left(\frac{\cot(x)}{3a\left(a+b\left(\cot^2(x)\right)\right)^{\frac{3}{2}}}+\frac{2\cot(x)}{3a^2\sqrt{a+b(\cot^2(x))}}\right)}{a-b}+$
default	$-\frac{\cot(x)}{3a(a+b(\cot^2(x)))^{\frac{3}{2}}}-\frac{2\cot(x)}{3a^2\sqrt{a+b(\cot^2(x))}}-\frac{b\left(\frac{\cot(x)}{3a\left(a+b\left(\cot^2(x)\right)\right)^{\frac{3}{2}}}+\frac{2\cot(x)}{3a^2\sqrt{a+b(\cot^2(x))}}\right)}{a-b}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/(a+b*cot(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*cot(x)/a/(a+b*cot(x)^2)^(3/2)-2/3/a^2*cot(x)/(a+b*cot(x)^2)^(1/2)-1/(a
-b)*b*(1/3*cot(x)/a/(a+b*cot(x)^2)^(3/2)+2/3/a^2*cot(x)/(a+b*cot(x)^2)^(1/2
))+1/(a-b)^3*(b^4*(a-b))^(1/2)/b^2*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*
cot(x)^2)^(1/2)*cot(x))-1/(a-b)^2*b*cot(x)/a/(a+b*cot(x)^2)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+b*cot(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(80) = 160$.
time = 3.25, size = 720, normalized size = 7.66

$$\left[-\frac{3\left(a^3+2a^2b+ab^2+(a^3-2a^2b+ab^2)\cos(2x)^2-2(a^3-ab^2)\cos(2x)\right)\sqrt{-a+b}\log\left(-2(a^2-2ab+b^2)\cos(2x)^2+2((a-b)\cos(2x)-b)\sqrt{-a+b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\sin(2x)+a^2-2b^2+4(ab-b^2)\cos(2x)\right)+4\left(3a^3-a^2b-ab^2-b^3-(3a^3-5a^2b+ab^2+b^3)\cos(2x)\right)\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\sin(2x)}{12\left(a^6-a^5b-2a^4b^2+2a^3b^3+a^2b^4-ab^5+(a^6-5a^5b+10a^4b^2-10a^3b^3+5a^2b^4-ab^5)\cos(2x)^2-2(a^6-3a^5b+2a^4b^2+2a^3b^3-3a^2b^4+ab^5)\cos(2x)\right)}, \frac{3\left(a^3+2a^2b+ab^2+(a^3-2a^2b+ab^2)\cos(2x)^2-2(a^3-ab^2)\cos(2x)\right)\sqrt{a-b}\arctan\left(-\frac{\sqrt{a-b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\sin(2x)}{(a-b)\cos(2x)-b}\right)-2\left(3a^3-a^2b-ab^2-b^3-(3a^3-5a^2b+ab^2+b^3)\cos(2x)\right)\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}\sin(2x)}{6\left(a^6-a^5b-2a^4b^2+2a^3b^3+a^2b^4-ab^5+(a^6-5a^5b+10a^4b^2-10a^3b^3+5a^2b^4-ab^5)\cos(2x)^2-2(a^6-3a^5b+2a^4b^2+2a^3b^3-3a^2b^4+ab^5)\cos(2x)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+b*cot(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(a^3 + 2*a^2*b + a*b^2 + (a^3 - 2*a^2*b + a*b^2)*cos(2*x)^2 - 2*(
a^3 - a*b^2)*cos(2*x))*sqrt(-a + b)*log(-2*(a^2 - 2*a*b + b^2)*cos(2*x)^2 +
```

```
 2*((a - b)*cos(2*x) - b)*sqrt(-a + b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos
(2*x) - 1))*sin(2*x) + a^2 - 2*b^2 + 4*(a*b - b^2)*cos(2*x)) + 4*(3*a^3 - a
^2*b - a*b^2 - b^3 - (3*a^3 - 5*a^2*b + a*b^2 + b^3)*cos(2*x))*sqrt(((a - b
)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x))/(a^6 - a^5*b - 2*a^4*b^2 + 2*
a^3*b^3 + a^2*b^4 - a*b^5 + (a^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^
2*b^4 - a*b^5)*cos(2*x)^2 - 2*(a^6 - 3*a^5*b + 2*a^4*b^2 + 2*a^3*b^3 - 3*a^
2*b^4 + a*b^5)*cos(2*x)), 1/6*(3*(a^3 + 2*a^2*b + a*b^2 + (a^3 - 2*a^2*b +
a*b^2)*cos(2*x)^2 - 2*(a^3 - a*b^2)*cos(2*x))*sqrt(a - b)*arctan(-sqrt(a -
b)*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x)/((a - b)*cos(2*
x) - b)) - 2*(3*a^3 - a^2*b - a*b^2 - b^3 - (3*a^3 - 5*a^2*b + a*b^2 + b^3)
*cos(2*x))*sqrt(((a - b)*cos(2*x) - a - b)/(cos(2*x) - 1))*sin(2*x))/(a^6 -
 a^5*b - 2*a^4*b^2 + 2*a^3*b^3 + a^2*b^4 - a*b^5 + (a^6 - 5*a^5*b + 10*a^4*
b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*cos(2*x)^2 - 2*(a^6 - 3*a^5*b + 2*a^4
*b^2 + 2*a^3*b^3 - 3*a^2*b^4 + a*b^5)*cos(2*x))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(x)}{(a + b\cot^2(x))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2/(a+b*cot(x)**2)**(5/2),x)
```

```
[Out] Integral(cot(x)**2/(a + b*cot(x)**2)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(80) = 160.
time = 0.52, size = 278, normalized size = 2.96

$$-\frac{\left(3\, a\sqrt{-a+b}\, \sqrt{b}\, \log\left(\left|-\sqrt{-a+b} + \sqrt{b}\right|\right) - 2\, a^2 + ab + b^2\right)\mathrm{sgn}(\sin(x))}{3\left(a^4\sqrt{b} - 3\, a^3 b^{\frac{3}{2}} + 3\, a^2 b^{\frac{5}{2}} - ab^{\frac{7}{2}}\right)} - \frac{\frac{\left(\frac{\left(3\, a^3 - 5\, a^2 b + ab^2 + b^3\right)\cos(x)^2}{a^4 - 3\, a^3 b + 3\, a^2 b^2 - ab^3} - \frac{3\left(a^3 - a^2 b\right)}{a^4 - 3\, a^3 b + 3\, a^2 b^2 - ab^3}\right)\cos(x)}{\left(a\cos(x)^2 - b\cos(x)^2 - a\right)\sqrt{-a\cos(x)^2 + b\cos(x)^2 + a}} - \frac{3\sqrt{-a+b}\, \log\left(\left|-\sqrt{-a+b}\, \cos(x) + \sqrt{-a\cos(x)^2 + b\cos(x)^2 + a}\right|\right)}{a^3 - 3\, a^2 b + 3\, ab^2 - b^3}}{3\,\mathrm{sgn}(\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+b*cot(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*a*sqrt(-a + b)*sqrt(b)*log(abs(-sqrt(-a + b) + sqrt(b))) - 2*a^2 +
a*b + b^2)*sgn(sin(x))/(a^4*sqrt(b) - 3*a^3*b^(3/2) + 3*a^2*b^(5/2) - a*b^(
7/2)) - 1/3*(((3*a^3 - 5*a^2*b + a*b^2 + b^3)*cos(x)^2/(a^4 - 3*a^3*b + 3*a
^2*b^2 - a*b^3) - 3*(a^3 - a^2*b)/(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3))*cos(
x)/((a*cos(x)^2 - b*cos(x)^2 - a)*sqrt(-a*cos(x)^2 + b*cos(x)^2 + a)) - 3*s
qrt(-a + b)*log(abs(-sqrt(-a + b)*cos(x) + sqrt(-a*cos(x)^2 + b*cos(x)^2 +
a)))/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/sgn(sin(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(x)^2}{\left(b\cot\left(x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/(a + b*cot(x)^2)^(5/2),x)
```

```
[Out] int(cot(x)^2/(a + b*cot(x)^2)^(5/2), x)
```

3.56 $\int \frac{\cot(x)}{\left(a+b\cot^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=78

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}}-\frac{1}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}}-\frac{1}{(a-b)^2\sqrt{a+b\cot^2(x)}}$$

```
[Out] arctanh((a+b*cot(x)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)-1/3/(a-b)/(a+b*cot(x)
^2)^(3/2)-1/(a-b)^2/(a+b*cot(x)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{3751, 455, 53, 65, 214\}$

$$-\frac{1}{(a-b)^2\sqrt{a+b\cot^2(x)}}-\frac{1}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(a + b*Cot[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a - b]]/(a - b)^(5/2) - 1/(3*(a - b)*(a +
 b*Cot[x]^2)^(3/2)) - 1/((a - b)^2*Sqrt[a + b*Cot[x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot(x)}{\left(a+b\cot^2(x)\right)^{5/2}}\,dx = -\text{Subst}\left(\int \frac{x}{(1+x^2)\,(a+bx^2)^{5/2}}\,dx, x, \cot(x)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{5/2}}\,dx, x, \cot^2(x)\right)\right)$$
$$= -\frac{1}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{3/2}}\,dx, x, \cot^2(x)\right)}{2(a-b)}$$
$$= -\frac{1}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}} - \frac{1}{(a-b)^2\sqrt{a+b\cot^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\right.}{2(a-b)}$$
$$= -\frac{1}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}} - \frac{1}{(a-b)^2\sqrt{a+b\cot^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x,\right.}{(a-b)}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}} - \frac{1}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}} - \frac{1}{(a-b)^2\sqrt{a+b\cot^2(}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, size = 47, normalized size = 0.60

$$-\frac{{}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; \frac{a+b\cot^2(x)}{a-b}\right)}{3(a-b)\left(a+b\cot^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(a + b*Cot[x]^2)^(5/2),x]
```

```
[Out] -1/3*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Cot[x]^2)/(a - b)]/((a - b)*(a
 + b*Cot[x]^2)^(3/2))
```

Maple [A]
time = 0.12, size = 75, normalized size = 0.96

method	result	size

derivativedivides	$-\frac{1}{3(a-b)(a+b(\cot^2(x)))^{\frac{3}{2}}}-\frac{1}{(a-b)^2\sqrt{a+b(\cot^2(x))}}-\frac{\arctan\left(\frac{\sqrt{a+b(\cot^2(x))}}{\sqrt{-a+b}}\right)}{(a-b)^2\sqrt{-a+b}}$	75
default	$-\frac{1}{3(a-b)(a+b(\cot^2(x)))^{\frac{3}{2}}}-\frac{1}{(a-b)^2\sqrt{a+b(\cot^2(x))}}-\frac{\arctan\left(\frac{\sqrt{a+b(\cot^2(x))}}{\sqrt{-a+b}}\right)}{(a-b)^2\sqrt{-a+b}}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a+b*cot(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/(a-b)/(a+b*cot(x)^2)^(3/2)-1/(a-b)^2/(a+b*cot(x)^2)^(1/2)-1/(a-b)^2/(-
a+b)^(1/2)*arctan((a+b*cot(x)^2)^(1/2)/(-a+b)^(1/2))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(66) = 132$.
time = 3.28, size = 627, normalized size = 8.04

$$\left[\frac{3\left((a^2-2ab+b^2)\cos(2x)^2+a^2+2ab+b^2-2(a^2-b^2)\cos(2x)\right)\sqrt{a-b}\log\left(-\sqrt{a-b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}(\cos(2x)-1)-(a-b)\cos(2x)+a\right)-4\left(2(a^2-2ab+b^2)\cos(2x)^2+2a^2-ab-b^2-(4a^2-5ab+b^2)\cos(2x)\right)\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}}{6\left(a^5-a^4b-2a^3b^2+2a^2b^3+ab^4-b^5+(a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5)\cos(2x)^2-2(a^5-3a^4b+2a^3b^2+2a^2b^3-3ab^4+b^5)\cos(2x)\right)}, \frac{3\left((a^2-2ab+b^2)\cos(2x)^2+a^2+2ab+b^2-2(a^2-b^2)\cos(2x)\right)\sqrt{-a+b}\arctan\left(-\frac{\sqrt{-a+b}\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}}{a-b}\right)-2\left(2(a^2-2ab+b^2)\cos(2x)^2+2a^2-ab-b^2-(4a^2-5ab+b^2)\cos(2x)\right)\sqrt{\frac{(a-b)\cos(2x)-a-b}{\cos(2x)-1}}}{3\left(a^5-a^4b-2a^3b^2+2a^2b^3+ab^4-b^5+(a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5)\cos(2x)^2-2(a^5-3a^4b+2a^3b^2+2a^2b^3-3ab^4+b^5)\cos(2x)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*((a^2 - 2*a*b + b^2)*cos(2*x)^2 + a^2 + 2*a*b + b^2 - 2*(a^2 - b^2)
*cos(2*x))*sqrt(a - b)*log(-sqrt(a - b)*sqrt(((a - b)*cos(2*x) - a - b)/(co
s(2*x) - 1))*(cos(2*x) - 1) - (a - b)*cos(2*x) + a) - 4*(2*(a^2 - 2*a*b + b
^2)*cos(2*x)^2 + 2*a^2 - a*b - b^2 - (4*a^2 - 5*a*b + b^2)*cos(2*x))*sqrt((
(a - b)*cos(2*x) - a - b)/(cos(2*x) - 1)))/(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2
*b^3 + a*b^4 - b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b
^5)*cos(2*x)^2 - 2*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5)*
```

```
cos(2*x)), 1/3*(3*((a^2 - 2*a*b + b^2)*cos(2*x)^2 + a^2 + 2*a*b + b^2 - 2*(
a^2 - b^2)*cos(2*x))*sqrt(-a + b)*arctan(-sqrt(-a + b)*sqrt(((a - b)*cos(2*
x) - a - b)/(cos(2*x) - 1))/(a - b)) - 2*(2*(a^2 - 2*a*b + b^2)*cos(2*x)^2
+ 2*a^2 - a*b - b^2 - (4*a^2 - 5*a*b + b^2)*cos(2*x))*sqrt(((a - b)*cos(2*x
) - a - b)/(cos(2*x) - 1)))/(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 -
b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*cos(2*x)^2
- 2*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5)*cos(2*x))]
```

Sympy [A]
time = 8.21, size = 70, normalized size = 0.90

$$-\frac{1}{3\left(a-b\right)\left(a+b\cot^{2}\left(x\right)\right)^{\frac{3}{2}}}-\frac{1}{\left(a-b\right)^{2}\sqrt{a+b\cot^{2}\left(x\right)}}-\frac{\operatorname{atan}\left(\frac{\sqrt{a+b\cot^{2}\left(x\right)}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}\left(a-b\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)**2)**(5/2),x)
```

```
[Out] -1/(3*(a - b)*(a + b*cot(x)**2)**(3/2)) - 1/((a - b)**2*sqrt(a + b*cot(x)**
2)) - atan(sqrt(a + b*cot(x)**2)/sqrt(-a + b))/(sqrt(-a + b)*(a - b)**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(66) = 132.
time = 0.46, size = 215, normalized size = 2.76

$$\frac{\log\left(|b|\right)\operatorname{sgn}(\sin\left(x\right))}{2\left(\sqrt{a-b}\,a^{2}-2\sqrt{a-b}\,ab+\sqrt{a-b}\,b^{2}\right)}-\frac{\frac{\left(\frac{4\left(a^{2}b-2ab^{2}+b^{3}\right)\sin(x)^{2}}{a^{3}b-3a^{2}b^{2}+3ab^{3}-b^{4}}+\frac{3\left(ab^{2}-b^{3}\right)}{a^{3}b-3a^{2}b^{2}+3ab^{3}-b^{4}}\right)\sin(x)}{\left(a\sin(x)^{2}-b\sin(x)^{2}+b\right)^{\frac{3}{2}}}+\frac{3\log\left(\left|-\sqrt{a-b}\sin(x)+\sqrt{a\sin\left(x\right)^{2}-b\sin\left(x\right)^{2}+b}\right|\right)}{\left(a^{2}-2ab+b^{2}\right)\sqrt{a-b}}}{3\operatorname{sgn}\left(\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b))*sgn(sin(x))/(sqrt(a - b)*a^2 - 2*sqrt(a - b)*a*b + sqrt(a -
 b)*b^2) - 1/3*((4*(a^2*b - 2*a*b^2 + b^3)*sin(x)^2/(a^3*b - 3*a^2*b^2 + 3*
a*b^3 - b^4) + 3*(a*b^2 - b^3)/(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4))*sin(x)/
(a*sin(x)^2 - b*sin(x)^2 + b)^(3/2) + 3*log(abs(-sqrt(a - b)*sin(x) + sqrt(
a*sin(x)^2 - b*sin(x)^2 + b)))/((a^2 - 2*a*b + b^2)*sqrt(a - b)))/sgn(sin(x
))
```

Mupad [B]
time = 4.46, size = 82, normalized size = 1.05

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b\cot\left(x\right)^{2}+a}\,\left(2a^{2}-4ab+2b^{2}\right)}{2\left(a-b\right)^{5/2}}\right)}{\left(a-b\right)^{5/2}}-\frac{\frac{1}{3\left(a-b\right)}+\frac{b\cot(x)^{2}+a}{\left(a-b\right)^{2}}}{\left(b\cot\left(x\right)^{2}+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a + b*cot(x)^2)^(5/2),x)
```

```
[Out] atanh(((a + b*cot(x)^2)^(1/2)*(2*a^2 - 4*a*b + 2*b^2))/(2*(a - b)^(5/2)))/(
a - b)^(5/2) - (1/(3*(a - b)) + (a + b*cot(x)^2)/(a - b)^2)/(a + b*cot(x)^2
)^(3/2)
```

3.57 $\int \frac{\tan(x)}{\left(a+b\cot^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=118

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}}+\frac{b}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}}+\frac{(2a-b)b}{a^2(a-b)^2\sqrt{a+b}}$$

```
[Out] arctanh((a+b*cot(x)^2)^(1/2)/a^(1/2))/a^(5/2)-arctanh((a+b*cot(x)^2)^(1/2)/
(a-b)^(1/2))/(a-b)^(5/2)+1/3*b/a/(a-b)/(a+b*cot(x)^2)^(3/2)+(2*a-b)*b/a^2/(
a-b)^2/(a+b*cot(x)^2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3751, 457, 87, 157, 162, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{b(2a-b)}{a^2(a-b)^2\sqrt{a+b\cot^2(x)}}+\frac{b}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(a + b*Cot[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Cot[x]^2]/Sqrt[a]]/a^(5/2) - ArcTanh[Sqrt[a + b*Cot[x]^2
]/Sqrt[a - b]]/(a - b)^(5/2) + b/(3*a*(a - b)*(a + b*Cot[x]^2)^(3/2)) + ((2
*a - b)*b)/(a^2*(a - b)^2*Sqrt[a + b*Cot[x]^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[(b*d*e - b*c*f - a*d*f - b*d*f*x
)*((e + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan(x)}{\left(a+b\cot^2(x)\right)^{5/2}}\,dx = -\text{Subst}\left(\int \frac{1}{x\,(1+x^2)\,(a+bx^2)^{5/2}}\,dx, x, \cot(x)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)(a+bx)^{5/2}}\,dx, x, \cot^2(x)\right)\right)$$
$$= \frac{b}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{a-b-bx}{x(1+x)(a+bx)^{3/2}}\,dx, x, \cot^2(x)\right)}{2a(a-b)}$$
$$= \frac{b}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}} + \frac{(2a-b)b}{a^2(a-b)^2\sqrt{a+b\cot^2(x)}} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(a-b)^2+\frac{1}{2}(2}{x(1+x)\sqrt{a+}}\right.}{a^2(a}$$
$$= \frac{b}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}} + \frac{(2a-b)b}{a^2(a-b)^2\sqrt{a+b\cot^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\right.}{2a^2}$$
$$= \frac{b}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}} + \frac{(2a-b)b}{a^2(a-b)^2\sqrt{a+b\cot^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x\right.}{a^2b}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cot^2(x)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}} + \frac{b}{3a(a-b)\left(a+b\,\text{co}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.07, size = 78, normalized size = 0.66

$$\frac{a\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\cot^2(x)}{a-b}\right)+(-a+b)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1+\frac{b\cot^2(x)}{a}\right)}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(a + b*Cot[x]^2)^(5/2),x]
```

```
[Out] (a*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Cot[x]^2)/(a - b)] + (-a + b)*Hy
pergeometric2F1[-3/2, 1, -1/2, 1 + (b*Cot[x]^2)/a])/(3*a*(a - b)*(a + b*Cot
[x]^2)^(3/2))
```

Maple [F]

time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{(a+b(\cot^2(x)))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a+b*cot(x)^2)^(5/2),x)
[Out] int(tan(x)/(a+b*cot(x)^2)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cot(x)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(tan(x)/(b*cot(x)^2 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(100) = 200$.
time = 3.34, size = 1531, normalized size = 12.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cot(x)^2)^(5/2),x, algorithm="fricas")
[Out] [1/6*(3*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5 + (a^5 - 3*a^4*b + 3*a^3*b^2 -
 a^2*b^3)*tan(x)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*tan(x)^2)*sq
rt(a)*log(2*a*tan(x)^2 + 2*sqrt(a)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2
 + b) + 3*(a^5*tan(x)^4 + 2*a^4*b*tan(x)^2 + a^3*b^2)*sqrt(a - b)*log(((2*a
 - b)*tan(x)^2 - 2*sqrt(a - b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2 + b
)/(tan(x)^2 + 1)) + 2*((7*a^4*b - 11*a^3*b^2 + 4*a^2*b^3)*tan(x)^4 + 3*(2*a
^3*b^2 - 3*a^2*b^3 + a*b^4)*tan(x)^2)*sqrt((a*tan(x)^2 + b)/tan(x)^2))/(a^6
*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5 + (a^8 - 3*a^7*b + 3*a^6*b^2 - a^5*b
^3)*tan(x)^4 + 2*(a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*tan(x)^2), -1/6*
(6*(a^5*tan(x)^4 + 2*a^4*b*tan(x)^2 + a^3*b^2)*sqrt(-a + b)*arctan(-sqrt(-a
 + b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)/(a - b)) - 3*(a^3*b^2 - 3*a^2*b^3 + 3
*a*b^4 - b^5 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*tan(x)^4 + 2*(a^4*b -
3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*tan(x)^2)*sqrt(a)*log(2*a*tan(x)^2 + 2*sqrt(
a)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2 + b) - 2*((7*a^4*b - 11*a^3*b^2
 + 4*a^2*b^3)*tan(x)^4 + 3*(2*a^3*b^2 - 3*a^2*b^3 + a*b^4)*tan(x)^2)*sqrt((
a*tan(x)^2 + b)/tan(x)^2))/(a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5 + (a^
```

```
8 - 3*a^7*b + 3*a^6*b^2 - a^5*b^3)*tan(x)^4 + 2*(a^7*b - 3*a^6*b^2 + 3*a^5*
b^3 - a^4*b^4)*tan(x)^2), -1/6*(6*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5 + (a
^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*tan(x)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2
*b^3 - a*b^4)*tan(x)^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*tan(x)^2 + b)/tan(
x)^2)/a) - 3*(a^5*tan(x)^4 + 2*a^4*b*tan(x)^2 + a^3*b^2)*sqrt(a - b)*log(((
2*a - b)*tan(x)^2 - 2*sqrt(a - b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)^2
+ b)/(tan(x)^2 + 1)) - 2*((7*a^4*b - 11*a^3*b^2 + 4*a^2*b^3)*tan(x)^4 + 3*(
2*a^3*b^2 - 3*a^2*b^3 + a*b^4)*tan(x)^2)*sqrt((a*tan(x)^2 + b)/tan(x)^2))/(
a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5 + (a^8 - 3*a^7*b + 3*a^6*b^2 - a^
5*b^3)*tan(x)^4 + 2*(a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*tan(x)^2), -1
/3*(3*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a
^2*b^3)*tan(x)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*tan(x)^2)*sqrt
(-a)*arctan(sqrt(-a)*sqrt((a*tan(x)^2 + b)/tan(x)^2)/a) + 3*(a^5*tan(x)^4 +
 2*a^4*b*tan(x)^2 + a^3*b^2)*sqrt(-a + b)*arctan(-sqrt(-a + b)*sqrt((a*tan(
x)^2 + b)/tan(x)^2)/(a - b)) - ((7*a^4*b - 11*a^3*b^2 + 4*a^2*b^3)*tan(x)^4
 + 3*(2*a^3*b^2 - 3*a^2*b^3 + a*b^4)*tan(x)^2)*sqrt((a*tan(x)^2 + b)/tan(x)
^2))/(a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5 + (a^8 - 3*a^7*b + 3*a^6*b^
2 - a^5*b^3)*tan(x)^4 + 2*(a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*tan(x)^
2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{(a+b\cot^2(x))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cot(x)**2)**(5/2),x)
```

```
[Out] Integral(tan(x)/(a + b*cot(x)**2)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 527 vs. 2(100) = 200.
time = 0.50, size = 527, normalized size = 4.47

$$-\frac{\left(2\sqrt{a-b}\,a^3\arctan\left(-\frac{a-b}{\sqrt{-a^2+ab}}\right)-6\sqrt{a-b}\,a^2b\arctan\left(-\frac{a-b}{\sqrt{-a^2+ab}}\right)+6\sqrt{a-b}\,ab^2\arctan\left(-\frac{a-b}{\sqrt{-a^2+ab}}\right)-2\sqrt{a-b}\,b^3\arctan\left(-\frac{a-b}{\sqrt{-a^2+ab}}\right)+\sqrt{-a^2+ab}\sqrt{a-b}\,a^2\log(b)\right)\mathrm{sgn}(\sin(x))}{2\left(\sqrt{-a^2+ab}\,a^5-3\sqrt{-a^2+ab}\,a^4b+3\sqrt{-a^2+ab}\,a^3b^2-\sqrt{-a^2+ab}\,a^2b^3\right)}+\frac{\frac{3\sqrt{a-b}\log\left(\left(\sqrt{a-b}\sin(x)-\sqrt{a\sin(x)^2-b\sin(x)^2+b}\right)^2\right)}{a^3-3a^2b+3ab^2-b^3}+\frac{2\left(\frac{(7a^5b^2-17a^4b^3+13a^3b^4-3a^2b^5)\sin(x)^2}{a^7b-3a^6b^2+3a^5b^3-a^4b^4}+\frac{3(2a^4b^3-3a^3b^4+a^2b^5)}{a^7b-3a^6b^2+3a^5b^3-a^4b^4}\right)\sin(x)}{(a\sin(x)^2-b\sin(x)^2+b)^{\frac{3}{2}}}+\frac{6\sqrt{a-b}\arctan\left(\frac{\left(\sqrt{a-b}\sin(x)-\sqrt{a\sin(x)^2-b\sin(x)^2+b}\right)^2-2a+b}{2\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,a^2}}{6\,\mathrm{sgn}(\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cot(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/2*(2*sqrt(a - b)*a^3*arctan(-(a - b)/sqrt(-a^2 + a*b)) - 6*sqrt(a - b)*a
^2*b*arctan(-(a - b)/sqrt(-a^2 + a*b)) + 6*sqrt(a - b)*a*b^2*arctan(-(a - b
)/sqrt(-a^2 + a*b)) - 2*sqrt(a - b)*b^3*arctan(-(a - b)/sqrt(-a^2 + a*b)) +
 sqrt(-a^2 + a*b)*sqrt(a - b)*a^2*log(b))*sgn(sin(x))/(sqrt(-a^2 + a*b)*a^5
 - 3*sqrt(-a^2 + a*b)*a^4*b + 3*sqrt(-a^2 + a*b)*a^3*b^2 - sqrt(-a^2 + a*b)
```

```
*a^2*b^3) + 1/6*(3*sqrt(a - b)*log((sqrt(a - b)*sin(x) - sqrt(a*sin(x)^2 -
b*sin(x)^2 + b))^2)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 2*((7*a^5*b^2 - 17*a^
4*b^3 + 13*a^3*b^4 - 3*a^2*b^5)*sin(x)^2/(a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a
^4*b^4) + 3*(2*a^4*b^3 - 3*a^3*b^4 + a^2*b^5)/(a^7*b - 3*a^6*b^2 + 3*a^5*b^
3 - a^4*b^4))*sin(x)/(a*sin(x)^2 - b*sin(x)^2 + b)^(3/2) + 6*sqrt(a - b)*ar
ctan(1/2*((sqrt(a - b)*sin(x) - sqrt(a*sin(x)^2 - b*sin(x)^2 + b))^2 - 2*a
+ b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a*b)*a^2))/sgn(sin(x))
```

Mupad [B]
time = 1.05, size = 2500, normalized size = 21.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a + b*cot(x)^2)^(5/2),x)
```

```
[Out] atanh((2*a^5*b^13*(a + b/tan(x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13 - 22*a^4
*b^12 + 110*a^5*b^11 - 330*a^6*b^10 + 660*a^7*b^9 - 922*a^8*b^8 + 912*a^9*b
^7 - 630*a^10*b^6 + 290*a^11*b^5 - 80*a^12*b^4 + 10*a^13*b^3)) - (22*a^6*b^
12*(a + b/tan(x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13 - 22*a^4*b^12 + 110*a^5
*b^11 - 330*a^6*b^10 + 660*a^7*b^9 - 922*a^8*b^8 + 912*a^9*b^7 - 630*a^10*b
^6 + 290*a^11*b^5 - 80*a^12*b^4 + 10*a^13*b^3)) + (110*a^7*b^11*(a + b/tan(
x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13 - 22*a^4*b^12 + 110*a^5*b^11 - 330*a^
6*b^10 + 660*a^7*b^9 - 922*a^8*b^8 + 912*a^9*b^7 - 630*a^10*b^6 + 290*a^11*
b^5 - 80*a^12*b^4 + 10*a^13*b^3)) - (330*a^8*b^10*(a + b/tan(x)^2)^(1/2))/(
(a^5)^(1/2)*(2*a^3*b^13 - 22*a^4*b^12 + 110*a^5*b^11 - 330*a^6*b^10 + 660*a
^7*b^9 - 922*a^8*b^8 + 912*a^9*b^7 - 630*a^10*b^6 + 290*a^11*b^5 - 80*a^12*
b^4 + 10*a^13*b^3)) + (660*a^9*b^9*(a + b/tan(x)^2)^(1/2))/((a^5)^(1/2)*(2*
a^3*b^13 - 22*a^4*b^12 + 110*a^5*b^11 - 330*a^6*b^10 + 660*a^7*b^9 - 922*a^
8*b^8 + 912*a^9*b^7 - 630*a^10*b^6 + 290*a^11*b^5 - 80*a^12*b^4 + 10*a^13*b
^3)) - (922*a^10*b^8*(a + b/tan(x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13 - 22*
a^4*b^12 + 110*a^5*b^11 - 330*a^6*b^10 + 660*a^7*b^9 - 922*a^8*b^8 + 912*a^
9*b^7 - 630*a^10*b^6 + 290*a^11*b^5 - 80*a^12*b^4 + 10*a^13*b^3)) + (912*a^
11*b^7*(a + b/tan(x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13 - 22*a^4*b^12 + 110
*a^5*b^11 - 330*a^6*b^10 + 660*a^7*b^9 - 922*a^8*b^8 + 912*a^9*b^7 - 630*a^
10*b^6 + 290*a^11*b^5 - 80*a^12*b^4 + 10*a^13*b^3)) - (630*a^12*b^6*(a + b/
tan(x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13 - 22*a^4*b^12 + 110*a^5*b^11 - 33
0*a^6*b^10 + 660*a^7*b^9 - 922*a^8*b^8 + 912*a^9*b^7 - 630*a^10*b^6 + 290*a
^11*b^5 - 80*a^12*b^4 + 10*a^13*b^3)) + (290*a^13*b^5*(a + b/tan(x)^2)^(1/2
))/((a^5)^(1/2)*(2*a^3*b^13 - 22*a^4*b^12 + 110*a^5*b^11 - 330*a^6*b^10 + 6
60*a^7*b^9 - 922*a^8*b^8 + 912*a^9*b^7 - 630*a^10*b^6 + 290*a^11*b^5 - 80*a
^12*b^4 + 10*a^13*b^3)) - (80*a^14*b^4*(a + b/tan(x)^2)^(1/2))/((a^5)^(1/2)
*(2*a^3*b^13 - 22*a^4*b^12 + 110*a^5*b^11 - 330*a^6*b^10 + 660*a^7*b^9 - 92
2*a^8*b^8 + 912*a^9*b^7 - 630*a^10*b^6 + 290*a^11*b^5 - 80*a^12*b^4 + 10*a^
13*b^3)) + (10*a^15*b^3*(a + b/tan(x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13 -
```

22*a^4*b^12 + 110*a^5*b^11 - 330*a^6*b^10 + 660*a^7*b^9 - 922*a^8*b^8 + 912
*a^9*b^7 - 630*a^10*b^6 + 290*a^11*b^5 - 80*a^12*b^4 + 10*a^13*b^3)))/(a^5)
^(1/2) - (b/(3*(a*b - a^2)) - (b*(a + b/tan(x)^2)*(2*a - b))/(a*b - a^2)^2)
/(a + b/tan(x)^2)^(3/2) + (atan((((((a + b/tan(x)^2)^(1/2)*(2*a^6*b^12 - 20
*a^7*b^11 + 90*a^8*b^10 - 240*a^9*b^9 + 422*a^10*b^8 - 516*a^11*b^7 + 450*a
^12*b^6 - 280*a^13*b^5 + 120*a^14*b^4 - 32*a^15*b^3 + 4*a^16*b^2))/2 - (((a
 - b)^5)^(1/2)*(2*a^8*b^13 - 22*a^9*b^12 + 110*a^10*b^11 - 328*a^11*b^10 +
644*a^12*b^9 - 868*a^13*b^8 + 812*a^14*b^7 - 520*a^15*b^6 + 218*a^16*b^5 -
54*a^17*b^4 + 6*a^18*b^3 - ((a + b/tan(x)^2)^(1/2)*((a - b)^5)^(1/2)*(8*a^1
0*b^13 - 96*a^11*b^12 + 520*a^12*b^11 - 1680*a^13*b^10 + 3600*a^14*b^9 - 53
76*a^15*b^8 + 5712*a^16*b^7 - 4320*a^17*b^6 + 2280*a^18*b^5 - 800*a^19*b^4
+ 168*a^20*b^3 - 16*a^21*b^2))/(4*(a - b)^5)))/(2*(a - b)^5))*((a - b)^5)^(
1/2)*1i)/(a - b)^5 + ((((a + b/tan(x)^2)^(1/2)*(2*a^6*b^12 - 20*a^7*b^11 +
90*a^8*b^10 - 240*a^9*b^9 + 422*a^10*b^8 - 516*a^11*b^7 + 450*a^12*b^6 - 28
0*a^13*b^5 + 120*a^14*b^4 - 32*a^15*b^3 + 4*a^16*b^2))/2 + (((a - b)^5)^(1/
2)*(2*a^8*b^13 - 22*a^9*b^12 + 110*a^10*b^11 - 328*a^11*b^10 + 644*a^12*b^9
 - 868*a^13*b^8 + 812*a^14*b^7 - 520*a^15*b^6 + 218*a^16*b^5 - 54*a^17*b^4
+ 6*a^18*b^3 + ((a + b/tan(x)^2)^(1/2)*((a - b)^5)^(1/2)*(8*a^10*b^13 - 96*
a^11*b^12 + 520*a^12*b^11 - 1680*a^13*b^10 + 3600*a^14*b^9 - 5376*a^15*b^8
+ 5712*a^16*b^7 - 4320*a^17*b^6 + 2280*a^18*b^5 - 800*a^19*b^4 + 168*a^20*b
^3 - 16*a^21*b^2))/(4*(a - b)^5)))/(2*(a - b)^5))*((a - b)^5)^(1/2)*1i)/(a
- b)^5)/(2*a^6*b^10 - 16*a^7*b^9 + 54*a^8*b^8 - 100*a^9*b^7 + 110*a^10*b^6
- 72*a^11*b^5 + 26*a^12*b^4 - 4*a^13*b^3 + ((((a + b/tan(x)^2)^(1/2)*(2*a^6
*b^12 - 20*a^7*b^11 + 90*a^8*b^10 - 240*a^9*b^9 + 422*a^10*b^8 - 516*a^11*b
^7 + 450*a^12*b^6 - 280*a^13*b^5 + 120*a^14*b^4 - 32*a^15*b^3 + 4*a^16*b^2)
)/2 - (((a - b)^5)^(1/2)*(2*a^8*b^13 - 22*a^9*b^12 + 110*a^10*b^11 - 328*a^
11*b^10 + 644*a^12*b^9 - 868*a^13*b^8 + 812*a^14*b^7 - 520*a^15*b^6 + 218*a
^16*b^5 - 54*a^17*b^4 + 6*a^18*b^3 - ((a + b/tan(x)^2)^(1/2)*((a - b)^5)^(1
/2)*(8*a^10*b^13 - 96*a^11*b^12 + 520*a^12*b^11 - 1680*a^13*b^10 + 3600*a^1
4*b^9 - 5376*a^15*b^8 + 5712*a^16*b^7 - 4320*a^17*b^6 + 2280*a^18*b^5 - 800
*a^19*b^4 + 168*a^20*b^3 - 16*a^21*b^2))/(4*(a - b)^5)))/(2*(a - b)^5))*((a
 - b)^5)^(1/2))/(a - b)^5 - ((((a + b/tan(x)^2)^(1/2)*(2*a^6*b^12 - 20*a^7*
b^11 + 90*a^8*b^10 - 240*a^9*b^9 + 422*a^10*b^8 - 516*a^11*b^7 + 450*a^12*b
^6 - 280*a^13*b^5 + 120*a^14*b^4 - 32*a^15*b^3 + 4*a^16*b^2))/2 + (((a - b)
^5)^(1/2)*(2*a^8*b^13 - 22*a^9*b^12 + 110*a^10*b^11 - 328*a^11*b^10 + 644*a
^12*b^9 - 868*a^13*b^8 + 812*a^14*b^7 - 520*a^15*b^6 + 218*a^16*b^5 - 54*a^
17*b^4 + 6*a^18*b^3 + ((a + b/tan(x)^2)^(1/2)*((a - b)^5)^(1/2)*(8*a^10*b^1
3 - 96*a^11*b^12 + 520*a^12*b^11 - 1680*a^13*b^...

3.58 $\int \frac{\tan^2(x)}{\left(a+b\cot^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=141

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{(a-b)^{5/2}}+\frac{b\tan(x)}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}}+\frac{(7a-4b)b\tan(x)}{3a^2(a-b)^2\sqrt{a+b\cot^2(x)}}+\frac{(a-4b)(3a-2b}{3a}$$

```
[Out] arctan(cot(x)*(a-b)^(1/2)/(a+b*cot(x)^2)^(1/2))/(a-b)^(5/2)+1/3*b*tan(x)/a/
(a-b)/(a+b*cot(x)^2)^(3/2)+1/3*(7*a-4*b)*b*tan(x)/a^2/(a-b)^2/(a+b*cot(x)^2
)^(1/2)+1/3*(a-4*b)*(3*a-2*b)*(a+b*cot(x)^2)^(1/2)*tan(x)/a^3/(a-b)^2
```

Rubi [A]
time = 0.16, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3751, 483, 593, 597, 12, 385, 209}

$$\frac{(a-4b)(3a-2b)\tan(x)\sqrt{a+b\cot^2(x)}}{3a^3(a-b)^2}+\frac{b(7a-4b)\tan(x)}{3a^2(a-b)^2\sqrt{a+b\cot^2(x)}}+\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\,\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{(a-b)^{5/2}}+\frac{b\tan(x)}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^2/(a + b*Cot[x]^2)^(5/2),x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Cot[x])/Sqrt[a + b*Cot[x]^2]]/(a - b)^(5/2) + (b*Tan[x]
)/(3*a*(a - b)*(a + b*Cot[x]^2)^(3/2)) + ((7*a - 4*b)*b*Tan[x])/(3*a^2*(a -
 b)^2*Sqrt[a + b*Cot[x]^2]) + ((a - 4*b)*(3*a - 2*b)*Sqrt[a + b*Cot[x]^2]*T
an[x])/(3*a^3*(a - b)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^2(x)}{\left(a+b\cot^2(x)\right)^{5/2}}\,dx = -\text{Subst}\left(\int \frac{1}{x^2\left(1+x^2\right)\left(a+bx^2\right)^{5/2}}\,dx, x, \cot(x)\right)$$

$$= \frac{b\tan(x)}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{3a-4b-4bx^2}{x^2(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \cot(x)\right)}{3a(a-b)}$$

$$= \frac{b\tan(x)}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}} + \frac{(7a-4b)b\tan(x)}{3a^2(a-b)^2\sqrt{a+b\cot^2(x)}} - \frac{\text{Subst}\left(\int \frac{(a-4b)(3a-}{x^2(1+x^2)}\right.}{3}$$

$$= \frac{b\tan(x)}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}} + \frac{(7a-4b)b\tan(x)}{3a^2(a-b)^2\sqrt{a+b\cot^2(x)}} + \frac{(a-4b)(3a-2b)\sqrt{}}{3a^3(}$$

$$= \frac{b\tan(x)}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}} + \frac{(7a-4b)b\tan(x)}{3a^2(a-b)^2\sqrt{a+b\cot^2(x)}} + \frac{(a-4b)(3a-2b)\sqrt{}}{3a^3(}$$

$$= \frac{b\tan(x)}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}} + \frac{(7a-4b)b\tan(x)}{3a^2(a-b)^2\sqrt{a+b\cot^2(x)}} + \frac{(a-4b)(3a-2b)\sqrt{}}{3a^3(}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a+b\cot^2(x)}}\right)}{(a-b)^{5/2}} + \frac{b\tan(x)}{3a(a-b)\left(a+b\cot^2(x)\right)^{3/2}} + \frac{(7a-4b)b\tan}{3a^2(a-b)^2\sqrt{a+b}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 8.08, size = 1450, normalized size = 10.28

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[x]^2/(a + b*Cot[x]^2)^(5/2),x]
```

```
[Out] (Sin[x]^2*((-16*b^3*(Cot[x] + Cot[x]^3)^2)/(a*(a - b)^2) + (40*b*Csc[x]^2)/
(a - b) + (160*b^2*Cot[x]^2*Csc[x]^2)/(3*a*(a - b)) + (64*b^3*Cot[x]^4*Csc[
x]^2)/(3*a^2*(a - b)) - (40*b^2*Csc[x]^4)/(a - b)^2 + (92*(a - b)*Cos[x]^2*
Hypergeometric2F1[2, 2, 9/2, ((a - b)*Cos[x]^2)/a])/(105*a) + (124*(a - b)*
```

```
b*Cos[x]^2*Cot[x]^2*Hypergeometric2F1[2, 2, 9/2, ((a - b)*Cos[x]^2)/a])/(35
*a^2) + (152*(a - b)*b^2*Cos[x]^2*Cot[x]^4*Hypergeometric2F1[2, 2, 9/2, ((a
 - b)*Cos[x]^2)/a])/(35*a^3) + (176*(a - b)*b^3*Cos[x]^2*Cot[x]^6*Hypergeom
etric2F1[2, 2, 9/2, ((a - b)*Cos[x]^2)/a])/(105*a^4) + (24*(a - b)*Cos[x]^2
*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Cos[x]^2)/a])/(35*a) + (16
*(a - b)*b*Cos[x]^2*Cot[x]^2*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b
)*Cos[x]^2)/a])/(7*a^2) + (88*(a - b)*b^2*Cos[x]^2*Cot[x]^4*HypergeometricP
FQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Cos[x]^2)/a])/(35*a^3) + (32*(a - b)*b^3*C
os[x]^2*Cot[x]^6*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Cos[x]^2)/
a])/(35*a^4) + (16*(a - b)*Cos[x]^2*HypergeometricPFQ[{2, 2, 2, 2}, {1, 1,
9/2}, ((a - b)*Cos[x]^2)/a])/(105*a) + (16*(a - b)*b*Cos[x]^2*Cot[x]^2*Hype
rgeometricPFQ[{2, 2, 2, 2}, {1, 1, 9/2}, ((a - b)*Cos[x]^2)/a])/(35*a^2) +
(16*(a - b)*b^2*Cos[x]^2*Cot[x]^4*HypergeometricPFQ[{2, 2, 2, 2}, {1, 1, 9/
2}, ((a - b)*Cos[x]^2)/a])/(35*a^3) + (16*(a - b)*b^3*Cos[x]^2*Cot[x]^6*Hyp
ergeometricPFQ[{2, 2, 2, 2}, {1, 1, 9/2}, ((a - b)*Cos[x]^2)/a])/(105*a^4)
+ (20*a*Sec[x]^2)/(3*(a - b)) - (30*a*b*Csc[x]^2*Sec[x]^2)/(a - b)^2 - (5*a
^2*Sec[x]^4)/(a - b)^2 + (5*ArcSin[Sqrt[((a - b)*Cos[x]^2)/a]])/((((a - b)*
Cos[x]^2)/a)^(5/2)*Sqrt[((a + b*Cot[x]^2)*Sin[x]^2)/a]) + (30*b*ArcSin[Sqrt
[((a - b)*Cos[x]^2)/a]]*Cot[x]^2)/(a*(((a - b)*Cos[x]^2)/a)^(5/2)*Sqrt[((a
+ b*Cot[x]^2)*Sin[x]^2)/a]) + (40*b^2*ArcSin[Sqrt[((a - b)*Cos[x]^2)/a]]*Co
t[x]^4)/(a^2*(((a - b)*Cos[x]^2)/a)^(5/2)*Sqrt[((a + b*Cot[x]^2)*Sin[x]^2)/
a]) + (16*b^3*ArcSin[Sqrt[((a - b)*Cos[x]^2)/a]]*Cot[x]^6)/(a^3*(((a - b)*C
os[x]^2)/a)^(5/2)*Sqrt[((a + b*Cot[x]^2)*Sin[x]^2)/a]) + (5*ArcSin[Sqrt[((a
 - b)*Cos[x]^2)/a]])/Sqrt[((a - b)*Cos[x]^2*(a + b*Cot[x]^2)*Sin[x]^2)/a^2]
 + (30*b*ArcSin[Sqrt[((a - b)*Cos[x]^2)/a]]*Cot[x]^2)/(a*Sqrt[((a - b)*Cos[
x]^2*(a + b*Cot[x]^2)*Sin[x]^2)/a^2]) + (40*b^2*ArcSin[Sqrt[((a - b)*Cos[x]
^2)/a]]*Cot[x]^4)/(a^2*Sqrt[((a - b)*Cos[x]^2*(a + b*Cot[x]^2)*Sin[x]^2)/a^
2]) + (16*b^3*ArcSin[Sqrt[((a - b)*Cos[x]^2)/a]]*Cot[x]^6)/(a^3*Sqrt[((a -
b)*Cos[x]^2*(a + b*Cot[x]^2)*Sin[x]^2)/a^2]) - (60*b*ArcSin[Sqrt[((a - b)*C
os[x]^2)/a]]*Csc[x]^2)/((a - b)*Sqrt[((a - b)*Cos[x]^2*(a + b*Cot[x]^2)*Sin
[x]^2)/a^2]) - (80*b^2*ArcSin[Sqrt[((a - b)*Cos[x]^2)/a]]*Cot[x]^2*Csc[x]^2
)/(a*(a - b)*Sqrt[((a - b)*Cos[x]^2*(a + b*Cot[x]^2)*Sin[x]^2)/a^2]) - (32*
b^3*ArcSin[Sqrt[((a - b)*Cos[x]^2)/a]]*Cot[x]^4*Csc[x]^2)/(a^2*(a - b)*Sqrt
[((a - b)*Cos[x]^2*(a + b*Cot[x]^2)*Sin[x]^2)/a^2]) - (10*a*ArcSin[Sqrt[((a
 - b)*Cos[x]^2)/a]]*Sec[x]^2)/((a - b)*Sqrt[((a - b)*Cos[x]^2*(a + b*Cot[x]
^2)*Sin[x]^2)/a^2]))*Tan[x])/(a^2*Sqrt[a + b*Cot[x]^2]*(1 + (b*Cot[x]^2)/a)
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1039 vs. $2(123) = 246$.

time = 2.14, size = 1040, normalized size = 7.38

method	result	size
default	Expression too large to display	1040

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(a+b*cot(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-1+cos(x))^2*(cos(x)+1)^2*(cos(x)^2*a-b*cos(x)^2-a)*(-3*cos(x)^4*ln(4
*cos(x)*(-a+b)^(1/2)*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)-4*a*co
s(x)+4*b*cos(x)+4*(-a+b)^(1/2)*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1
/2))*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*a^4+3*cos(x)^4*ln(4*co
s(x)*(-a+b)^(1/2)*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)-4*a*cos(x
)+4*b*cos(x)+4*(-a+b)^(1/2)*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)
)*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*a^3*b+3*cos(x)^4*(-a+b)^(
1/2)*a^4-12*cos(x)^4*(-a+b)^(1/2)*a^3*b+27*cos(x)^4*(-a+b)^(1/2)*a^2*b^2-26
*cos(x)^4*(-a+b)^(1/2)*a*b^3+8*cos(x)^4*(-a+b)^(1/2)*b^4-3*cos(x)^3*ln(4*co
s(x)*(-a+b)^(1/2)*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)-4*a*cos(x
)+4*b*cos(x)+4*(-a+b)^(1/2)*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)
)*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*a^4+3*cos(x)^3*ln(4*cos(x
)*(-a+b)^(1/2)*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)-4*a*cos(x)+4
*b*cos(x)+4*(-a+b)^(1/2)*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2))*(
-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*a^3*b+3*cos(x)^2*ln(4*cos(x)
*(-a+b)^(1/2)*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)-4*a*cos(x)+4*
b*cos(x)+4*(-a+b)^(1/2)*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2))*(-
(cos(x)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2)*a^4-6*cos(x)^2*(-a+b)^(1/2)*a
^4+18*cos(x)^2*(-a+b)^(1/2)*a^3*b-27*cos(x)^2*(-a+b)^(1/2)*a^2*b^2+12*cos(x
)^2*(-a+b)^(1/2)*a*b^3+3*cos(x)*ln(4*cos(x)*(-a+b)^(1/2)*(-(cos(x)^2*a-b*co
s(x)^2-a)/(cos(x)+1)^2)^(1/2)-4*a*cos(x)+4*b*cos(x)+4*(-a+b)^(1/2)*(-(cos(x
)^2*a-b*cos(x)^2-a)/(cos(x)+1)^2)^(1/2))*(-(cos(x)^2*a-b*cos(x)^2-a)/(cos(x
)+1)^2)^(1/2)*a^4+3*(-a+b)^(1/2)*a^4-6*(-a+b)^(1/2)*a^3*b+3*(-a+b)^(1/2)*a^
2*b^2)*b^2/cos(x)/((cos(x)^2*a-b*cos(x)^2-a)/(cos(x)^2-1))^(5/2)/sin(x)^9/(
(a*(a-b))^(1/2)+a-b)^2/a^3/((a*(a-b))^(1/2)-a+b)^2/(-a+b)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+b*cot(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)^2/(b*cot(x)^2 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 297 vs. 2(123) = 246.
time = 2.69, size = 647, normalized size = 4.59

$$\left[-\frac{3\left(a^{5}\tan\left(x\right)^{4}+2\,a^{4}b\tan\left(x\right)^{2}+a^{3}b^{2}\right)\sqrt{-a+b}\log\left(-\frac{a^{2}\tan\left(x\right)^{4}-2\left(3\,a^{2}-4\,ab\right)\tan\left(x\right)^{2}+a^{2}-8\,ab+8\,b^{2}+4\left(a\tan\left(x\right)^{3}-\left(a-2\,b\right)\tan\left(x\right)\right)\sqrt{-a+b}\sqrt{\frac{a\tan\left(x\right)^{2}+b}{\tan\left(x\right)^{2}}}}{\tan\left(x\right)^{4}+2\tan\left(x\right)^{2}+1}\right)-4\left(3\left(a^{5}-3\,a^{4}b+3\,a^{3}b^{2}-a^{2}b^{3}\right)\tan\left(x\right)^{5}+3\left(2\,a^{4}b-9\,a^{3}b^{2}+11\,a^{2}b^{3}-4\,ab^{4}\right)\tan\left(x\right)^{3}+\left(3\,a^{3}b^{2}-17\,a^{2}b^{3}+22\,ab^{4}-8\,b^{5}\right)\tan\left(x\right)\right)\sqrt{\frac{a\tan\left(x\right)^{2}+b}{\tan\left(x\right)^{2}}}}{12\left(a^{6}b^{2}-3\,a^{5}b^{3}+3\,a^{4}b^{4}-a^{3}b^{5}+\left(a^{8}-3\,a^{7}b+3\,a^{6}b^{2}-a^{5}b^{3}\right)\tan\left(x\right)^{4}+2\left(a^{7}b-3\,a^{6}b^{2}+3\,a^{5}b^{3}-a^{4}b^{4}\right)\tan\left(x\right)^{2}\right)}, \frac{3\left(a^{5}\tan\left(x\right)^{4}+2\,a^{4}b\tan\left(x\right)^{2}+a^{3}b^{2}\right)\sqrt{a-b}\arctan\left(\frac{2\sqrt{a-b}\sqrt{\frac{a\tan\left(x\right)^{2}+b}{\tan\left(x\right)^{2}}}\tan\left(x\right)}{a\tan\left(x\right)^{2}-a+2\,b}\right)+2\left(3\left(a^{5}-3\,a^{4}b+3\,a^{3}b^{2}-a^{2}b^{3}\right)\tan\left(x\right)^{5}+3\left(2\,a^{4}b-9\,a^{3}b^{2}+11\,a^{2}b^{3}-4\,ab^{4}\right)\tan\left(x\right)^{3}+\left(3\,a^{3}b^{2}-17\,a^{2}b^{3}+22\,ab^{4}-8\,b^{5}\right)\tan\left(x\right)\right)\sqrt{\frac{a\tan\left(x\right)^{2}+b}{\tan\left(x\right)^{2}}}}{6\left(a^{6}b^{2}-3\,a^{5}b^{3}+3\,a^{4}b^{4}-a^{3}b^{5}+\left(a^{8}-3\,a^{7}b+3\,a^{6}b^{2}-a^{5}b^{3}\right)\tan\left(x\right)^{4}+2\left(a^{7}b-3\,a^{6}b^{2}+3\,a^{5}b^{3}-a^{4}b^{4}\right)\tan\left(x\right)^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x)^2/(a+b*cot(x)^2)^(5/2),x, algorithm="fricas")

[Out] [-1/12*(3*(a^5*tan(x)^4 + 2*a^4*b*tan(x)^2 + a^3*b^2)*sqrt(-a + b)*log(-(a^2*tan(x)^4 - 2*(3*a^2 - 4*a*b)*tan(x)^2 + a^2 - 8*a*b + 8*b^2 + 4*(a*tan(x)^3 - (a - 2*b)*tan(x))*sqrt(-a + b)*sqrt((a*tan(x)^2 + b)/tan(x)^2))/(tan(x)^4 + 2*tan(x)^2 + 1)) - 4*(3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*tan(x)^5 + 3*(2*a^4*b - 9*a^3*b^2 + 11*a^2*b^3 - 4*a*b^4)*tan(x)^3 + (3*a^3*b^2 - 17*a^2*b^3 + 22*a*b^4 - 8*b^5)*tan(x))*sqrt((a*tan(x)^2 + b)/tan(x)^2))/(a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5 + (a^8 - 3*a^7*b + 3*a^6*b^2 - a^5*b^3)*tan(x)^4 + 2*(a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*tan(x)^2), 1/6*(3*(a^5*tan(x)^4 + 2*a^4*b*tan(x)^2 + a^3*b^2)*sqrt(a - b)*arctan(2*sqrt(a - b)*sqrt((a*tan(x)^2 + b)/tan(x)^2)*tan(x)/(a*tan(x)^2 - a + 2*b)) + 2*(3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*tan(x)^5 + 3*(2*a^4*b - 9*a^3*b^2 + 11*a^2*b^3 - 4*a*b^4)*tan(x)^3 + (3*a^3*b^2 - 17*a^2*b^3 + 22*a*b^4 - 8*b^5)*tan(x))*sqrt((a*tan(x)^2 + b)/tan(x)^2))/(a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5 + (a^8 - 3*a^7*b + 3*a^6*b^2 - a^5*b^3)*tan(x)^4 + 2*(a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*tan(x)^2)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(x)}{(a + b\cot^2(x))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x)**2/(a+b*cot(x)**2)**(5/2),x)

[Out] Integral(tan(x)**2/(a + b*cot(x)**2)**(5/2), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 537 vs. 2(123) = 246.
time = 0.53, size = 537, normalized size = 3.81

$$\frac{\left(3\,a^4\sqrt{b}\log\left(-a-2\sqrt{-a+b}\sqrt{b}+2\,b\right)+3\,a^3\sqrt{-a+b}\,b\log\left(-a-2\sqrt{-a+b}\sqrt{b}+2\,b\right)-3\,a^3b^{\frac{3}{2}}\log\left(-a-2\sqrt{-a+b}\sqrt{b}+2\,b\right)+6\,a^4\sqrt{b}-18\,a^3b^{\frac{3}{2}}+16\,a^2\sqrt{-a+b}\,b^2+2\,a^2b^{\frac{5}{2}}-26\,a\sqrt{-a+b}\,b^3+20\,ab^{\frac{7}{2}}+10\sqrt{-a+b}\,b^4-10\,b^{\frac{9}{2}}\right)\mathrm{sgn}(\sin(x))}{6\left(a^6\sqrt{-a+b}\sqrt{b}-a^6b-3\,a^5\sqrt{-a+b}\,b^{\frac{3}{2}}+3\,a^5b^2+3\,a^4\sqrt{-a+b}\,b^{\frac{5}{2}}-3\,a^4b^3-a^3\sqrt{-a+b}\,b^{\frac{7}{2}}+a^3b^4\right)}-\frac{\frac{2\left(\frac{\left(9\,a^5b^2-23\,a^4b^3+19\,a^3b^4-5\,a^2b^5\right)\cos(x)^2}{a^8-3\,a^7b+3\,a^6b^2-a^5b^3}-\frac{3\left(3\,a^5b^2-5\,a^4b^3+2\,a^3b^4\right)}{a^8-3\,a^7b+3\,a^6b^2-a^5b^3}\right)\cos(x)}{\left(a\cos(x)^2-b\cos(x)^2-a\right)\sqrt{-a\cos(x)^2+b\cos(x)^2+a}}+\frac{3\log\left(\left(\sqrt{-a+b}\cos(x)-\sqrt{-a\cos(x)^2+b\cos(x)^2+a}\right)^2\right)}{\left(a^2-2\,ab+b^2\right)\sqrt{-a+b}}+\frac{12\sqrt{-a+b}}{\left(\left(\sqrt{-a+b}\cos(x)-\sqrt{-a\cos(x)^2+b\cos(x)^2+a}\right)^2-a\right)a^2}}{6\,\mathrm{sgn}(\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x)^2/(a+b*cot(x)^2)^(5/2),x, algorithm="giac")

[Out] 1/6*(3*a^4*sqrt(b)*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b) + 3*a^3*sqrt(-a + b)*b*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b) - 3*a^3*b^(3/2)*log(-a - 2*sqrt(-a + b)*sqrt(b) + 2*b) + 6*a^4*sqrt(b) - 18*a^3*b^(3/2) + 16*a^2*sqrt(-a + b)*b^2 + 2*a^2*b^(5/2) - 26*a*sqrt(-a + b)*b^3 + 20*a*b^(7/2) + 10*sqrt(-a + b)*b^4 - 10*b^(9/2))*sgn(sin(x))/(a^6*sqrt(-a + b)*sqrt(b) - a^6*b - 3*a^5*sqrt(-a + b)*b^(3/2) + 3*a^5*b^2 + 3*a^4*sqrt(-a + b)*b^(5/2) - 3*a^4*b^3 - a^3*sqrt(-a + b)*b^(7/2) + a^3*b^4) - 1/6*(2*((9*a^5*b^2 - 23*a^4*b^3 + 19*a^3*b^4 - 5*a^2*b^5)*cos(x)^2/(a^8 - 3*a^7*b + 3*a^6*b^2 - a^5*b^3) -

```
3*(3*a^5*b^2 - 5*a^4*b^3 + 2*a^3*b^4)/(a^8 - 3*a^7*b + 3*a^6*b^2 - a^5*b^3)
)*cos(x)/((a*cos(x)^2 - b*cos(x)^2 - a)*sqrt(-a*cos(x)^2 + b*cos(x)^2 + a))
 + 3*log((sqrt(-a + b)*cos(x) - sqrt(-a*cos(x)^2 + b*cos(x)^2 + a))^2)/((a^
2 - 2*a*b + b^2)*sqrt(-a + b)) + 12*sqrt(-a + b)/(((sqrt(-a + b)*cos(x) - s
qrt(-a*cos(x)^2 + b*cos(x)^2 + a))^2 - a)*a^2))/sgn(sin(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(x)^2}{\left(b\cot\left(x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(a + b*cot(x)^2)^(5/2),x)
```

```
[Out] int(tan(x)^2/(a + b*cot(x)^2)^(5/2), x)
```

3.59 $\int \frac{1}{1+\cot^3(x)}\,dx$

Optimal. Leaf size=37

$$\frac{x}{2}-\frac{1}{6}\log(1+\cot(x))+\frac{1}{3}\log\left(1-\cot(x)+\cot^2(x)\right)+\frac{1}{2}\log(\sin(x))$$

```
[Out] 1/2*x-1/6*ln(1+cot(x))+1/3*ln(1-cot(x)+cot(x)^2)+1/2*ln(sin(x))
```

Rubi [A]
time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3742, 2099, 649, 209, 266, 642}

$$\frac{x}{2}+\frac{1}{2}\log(\sin(x))+\frac{1}{3}\log\left(\cot^2(x)-\cot(x)+1\right)-\frac{1}{6}\log(\cot(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cot[x]^3)^(-1),x]
```

```
[Out] x/2 - Log[1 + Cot[x]]/6 + Log[1 - Cot[x] + Cot[x]^2]/3 + Log[Sin[x]]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1+\cot^3(x)}\,dx &= -\text{Subst}\bigg(\int \frac{1}{(1+x^2)(1+x^3)}\,dx, x, \cot(x)\bigg)\\
&= -\text{Subst}\bigg(\int \bigg(\frac{1}{6(1+x)}+\frac{1+x}{2(1+x^2)}+\frac{1-2x}{3(1-x+x^2)}\bigg)\,dx, x, \cot(x)\bigg)\\
&= -\frac{1}{6}\log(1+\cot(x))-\frac{1}{3}\text{Subst}\bigg(\int \frac{1-2x}{1-x+x^2}\,dx, x, \cot(x)\bigg)-\frac{1}{2}\text{Subst}\bigg(\int \frac{1+x}{1+x^2}\,dx, x
\\
&= -\frac{1}{6}\log(1+\cot(x))+\frac{1}{3}\log\left(1-\cot(x)+\cot^2(x)\right)-\frac{1}{2}\text{Subst}\bigg(\int \frac{1}{1+x^2}\,dx, x, \cot(x)\bigg)\\
&= \frac{x}{2}-\frac{1}{6}\log(1+\cot(x))+\frac{1}{3}\log\left(1-\cot(x)+\cot^2(x)\right)+\frac{1}{2}\log(\sin(x))
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 43, normalized size = 1.16

$$-\frac{1}{2}\text{ArcTan}(\cot(x))-\frac{1}{6}\log(1+\cot(x))-\frac{1}{4}\log\left(1+\cot^2(x)\right)+\frac{1}{3}\log\left(1-\cot(x)+\cot^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cot[x]^3)^(-1),x]
```

```
[Out] -1/2*ArcTan[Cot[x]] - Log[1 + Cot[x]]/6 - Log[1 + Cot[x]^2]/4 + Log[1 - Cot
[x] + Cot[x]^2]/3
```

Maple [A]
time = 0.14, size = 39, normalized size = 1.05

method	result	size

norman	$\frac{x}{2}-\frac{\ln(1+\tan(x))}{6}-\frac{\ln(1+\tan^2(x))}{4}+\frac{\ln(1-\tan(x)+\tan^2(x))}{3}$	34
risch	$\frac{x}{2}-\frac{ix}{2}-\frac{\ln(e^{2ix}+i)}{6}+\frac{\ln(e^{4ix}-4ie^{2ix}-1)}{3}$	38
derivativedivides	$-\frac{\ln(1+\cot(x))}{6}+\frac{\ln(1-\cot(x)+\cot^2(x))}{3}-\frac{\ln(1+\cot^2(x))}{4}-\frac{\pi}{4}+\frac{\operatorname{arccot}(\cot(x))}{2}$	39
default	$-\frac{\ln(1+\cot(x))}{6}+\frac{\ln(1-\cot(x)+\cot^2(x))}{3}-\frac{\ln(1+\cot^2(x))}{4}-\frac{\pi}{4}+\frac{\operatorname{arccot}(\cot(x))}{2}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cot(x)^3),x,method=_RETURNVERBOSE)
[Out] -1/6*ln(1+cot(x))+1/3*ln(1-cot(x)+cot(x)^2)-1/4*ln(1+cot(x)^2)-1/4*Pi+1/2*a
rccot(cot(x))
```

Maxima [A]
time = 0.52, size = 33, normalized size = 0.89

$$\frac{1}{2}\,x+\frac{1}{3}\,\log\left(\tan\left(x\right)^{2}-\tan\left(x\right)+1\right)-\frac{1}{4}\,\log\left(\tan\left(x\right)^{2}+1\right)-\frac{1}{6}\,\log\left(\tan\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cot(x)^3),x, algorithm="maxima")
[Out] 1/2*x + 1/3*log(tan(x)^2 - tan(x) + 1) - 1/4*log(tan(x)^2 + 1) - 1/6*log(ta
n(x) + 1)
```

Fricas [A]
time = 3.22, size = 24, normalized size = 0.65

$$\frac{1}{2}\,x-\frac{1}{12}\,\log\left(\sin\left(2\,x\right)+1\right)+\frac{1}{3}\,\log\left(-\frac{1}{2}\,\sin\left(2\,x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cot(x)^3),x, algorithm="fricas")
[Out] 1/2*x - 1/12*log(sin(2*x) + 1) + 1/3*log(-1/2*sin(2*x) + 1)
```

Sympy [A]
time = 0.12, size = 34, normalized size = 0.92

$$\frac{x}{2}-\frac{\log\left(\tan\left(x\right)+1\right)}{6}-\frac{\log\left(\tan^{2}\left(x\right)+1\right)}{4}+\frac{\log\left(\tan^{2}\left(x\right)-\tan\left(x\right)+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cot(x)**3),x)
```

```
[Out] x/2 - log(tan(x) + 1)/6 - log(tan(x)**2 + 1)/4 + log(tan(x)**2 - tan(x) + 1
)/3
```

Giac [A]
time = 0.45, size = 34, normalized size = 0.92

$$\frac{1}{2}\,x + \frac{1}{3}\,\log\left(\tan\left(x\right)^2 - \tan\left(x\right) + 1\right) - \frac{1}{4}\,\log\left(\tan\left(x\right)^2 + 1\right) - \frac{1}{6}\,\log\left(\left|\tan\left(x\right) + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cot(x)^3),x, algorithm="giac")
```

```
[Out] 1/2*x + 1/3*log(tan(x)^2 - tan(x) + 1) - 1/4*log(tan(x)^2 + 1) - 1/6*log(ab
s(tan(x) + 1))
```

Mupad [B]
time = 0.72, size = 37, normalized size = 1.00

$$x\left(\frac{1}{2} - \frac{1}{2}\mathrm{i}\right) - \frac{\ln\left(12\,\mathrm{e}^{x\,2\mathrm{i}} + 12\mathrm{i}\right)}{6} + \frac{\ln\left(\mathrm{e}^{x\,4\mathrm{i}} - 1 - \mathrm{e}^{x\,2\mathrm{i}}\,4\mathrm{i}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)^3 + 1),x)
```

```
[Out] x*(1/2 - 1i/2) - log(12*exp(x*2i) + 12i)/6 + log(exp(x*4i) - exp(x*2i)*4i -
 1)/3
```

3.60 $\int \cot(x)\sqrt{a+b\cot^4(x)}\,dx$

Optimal. Leaf size=90

$$\frac{1}{2}\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\cot^2(x)}{\sqrt{a+b\cot^4(x)}}\right)+\frac{1}{2}\sqrt{a+b}\,\tanh^{-1}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\,\sqrt{a+b\cot^4(x)}}\right)-\frac{1}{2}\sqrt{a+b\cot^4(x)}$$

```
[Out] 1/2*arctanh(cot(x)^2*b^(1/2)/(a+b*cot(x)^4)^(1/2))*b^(1/2)+1/2*arctanh((a-b
*cot(x)^2)/(a+b)^(1/2)/(a+b*cot(x)^4)^(1/2))*(a+b)^(1/2)-1/2*(a+b*cot(x)^4)
^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = $\{3751, 1262, 749, 858, 223, 212, 739\}$

$$-\frac{1}{2}\sqrt{a+b\cot^4(x)}+\frac{1}{2}\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\cot^2(x)}{\sqrt{a+b\cot^4(x)}}\right)+\frac{1}{2}\sqrt{a+b}\,\tanh^{-1}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\,\sqrt{a+b\cot^4(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*Sqrt[a + b*Cot[x]^4],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Cot[x]^2)/Sqrt[a + b*Cot[x]^4]])/2 + (Sqrt[a + b]
*ArcTanh[(a - b*Cot[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Cot[x]^4])])/2 - Sqrt[a +
 b*Cot[x]^4]/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
 2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \cot(x)\sqrt{a+b\cot^4(x)}\,dx &= -\mathrm{Subst}\left(\int \frac{x\sqrt{a+bx^4}}{1+x^2}\,dx, x, \cot(x)\right)\\
&= -\left(\frac{1}{2}\mathrm{Subst}\left(\int \frac{\sqrt{a+bx^2}}{1+x}\,dx, x, \cot^2(x)\right)\right)\\
&= -\frac{1}{2}\sqrt{a+b\cot^4(x)} - \frac{1}{2}\mathrm{Subst}\left(\int \frac{a-bx}{(1+x)\sqrt{a+bx^2}}\,dx, x, \cot^2(x)\right)\\
&= -\frac{1}{2}\sqrt{a+b\cot^4(x)} + \frac{1}{2}b\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \cot^2(x)\right) - \frac{1}{2}(a+b)\mathrm{Sub}\\
&= -\frac{1}{2}\sqrt{a+b\cot^4(x)} - \frac{1}{2}(-a-b)\mathrm{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{a-b\cot^2(x)}{\sqrt{a+b\cot^4(x)}}\right.\\
&= \frac{1}{2}\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\cot^2(x)}{\sqrt{a+b\cot^4(x)}}\right) + \frac{1}{2}\sqrt{a+b}\tanh^{-1}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4}}\right.
\end{aligned}
$$

Mathematica [A]
time = 0.17, size = 86, normalized size = 0.96

$$
\frac{1}{2}\left(\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\cot^2(x)}{\sqrt{a+b\cot^4(x)}}\right) + \sqrt{a+b}\tanh^{-1}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right) - \sqrt{a+b\cot^4(x)}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*Sqrt[a + b*Cot[x]^4],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Cot[x]^2)/Sqrt[a + b*Cot[x]^4]] + Sqrt[a + b]*Arc
Tanh[(a - b*Cot[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Cot[x]^4])] - Sqrt[a + b*Cot[
x]^4])/2
```

Maple [A]
time = 0.32, size = 139, normalized size = 1.54

method	result
derivativedivides	$-\frac{\sqrt{b\left(1+\cot^2(x)\right)^2-2b\left(1+\cot^2(x)\right)+a+b}}{2}+\frac{\sqrt{b}\,\ln\left(\frac{b\left(1+\cot^2(x)\right)-b}{\sqrt{b}}+\sqrt{b\left(1+\cot^2(x\right.}\right.}{2}$

default	$-\frac{\sqrt{b\left(1+\cot^{2}(x)\right)^{2}-2b\left(1+\cot^{2}(x)\right)+a+b}}{2}+\frac{\sqrt{b}\,\ln\left(\frac{b\left(1+\cot^{2}(x)\right)-b}{\sqrt{b}}+\sqrt{b\left(1+\cot^{2}\right.}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a+b*cot(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(b*(1+cot(x)^2)^2-2*b*(1+cot(x)^2)+a+b)^(1/2)+1/2*b^(1/2)*ln((b*(1+cot
(x)^2)-b)/b^(1/2)+(b*(1+cot(x)^2)^2-2*b*(1+cot(x)^2)+a+b)^(1/2))+1/2*(a+b)^
(1/2)*ln((2*a+2*b-2*b*(1+cot(x)^2)+2*(a+b)^(1/2)*(b*(1+cot(x)^2)^2-2*b*(1+c
ot(x)^2)+a+b)^(1/2))/(1+cot(x)^2))
```

<u>Maxima</u> [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*cot(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cot(x)^4 + a)*cot(x), x)
```

<u>Fricas</u> [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(72) = 144$.
time = 3.96, size = 1063, normalized size = 11.81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*cot(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a + b)*log(1/2*(a^2 + 2*a*b + b^2)*cos(2*x)^2 + 1/2*a^2 + 1/2*b^2
 + 1/2*((a + b)*cos(2*x)^2 - 2*a*cos(2*x) + a - b)*sqrt(a + b)*sqrt(((a + b
)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1)) -
 (a^2 - b^2)*cos(2*x)) + 1/4*sqrt(b)*log(-((a + 2*b)*cos(2*x)^2 - 2*(cos(2*
x)^2 - 1)*sqrt(b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(c
os(2*x)^2 - 2*cos(2*x) + 1)) - 2*(a - 2*b)*cos(2*x) + a + 2*b)/(cos(2*x)^2
- 2*cos(2*x) + 1)) - 1/2*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a
+ b)/(cos(2*x)^2 - 2*cos(2*x) + 1)), 1/2*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a
+ b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1)
)*(cos(2*x) - 1)/(b*cos(2*x) + b)) + 1/4*sqrt(a + b)*log(1/2*(a^2 + 2*a*b +
 b^2)*cos(2*x)^2 + 1/2*a^2 + 1/2*b^2 + 1/2*((a + b)*cos(2*x)^2 - 2*a*cos(2*
x) + a - b)*sqrt(a + b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a +
 b)/(cos(2*x)^2 - 2*cos(2*x) + 1)) - (a^2 - b^2)*cos(2*x)) - 1/2*sqrt(((a +
```

```
 b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1))
, -1/2*sqrt(-a - b)*arctan(((a + b)*cos(2*x)^2 - 2*a*cos(2*x) + a - b)*sqrt
(-a - b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2
 - 2*cos(2*x) + 1))/((a^2 + 2*a*b + b^2)*cos(2*x)^2 + a^2 + 2*a*b + b^2 - 2
*(a^2 - b^2)*cos(2*x))) + 1/4*sqrt(b)*log(-((a + 2*b)*cos(2*x)^2 - 2*(cos(2
*x)^2 - 1)*sqrt(b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(
cos(2*x)^2 - 2*cos(2*x) + 1)) - 2*(a - 2*b)*cos(2*x) + a + 2*b)/(cos(2*x)^2
 - 2*cos(2*x) + 1)) - 1/2*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a
 + b)/(cos(2*x)^2 - 2*cos(2*x) + 1)), -1/2*sqrt(-a - b)*arctan(((a + b)*cos
(2*x)^2 - 2*a*cos(2*x) + a - b)*sqrt(-a - b)*sqrt(((a + b)*cos(2*x)^2 - 2*(
a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1))/((a^2 + 2*a*b + b^2
)*cos(2*x)^2 + a^2 + 2*a*b + b^2 - 2*(a^2 - b^2)*cos(2*x))) + 1/2*sqrt(-b)*
arctan(sqrt(-b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos
(2*x)^2 - 2*cos(2*x) + 1))*(cos(2*x) - 1)/(b*cos(2*x) + b)) - 1/2*sqrt(((a
+ b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1)
)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\cot^4(x)}\,\cot(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*cot(x)**4)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cot(x)**4)*cot(x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(72) = 144.
time = 0.47, size = 204, normalized size = 2.27

$$-\frac{b\arctan\left(-\frac{\sqrt{a+b}\sin(x)^2-\sqrt{a\sin(x)^4+b\sin(x)^4-2\,b\sin(x)^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}\sin(x)^2-\sqrt{a\sin(x)^4+b\sin(x)^4-2\,b\sin(x)^2+b}\right)(a+b)+\sqrt{a+b}\,b\right|\right)-\frac{\left(\sqrt{a+b}\sin(x)^2-\sqrt{a\sin(x)^4+b\sin(x)^4-2\,b\sin(x)^2+b}\right)b-\sqrt{a+b}\,b}{\left(\sqrt{a+b}\sin(x)^2-\sqrt{a\sin(x)^4+b\sin(x)^4-2\,b\sin(x)^2+b}\right)^2-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*cot(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] -b*arctan(-(sqrt(a + b)*sin(x)^2 - sqrt(a*sin(x)^4 + b*sin(x)^4 - 2*b*sin(x
)^2 + b))/sqrt(-b))/sqrt(-b) - 1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*sin(x)
^2 - sqrt(a*sin(x)^4 + b*sin(x)^4 - 2*b*sin(x)^2 + b))*(a + b) + sqrt(a + b
)*b)) - ((sqrt(a + b)*sin(x)^2 - sqrt(a*sin(x)^4 + b*sin(x)^4 - 2*b*sin(x)^
2 + b))*b - sqrt(a + b)*b)/((sqrt(a + b)*sin(x)^2 - sqrt(a*sin(x)^4 + b*sin
(x)^4 - 2*b*sin(x)^2 + b))^2 - b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(x)\,\sqrt{b\cot(x)^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a + b*cot(x)^4)^(1/2),x)
[Out] int(cot(x)*(a + b*cot(x)^4)^(1/2), x)
```

3.61 $\int \cot(x)\left(a+b\cot^4(x)\right)^{3/2}\,dx$

Optimal. Leaf size=126

$$\frac{1}{4}\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\cot^2(x)}{\sqrt{a+b\cot^4(x)}}\right)+\frac{1}{2}(a+b)^{3/2}\tanh^{-1}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\,\sqrt{a+b\cot^4(x)}}\right)-\frac{1}{4}(2(a+b)-b$$

```
[Out] 1/2*(a+b)^(3/2)*arctanh((a-b*cot(x)^2)/(a+b)^(1/2)/(a+b*cot(x)^4)^(1/2))-1/
6*(a+b*cot(x)^4)^(3/2)+1/4*(3*a+2*b)*arctanh(cot(x)^2*b^(1/2)/(a+b*cot(x)^4
)^(1/2))*b^(1/2)-1/4*(2*a+2*b-b*cot(x)^2)*(a+b*cot(x)^4)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = $\{3751, 1262, 749, 829, 858, 223, 212, 739\}$

$$-\frac{1}{6}\left(a+b\cot^4(x)\right)^{3/2}-\frac{1}{4}\left(2(a+b)-b\cot^2(x)\right)\sqrt{a+b\cot^4(x)}+\frac{1}{2}(a+b)^{3/2}\tanh^{-1}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\,\sqrt{a+b\cot^4(x)}}\right)+\frac{1}{4}\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\cot^2(x)}{\sqrt{a+b\cot^4(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*(a + b*Cot[x]^4)^(3/2),x]
```

```
[Out] (Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Cot[x]^2)/Sqrt[a + b*Cot[x]^4]])/4 +
((a + b)^(3/2)*ArcTanh[(a - b*Cot[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Cot[x]^4])]
)/2 - ((2*(a + b) - b*Cot[x]^2)*Sqrt[a + b*Cot[x]^4])/4 - (a + b*Cot[x]^4)^
(3/2)/6
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
 2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot(x)\left(a+b\cot^4(x)\right)^{3/2}\,dx = -\text{Subst}\left(\int \frac{x(a+bx^4)^{3/2}}{1+x^2}\,dx, x, \cot(x)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{1+x}\,dx, x, \cot^2(x)\right)\right)$$
$$= -\frac{1}{6}\left(a+b\cot^4(x)\right)^{3/2} - \frac{1}{2}\text{Subst}\left(\int \frac{(a-bx)\sqrt{a+bx^2}}{1+x}\,dx, x, \cot^2(x)\right)$$
$$= -\frac{1}{4}\left(2(a+b)-b\cot^2(x)\right)\sqrt{a+b\cot^4(x)} - \frac{1}{6}\left(a+b\cot^4(x)\right)^{3/2} - \frac{\text{Subst}\Big(}{}$$
$$= -\frac{1}{4}\left(2(a+b)-b\cot^2(x)\right)\sqrt{a+b\cot^4(x)} - \frac{1}{6}\left(a+b\cot^4(x)\right)^{3/2} - \frac{1}{2}(a+b$$
$$= -\frac{1}{4}\left(2(a+b)-b\cot^2(x)\right)\sqrt{a+b\cot^4(x)} - \frac{1}{6}\left(a+b\cot^4(x)\right)^{3/2} + \frac{1}{2}(a+b$$
$$= \frac{1}{4}\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\cot^2(x)}{\sqrt{a+b\cot^4(x)}}\right) + \frac{1}{2}(a+b)^{3/2}\tanh^{-1}\left(\frac{a}{\sqrt{a+b}}\right.$$

Mathematica [A]
time = 4.63, size = 167, normalized size = 1.33

$$\frac{1}{12}\left(6\sqrt{b}\,(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\,\cot^2(x)}{\sqrt{a+b\cot^4(x)}}\right)+6(a+b)^{3/2}\tanh^{-1}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\,\sqrt{a+b\cot^4(x)}}\right)-\sqrt{a+b\cot^4(x)}\,\left(8a+6b-3b\cot^2(x)+2b\cot^4(x)\right)+\frac{3\sqrt{a}\,\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\cot^2(x)}{\sqrt{a}}\right)\sqrt{a+b\cot^4(x)}}{\sqrt{1+\frac{b\cot^4(x)}{a}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*(a + b*Cot[x]^4)^(3/2),x]
```

```
[Out] (6*Sqrt[b]*(a + b)*ArcTanh[(Sqrt[b]*Cot[x]^2)/Sqrt[a + b*Cot[x]^4]] + 6*(a
+ b)^(3/2)*ArcTanh[(a - b*Cot[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Cot[x]^4])] - S
qrt[a + b*Cot[x]^4]*(8*a + 6*b - 3*b*Cot[x]^2 + 2*b*Cot[x]^4) + (3*Sqrt[a]*
Sqrt[b]*ArcSinh[(Sqrt[b]*Cot[x]^2)/Sqrt[a]]*Sqrt[a + b*Cot[x]^4])/Sqrt[1 +
(b*Cot[x]^4)/a])/12
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(103) = 206$.
time = 0.24, size = 312, normalized size = 2.48

method	result

derivativedivides	$-\frac{b(\cot^4(x))\sqrt{a+b(\cot^4(x))}}{6}-\frac{2a\sqrt{a+b(\cot^4(x))}}{3}+\frac{b(\cot^2(x))\sqrt{a+b(\cot^4(x))}}{4}+\frac{3a}{}$
default	$-\frac{b(\cot^4(x))\sqrt{a+b(\cot^4(x))}}{6}-\frac{2a\sqrt{a+b(\cot^4(x))}}{3}+\frac{b(\cot^2(x))\sqrt{a+b(\cot^4(x))}}{4}+\frac{3a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a+b*cot(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*b*cot(x)^4*(a+b*cot(x)^4)^(1/2)-2/3*a*(a+b*cot(x)^4)^(1/2)+1/4*b*cot(x
)^2*(a+b*cot(x)^4)^(1/2)+3/4*a*b^(1/2)*ln(b^(1/2)*cot(x)^2+(a+b*cot(x)^4)^(
1/2))-1/2*b*(a+b*cot(x)^4)^(1/2)+1/2*b^(3/2)*ln(b^(1/2)*cot(x)^2+(a+b*cot(x
)^4)^(1/2))+1/2/(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+cot(x)^2)+2*(a+b)^(1/2)*(b*(
1+cot(x)^2)^2-2*b*(1+cot(x)^2)+a+b)^(1/2))/(1+cot(x)^2))*a^2+1/(a+b)^(1/2)*
ln((2*a+2*b-2*b*(1+cot(x)^2)+2*(a+b)^(1/2)*(b*(1+cot(x)^2)^2-2*b*(1+cot(x)^
2)+a+b)^(1/2))/(1+cot(x)^2))*a*b+1/2/(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+cot(x)^
2)+2*(a+b)^(1/2)*(b*(1+cot(x)^2)^2-2*b*(1+cot(x)^2)+a+b)^(1/2))/(1+cot(x)^2
))*b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*cot(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cot(x)^4 + a)^(3/2)*cot(x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(104) = 208$.
time = 3.30, size = 1486, normalized size = 11.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*cot(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(6*((a + b)*cos(2*x)^2 - 2*(a + b)*cos(2*x) + a + b)*sqrt(a + b)*log(
1/2*(a^2 + 2*a*b + b^2)*cos(2*x)^2 + 1/2*a^2 + 1/2*b^2 + 1/2*((a + b)*cos(2
```

```
*x)^2 - 2*a*cos(2*x) + a - b)*sqrt(a + b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a -
 b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1)) - (a^2 - b^2)*cos(2*x)
) + 3*((3*a + 2*b)*cos(2*x)^2 - 2*(3*a + 2*b)*cos(2*x) + 3*a + 2*b)*sqrt(b)
*log(-((a + 2*b)*cos(2*x)^2 - 2*(cos(2*x)^2 - 1)*sqrt(b)*sqrt(((a + b)*cos(
2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1)) - 2*(a
- 2*b)*cos(2*x) + a + 2*b)/(cos(2*x)^2 - 2*cos(2*x) + 1)) - 2*((8*a + 11*b)
*cos(2*x)^2 - 8*(2*a + b)*cos(2*x) + 8*a + 5*b)*sqrt(((a + b)*cos(2*x)^2 -
2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1)))/(cos(2*x)^2 - 2
*cos(2*x) + 1), 1/12*(3*((3*a + 2*b)*cos(2*x)^2 - 2*(3*a + 2*b)*cos(2*x) +
3*a + 2*b)*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*co
s(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1))*(cos(2*x) - 1)/(b*cos(2*x) +
 b)) + 3*((a + b)*cos(2*x)^2 - 2*(a + b)*cos(2*x) + a + b)*sqrt(a + b)*log(
1/2*(a^2 + 2*a*b + b^2)*cos(2*x)^2 + 1/2*a^2 + 1/2*b^2 + 1/2*((a + b)*cos(2
*x)^2 - 2*a*cos(2*x) + a - b)*sqrt(a + b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a -
 b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1)) - (a^2 - b^2)*cos(2*x)
) - ((8*a + 11*b)*cos(2*x)^2 - 8*(2*a + b)*cos(2*x) + 8*a + 5*b)*sqrt(((a +
 b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1))
)/(cos(2*x)^2 - 2*cos(2*x) + 1), -1/24*(12*((a + b)*cos(2*x)^2 - 2*(a + b)*
cos(2*x) + a + b)*sqrt(-a - b)*arctan(((a + b)*cos(2*x)^2 - 2*a*cos(2*x) +
a - b)*sqrt(-a - b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/
(cos(2*x)^2 - 2*cos(2*x) + 1))/((a^2 + 2*a*b + b^2)*cos(2*x)^2 + a^2 + 2*a*
b + b^2 - 2*(a^2 - b^2)*cos(2*x))) - 3*((3*a + 2*b)*cos(2*x)^2 - 2*(3*a + 2
*b)*cos(2*x) + 3*a + 2*b)*sqrt(b)*log(-((a + 2*b)*cos(2*x)^2 - 2*(cos(2*x)^
2 - 1)*sqrt(b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(
2*x)^2 - 2*cos(2*x) + 1)) - 2*(a - 2*b)*cos(2*x) + a + 2*b)/(cos(2*x)^2 - 2
*cos(2*x) + 1)) + 2*((8*a + 11*b)*cos(2*x)^2 - 8*(2*a + b)*cos(2*x) + 8*a +
 5*b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2 -
2*cos(2*x) + 1)))/(cos(2*x)^2 - 2*cos(2*x) + 1), -1/12*(6*((a + b)*cos(2*x)
^2 - 2*(a + b)*cos(2*x) + a + b)*sqrt(-a - b)*arctan(((a + b)*cos(2*x)^2 -
2*a*cos(2*x) + a - b)*sqrt(-a - b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos
(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1))/((a^2 + 2*a*b + b^2)*cos(2*x)
^2 + a^2 + 2*a*b + b^2 - 2*(a^2 - b^2)*cos(2*x))) - 3*((3*a + 2*b)*cos(2*x)
^2 - 2*(3*a + 2*b)*cos(2*x) + 3*a + 2*b)*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a
+ b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1)
)*(cos(2*x) - 1)/(b*cos(2*x) + b)) + ((8*a + 11*b)*cos(2*x)^2 - 8*(2*a + b)
*cos(2*x) + 8*a + 5*b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a +
b)/(cos(2*x)^2 - 2*cos(2*x) + 1)))/(cos(2*x)^2 - 2*cos(2*x) + 1)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\cot^{4}\left(x\right)\right)^{\frac{3}{2}}\cot\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*cot(x)**4)**(3/2),x)
```

```
[Out] Integral((a + b*cot(x)**4)**(3/2)*cot(x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(104) = 208$.
time = 0.65, size = 445, normalized size = 3.53

$$-\frac{(3\,ab+2\,b^2)\arctan\left(-\frac{\sqrt{a+b}\sin(x)^2-\sqrt{a\sin(x)^4+b\sin(x)^4-2\,b\sin(x)^2+b}}{\sqrt{-b}}\right)}{2\sqrt{-b}}-\frac{(a^2+2\,ab+b^2)\log\left(\left|-\left(\sqrt{a+b}\sin(x)^2-\sqrt{a\sin(x)^4+b\sin(x)^4-2\,b\sin(x)^2+b}\right)(a+b)+\sqrt{a+b}\,b\right|\right)}{2\sqrt{a+b}}-\frac{3\left(\sqrt{a+b}\sin(x)^2-\sqrt{a\sin(x)^4+b\sin(x)^4-2\,b\sin(x)^2+b}\right)^5(5\,ab+6\,b^2)+8\left(\sqrt{a+b}\sin(x)^2-\sqrt{a\sin(x)^4+b\sin(x)^4-2\,b\sin(x)^2+b}\right)^3b^3-12\left(\sqrt{a+b}\sin(x)^2-\sqrt{a\sin(x)^4+b\sin(x)^4-2\,b\sin(x)^2+b}\right)^4(ab+3\,b^2)\sqrt{a+b}+12\,(ab^2+b^3)\left(\sqrt{a+b}\sin(x)^2-\sqrt{a\sin(x)^4+b\sin(x)^4-2\,b\sin(x)^2+b}\right)^2\sqrt{a+b}+3\,(3\,ab^3+2\,b^4)\left(\sqrt{a+b}\sin(x)^2-\sqrt{a\sin(x)^4+b\sin(x)^4-2\,b\sin(x)^2+b}\right)-8\,(ab^3+b^4)\sqrt{a+b}}{6\left(\left(\sqrt{a+b}\sin(x)^2-\sqrt{a\sin(x)^4+b\sin(x)^4-2\,b\sin(x)^2+b}\right)^2-b\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*cot(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(3*a*b + 2*b^2)*arctan(-(sqrt(a + b)*sin(x)^2 - sqrt(a*sin(x)^4 + b*si
n(x)^4 - 2*b*sin(x)^2 + b))/sqrt(-b))/sqrt(-b) - 1/2*(a^2 + 2*a*b + b^2)*lo
g(abs(-(sqrt(a + b)*sin(x)^2 - sqrt(a*sin(x)^4 + b*sin(x)^4 - 2*b*sin(x)^2
+ b))*(a + b) + sqrt(a + b)*b))/sqrt(a + b) - 1/6*(3*(sqrt(a + b)*sin(x)^2
- sqrt(a*sin(x)^4 + b*sin(x)^4 - 2*b*sin(x)^2 + b))^5*(5*a*b + 6*b^2) + 8*(
sqrt(a + b)*sin(x)^2 - sqrt(a*sin(x)^4 + b*sin(x)^4 - 2*b*sin(x)^2 + b))^3*
b^3 - 12*(sqrt(a + b)*sin(x)^2 - sqrt(a*sin(x)^4 + b*sin(x)^4 - 2*b*sin(x)^
2 + b))^4*(a*b + 3*b^2)*sqrt(a + b) + 12*(a*b^2 + b^3)*(sqrt(a + b)*sin(x)^
2 - sqrt(a*sin(x)^4 + b*sin(x)^4 - 2*b*sin(x)^2 + b))^2*sqrt(a + b) + 3*(3*
a*b^3 + 2*b^4)*(sqrt(a + b)*sin(x)^2 - sqrt(a*sin(x)^4 + b*sin(x)^4 - 2*b*s
in(x)^2 + b)) - 8*(a*b^3 + b^4)*sqrt(a + b))/((sqrt(a + b)*sin(x)^2 - sqrt(
a*sin(x)^4 + b*sin(x)^4 - 2*b*sin(x)^2 + b))^2 - b)^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(x)\,\left(b\cot(x)^4+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a + b*cot(x)^4)^(3/2),x)
```

```
[Out] int(cot(x)*(a + b*cot(x)^4)^(3/2), x)
```

3.62 $\int \frac{\cot(x)}{\sqrt{a+b\cot^4(x)}}\,dx$

Optimal. Leaf size=41

$$\frac{\tanh^{-1}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)}{2\sqrt{a+b}}$$

```
[Out] 1/2*arctanh((a-b*cot(x)^2)/(a+b)^(1/2)/(a+b*cot(x)^4)^(1/2))/(a+b)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3751, 1262, 739, 212}

$$\frac{\tanh^{-1}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)}{2\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/Sqrt[a + b*Cot[x]^4],x]
```

```
[Out] ArcTanh[(a - b*Cot[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Cot[x]^4])]/(2*Sqrt[a + b]
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(x)}{\sqrt{a+b\cot^4(x)}}\,dx &= -\text{Subst}\left(\int \frac{x}{(1+x^2)\sqrt{a+bx^4}}\,dx, x, \cot(x)\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx^2}}\,dx, x, \cot^2(x)\right)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{a-b\cot^2(x)}{\sqrt{a+b\cot^4(x)}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)}{2\sqrt{a+b}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)}{2\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/Sqrt[a + b*Cot[x]^4],x]
[Out] ArcTanh[(a - b*Cot[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Cot[x]^4])]/(2*Sqrt[a + b]
)
```

Maple [A]
time = 0.37, size = 65, normalized size = 1.59

method	result	size
derivativedivides	$\frac{\ln\left(\frac{2a+2b-2b\left(1+\cot^2(x)\right)+2\sqrt{a+b}\,\sqrt{b\left(1+\cot^2(x)\right)^2-2b\left(1+\cot^2(x)\right)+a+b}}{1+\cot^2(x)}\right)}{2\sqrt{a+b}}$	65
default	$\frac{\ln\left(\frac{2a+2b-2b\left(1+\cot^2(x)\right)+2\sqrt{a+b}\,\sqrt{b\left(1+\cot^2(x)\right)^2-2b\left(1+\cot^2(x)\right)+a+b}}{1+\cot^2(x)}\right)}{2\sqrt{a+b}}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a+b*cot(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+cot(x)^2)+2*(a+b)^(1/2)*(b*(1+cot(x)^2)^
2-2*b*(1+cot(x)^2)+a+b)^(1/2))/(1+cot(x)^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(x)/sqrt(b*cot(x)^4 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(35) = 70$.
time = 3.80, size = 264, normalized size = 6.44

$$\left[\frac{\log\left(\frac{1}{2}\left(a^2+2ab+b^2\right)\cos(2x)^2+\frac{1}{2}a^2+\frac{1}{2}b^2+\frac{1}{2}\left((a+b)\cos(2x)^2-2a\cos(2x)+a-b\right)\sqrt{a+b}\sqrt{\frac{(a+b)\cos(2x)^2-2(a-b)\cos(2x)+a+b}{\cos(2x)^2-2\cos(2x)+1}}-\left(a^2-b^2\right)\cos(2x)\right)}{4\sqrt{a+b}}, -\frac{\sqrt{-a-b}\arctan\left(\frac{\left((a+b)\cos(2x)^2-2a\cos(2x)+a-b\right)\sqrt{-a-b}\sqrt{\frac{(a+b)\cos(2x)^2-2(a-b)\cos(2x)+a+b}{\cos(2x)^2-2\cos(2x)+1}}}{\left(a^2+2ab+b^2\right)\cos(2x)^2+a^2+2ab+b^2-2\left(a^2-b^2\right)\cos(2x)}\right)}{2(a+b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(1/2*(a^2 + 2*a*b + b^2)*cos(2*x)^2 + 1/2*a^2 + 1/2*b^2 + 1/2*((a +
 b)*cos(2*x)^2 - 2*a*cos(2*x) + a - b)*sqrt(a + b)*sqrt(((a + b)*cos(2*x)^2
 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1)) - (a^2 - b^2)
*cos(2*x))/sqrt(a + b), -1/2*sqrt(-a - b)*arctan(((a + b)*cos(2*x)^2 - 2*a*
cos(2*x) + a - b)*sqrt(-a - b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x
) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1))/((a^2 + 2*a*b + b^2)*cos(2*x)^2 +
 a^2 + 2*a*b + b^2 - 2*(a^2 - b^2)*cos(2*x)))/(a + b)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\sqrt{a+b\cot^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)**4)**(1/2),x)
```

```
[Out] Integral(cot(x)/sqrt(a + b*cot(x)**4), x)
```

Giac [**A**]
time = 0.47, size = 58, normalized size = 1.41

$$-\frac{\log\left(\left|-\left(\sqrt{a+b}\,\sin(x)^2-\sqrt{a\sin(x)^4+b\sin(x)^4-2\,b\sin(x)^2+b}\right)(a+b)+\sqrt{a+b}\,b\right|\right)}{2\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(-(sqrt(a + b)*sin(x)^2 - sqrt(a*sin(x)^4 + b*sin(x)^4 - 2*b*si
n(x)^2 + b))*(a + b) + sqrt(a + b)*b))/sqrt(a + b)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cot(x)}{\sqrt{b\cot(x)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a + b*cot(x)^4)^(1/2),x)
```

```
[Out] int(cot(x)/(a + b*cot(x)^4)^(1/2), x)
```

3.63 $\int \frac{\cot(x)}{\left(a+b\cot^4(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=74

$$\frac{\tanh^{-1}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)}{2(a+b)^{3/2}}-\frac{a+b\cot^2(x)}{2a(a+b)\sqrt{a+b\cot^4(x)}}$$

[Out] 1/2*arctanh((a-b*cot(x)^2)/(a+b)^(1/2)/(a+b*cot(x)^4)^(1/2))/(a+b)^(3/2)+1/2*(-a-b*cot(x)^2)/a/(a+b)/(a+b*cot(x)^4)^(1/2)

Rubi [A]
time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3751, 1262, 755, 12, 739, 212}

$$\frac{\tanh^{-1}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)}{2(a+b)^{3/2}}-\frac{a+b\cot^2(x)}{2a(a+b)\sqrt{a+b\cot^4(x)}}$$

Antiderivative was successfully verified.

[In] Int[Cot[x]/(a + b*Cot[x]^4)^(3/2),x]

[Out] ArcTanh[(a - b*Cot[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Cot[x]^4])]/(2*(a + b)^(3/2)) - (a + b*Cot[x]^2)/(2*a*(a + b)*Sqrt[a + b*Cot[x]^4])

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot(x)}{(a+b\cot^4(x))^{3/2}}\,dx &= -\text{Subst}\left(\int \frac{x}{(1+x^2)\,(a+bx^4)^{3/2}}\,dx, x, \cot(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\,(a+bx^2)^{3/2}}\,dx, x, \cot^2(x)\right)\right)\\
&= -\frac{a+b\cot^2(x)}{2a(a+b)\sqrt{a+b\cot^4(x)}} - \frac{\text{Subst}\left(\int \frac{a}{(1+x)\sqrt{a+bx^2}}\,dx, x, \cot^2(x)\right)}{2a(a+b)}\\
&= -\frac{a+b\cot^2(x)}{2a(a+b)\sqrt{a+b\cot^4(x)}} - \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx^2}}\,dx, x, \cot^2(x)\right)}{2(a+b)}\\
&= -\frac{a+b\cot^2(x)}{2a(a+b)\sqrt{a+b\cot^4(x)}} + \frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{a-b\cot^2(x)}{\sqrt{a+b\cot^4(x)}}\right)}{2(a+b)}\\
&= \frac{\tanh^{-1}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)}{2(a+b)^{3/2}} - \frac{a+b\cot^2(x)}{2a(a+b)\sqrt{a+b\cot^4(x)}}
\end{aligned}
$$

Mathematica [A]
time = 0.30, size = 73, normalized size = 0.99

$$
\frac{1}{2}\left(\frac{\tanh^{-1}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)}{(a+b)^{3/2}} - \frac{a+b\cot^2(x)}{a(a+b)\sqrt{a+b\cot^4(x)}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(a + b*Cot[x]^4)^(3/2),x]
```

```
[Out] (ArcTanh[(a - b*Cot[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Cot[x]^4])]/(a + b)^(3/2)
 - (a + b*Cot[x]^2)/(a*(a + b)*Sqrt[a + b*Cot[x]^4]))/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(65) = 130$.
time = 3.33, size = 248, normalized size = 3.35

method	result
derivativedivides	$-\frac{b\ln\left(\frac{2a+2b-2b\left(1+\cot^2(x)\right)+2\sqrt{a+b}\,\sqrt{b\left(1+\cot^2(x)\right)^2-2b\left(1+\cot^2(x)\right)+a+b}}{1+\cot^2(x)}\right)}{2\left(\sqrt{-ab}+b\right)\left(\sqrt{-ab}-b\right)\sqrt{a+b}}-\frac{\sqrt{b}}{}$
default	$-\frac{b\ln\left(\frac{2a+2b-2b\left(1+\cot^2(x)\right)+2\sqrt{a+b}\,\sqrt{b\left(1+\cot^2(x)\right)^2-2b\left(1+\cot^2(x)\right)+a+b}}{1+\cot^2(x)}\right)}{2\left(\sqrt{-ab}+b\right)\left(\sqrt{-ab}-b\right)\sqrt{a+b}}-\frac{\sqrt{b}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a+b*cot(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b/((-a*b)^(1/2)+b)/((-a*b)^(1/2)-b)/(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+cot
(x)^2)+2*(a+b)^(1/2)*(b*(1+cot(x)^2)^2-2*b*(1+cot(x)^2)+a+b)^(1/2))/(1+cot(
x)^2))-1/4/((-a*b)^(1/2)+b)/a/(cot(x)^2-(-a*b)^(1/2)/b)*(b*(cot(x)^2-(-a*b)
^(1/2)/b)^2+2*(-a*b)^(1/2)*(cot(x)^2-(-a*b)^(1/2)/b))^(1/2)+1/4/((-a*b)^(1/
2)-b)/a/(cot(x)^2+(-a*b)^(1/2)/b)*(b*(cot(x)^2+(-a*b)^(1/2)/b)^2-2*(-a*b)^(
1/2)*(cot(x)^2+(-a*b)^(1/2)/b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(x)/(b*cot(x)^4 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(64) = 128$.
time = 3.43, size = 670, normalized size = 9.05

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a^2 + a*b)*cos(2*x)^2 + a^2 + a*b - 2*(a^2 - a*b)*cos(2*x))*sqrt(a
+ b)*log(1/2*(a^2 + 2*a*b + b^2)*cos(2*x)^2 + 1/2*a^2 + 1/2*b^2 + 1/2*((a +
```

```
b)*cos(2*x)^2 - 2*a*cos(2*x) + a - b)*sqrt(a + b)*sqrt(((a + b)*cos(2*x)^2
 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1)) - (a^2 - b^2)
*cos(2*x)) - 2*((a^2 - b^2)*cos(2*x)^2 + a^2 + 2*a*b + b^2 - 2*(a^2 + a*b)*
cos(2*x))*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^
2 - 2*cos(2*x) + 1)))/(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 + (a^4 + 3*a^3*b +
 3*a^2*b^2 + a*b^3)*cos(2*x)^2 - 2*(a^4 + a^3*b - a^2*b^2 - a*b^3)*cos(2*x)
), -1/2*(((a^2 + a*b)*cos(2*x)^2 + a^2 + a*b - 2*(a^2 - a*b)*cos(2*x))*sqrt
(-a - b)*arctan(((a + b)*cos(2*x)^2 - 2*a*cos(2*x) + a - b)*sqrt(-a - b)*sq
rt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*
x) + 1))/((a^2 + 2*a*b + b^2)*cos(2*x)^2 + a^2 + 2*a*b + b^2 - 2*(a^2 - b^2
)*cos(2*x))) + ((a^2 - b^2)*cos(2*x)^2 + a^2 + 2*a*b + b^2 - 2*(a^2 + a*b)*
cos(2*x))*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^
2 - 2*cos(2*x) + 1)))/(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 + (a^4 + 3*a^3*b +
 3*a^2*b^2 + a*b^3)*cos(2*x)^2 - 2*(a^4 + a^3*b - a^2*b^2 - a*b^3)*cos(2*x)
)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\left(a + b\cot^4(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)**4)**(3/2),x)
```

```
[Out] Integral(cot(x)/(a + b*cot(x)**4)**(3/2), x)
```

Giac [A]
time = 0.47, size = 111, normalized size = 1.50

$$-\frac{\frac{(a-b)\sin(x)^2}{a^2+ab} + \frac{b}{a^2+ab}}{2\sqrt{a\sin(x)^4 + b\sin(x)^4 - 2b\sin(x)^2 + b}} - \frac{\log\left(\left|-\left(\sqrt{a+b}\sin(x)^2 - \sqrt{a\sin(x)^4 + b\sin(x)^4 - 2b\sin(x)^2 + b}\right)\sqrt{a+b} + b\right|\right)}{2(a+b)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*((a - b)*sin(x)^2/(a^2 + a*b) + b/(a^2 + a*b))/sqrt(a*sin(x)^4 + b*sin
(x)^4 - 2*b*sin(x)^2 + b) - 1/2*log(abs(-(sqrt(a + b)*sin(x)^2 - sqrt(a*sin
(x)^4 + b*sin(x)^4 - 2*b*sin(x)^2 + b))*sqrt(a + b) + b))/(a + b)^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(x)}{{\left(b\,{\cot\left(x\right)}^4+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a + b*cot(x)^4)^(3/2),x)
```

```
[Out] int(cot(x)/(a + b*cot(x)^4)^(3/2), x)
```

3.64 $\int \frac{\cot(x)}{\left(a+b\cot^4(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=117

$$\frac{\tanh^{-1}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)}{2(a+b)^{5/2}}-\frac{a+b\cot^2(x)}{6a(a+b)\left(a+b\cot^4(x)\right)^{3/2}}-\frac{3a^2+b(5a+2b)\cot^2(x)}{6a^2(a+b)^2\sqrt{a+b\cot^4(x)}}$$

```
[Out] 1/2*arctanh((a-b*cot(x)^2)/(a+b)^(1/2)/(a+b*cot(x)^4)^(1/2))/(a+b)^(5/2)+1/
6*(-a-b*cot(x)^2)/a/(a+b)/(a+b*cot(x)^4)^(3/2)+1/6*(-3*a^2-b*(5*a+2*b)*cot(
x)^2)/a^2/(a+b)^2/(a+b*cot(x)^4)^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3751, 1262, 755, 837, 12, 739, 212}

$$-\frac{3a^2+b(5a+2b)\cot^2(x)}{6a^2(a+b)^2\sqrt{a+b\cot^4(x)}}-\frac{a+b\cot^2(x)}{6a(a+b)\left(a+b\cot^4(x)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)}{2(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(a + b*Cot[x]^4)^(5/2),x]
```

```
[Out] ArcTanh[(a - b*Cot[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Cot[x]^4])]/(2*(a + b)^(5/
2)) - (a + b*Cot[x]^2)/(6*a*(a + b)*(a + b*Cot[x]^4)^(3/2)) - (3*a^2 + b*(5
*a + 2*b)*Cot[x]^2)/(6*a^2*(a + b)^2*Sqrt[a + b*Cot[x]^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot(x)}{\left(a+b\cot^4(x)\right)^{5/2}}\,dx = -\text{Subst}\left(\int \frac{x}{(1+x^2)\left(a+bx^4\right)^{5/2}}\,dx, x, \cot(x)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\left(a+bx^2\right)^{5/2}}\,dx, x, \cot^2(x)\right)\right)$$

$$= -\frac{a+b\cot^2(x)}{6a(a+b)\left(a+b\cot^4(x)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-3a-2b-2bx}{(1+x)(a+bx^2)^{3/2}}\,dx, x, \cot^2(x)\right)}{6a(a+b)}$$

$$= -\frac{a+b\cot^2(x)}{6a(a+b)\left(a+b\cot^4(x)\right)^{3/2}} - \frac{3a^2+b(5a+2b)\cot^2(x)}{6a^2(a+b)^2\sqrt{a+b\cot^4(x)}} - \frac{\text{Subst}\left(\int \frac{3a^2b}{(1+x)\sqrt{a+}}\right.}{6a^2b}$$

$$= -\frac{a+b\cot^2(x)}{6a(a+b)\left(a+b\cot^4(x)\right)^{3/2}} - \frac{3a^2+b(5a+2b)\cot^2(x)}{6a^2(a+b)^2\sqrt{a+b\cot^4(x)}} - \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+}}\right.}{2(a}$$

$$= -\frac{a+b\cot^2(x)}{6a(a+b)\left(a+b\cot^4(x)\right)^{3/2}} - \frac{3a^2+b(5a+2b)\cot^2(x)}{6a^2(a+b)^2\sqrt{a+b\cot^4(x)}} + \frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,d\right.}{2(}$$

$$= \frac{\tanh^{-1}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)}{2(a+b)^{5/2}} - \frac{a+b\cot^2(x)}{6a(a+b)\left(a+b\cot^4(x)\right)^{3/2}} - \frac{3a^2+b(5a}{6a^2(a+b)^2}$$

Mathematica [A]
time = 0.78, size = 114, normalized size = 0.97

$$\frac{\tanh^{-1}\left(\frac{a-b\cot^2(x)}{\sqrt{a+b}\sqrt{a+b\cot^4(x)}}\right)}{2(a+b)^{5/2}} - \frac{a^2(4a+b)+3ab(2a+b)\cot^2(x)+3a^2b\cot^4(x)+b^2(5a+2b)\cot^6(x)}{6a^2(a+b)^2\left(a+b\cot^4(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(a + b*Cot[x]^4)^(5/2),x]
```

```
[Out] ArcTanh[(a - b*Cot[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Cot[x]^4])]/(2*(a + b)^(5/
2)) - (a^2*(4*a + b) + 3*a*b*(2*a + b)*Cot[x]^2 + 3*a^2*b*Cot[x]^4 + b^2*(5
*a + 2*b)*Cot[x]^6)/(6*a^2*(a + b)^2*(a + b*Cot[x]^4)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(105) = 210$.
time = 3.33, size = 602, normalized size = 5.15

method	result
derivativedivides	$-\frac{\sqrt{b\left(\cot^{2}(x)+\frac{\sqrt{-ab}}{b}\right)^{2}-2\sqrt{-ab}\,\left(\cot^{2}(x)+\frac{\sqrt{-ab}}{b}\right)}}{24\left(\sqrt{-ab}-b\right)a\sqrt{-ab}\,\left(\cot^{2}(x)+\frac{\sqrt{-ab}}{b}\right)^{2}}+\frac{\sqrt{b\left(\cot^{2}(x)+\frac{\sqrt{-}}{}\right.}}{24\left(\sqrt{-}\right.}$
default	$-\frac{\sqrt{b\left(\cot^{2}(x)+\frac{\sqrt{-ab}}{b}\right)^{2}-2\sqrt{-ab}\,\left(\cot^{2}(x)+\frac{\sqrt{-ab}}{b}\right)}}{24\left(\sqrt{-ab}-b\right)a\sqrt{-ab}\,\left(\cot^{2}(x)+\frac{\sqrt{-ab}}{b}\right)^{2}}+\frac{\sqrt{b\left(\cot^{2}(x)+\frac{\sqrt{-}}{}\right.}}{24\left(\sqrt{-}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a+b*cot(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24/((-a*b)^(1/2)-b)/a/(-a*b)^(1/2)/(cot(x)^2+(-a*b)^(1/2)/b)^2*(b*(cot(x
)^2+(-a*b)^(1/2)/b)^2-2*(-a*b)^(1/2)*(cot(x)^2+(-a*b)^(1/2)/b))^(1/2)+1/24/
((-a*b)^(1/2)-b)/a^2/(cot(x)^2+(-a*b)^(1/2)/b)*(b*(cot(x)^2+(-a*b)^(1/2)/b)
^2-2*(-a*b)^(1/2)*(cot(x)^2+(-a*b)^(1/2)/b))^(1/2)+1/2*b^2/((-a*b)^(1/2)+b)
^2/((-a*b)^(1/2)-b)^2/(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+cot(x)^2)+2*(a+b)^(1/2
)*(b*(1+cot(x)^2)^2-2*b*(1+cot(x)^2)+a+b)^(1/2))/(1+cot(x)^2))-1/24/((-a*b)
^(1/2)+b)/a/(-a*b)^(1/2)/(cot(x)^2-(-a*b)^(1/2)/b)^2*(b*(cot(x)^2-(-a*b)^(1
/2)/b)^2+2*(-a*b)^(1/2)*(cot(x)^2-(-a*b)^(1/2)/b))^(1/2)-1/24/((-a*b)^(1/2)
+b)/a^2/(cot(x)^2-(-a*b)^(1/2)/b)*(b*(cot(x)^2-(-a*b)^(1/2)/b)^2+2*(-a*b)^(
1/2)*(cot(x)^2-(-a*b)^(1/2)/b))^(1/2)-1/8*(2*(-a*b)^(1/2)+b)/((-a*b)^(1/2)+
b)^2/a^2/(cot(x)^2-(-a*b)^(1/2)/b)*(b*(cot(x)^2-(-a*b)^(1/2)/b)^2+2*(-a*b)^
(1/2)*(cot(x)^2-(-a*b)^(1/2)/b))^(1/2)+1/8*(2*(-a*b)^(1/2)-b)/((-a*b)^(1/2)
-b)^2/a^2/(cot(x)^2+(-a*b)^(1/2)/b)*(b*(cot(x)^2+(-a*b)^(1/2)/b)^2-2*(-a*b)
^(1/2)*(cot(x)^2+(-a*b)^(1/2)/b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(x)/(b*cot(x)^4 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 686 vs. $2(103) = 206$.
time = 3.87, size = 1365, normalized size = 11.67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(2*x)^4 + a^4 + 2*a^3*b + a^2*b^2 -
4*(a^4 - a^2*b^2)*cos(2*x)^3 + 2*(3*a^4 - 2*a^3*b + 3*a^2*b^2)*cos(2*x)^2 -
 4*(a^4 - a^2*b^2)*cos(2*x))*sqrt(a + b)*log(1/2*(a^2 + 2*a*b + b^2)*cos(2*
x)^2 + 1/2*a^2 + 1/2*b^2 + 1/2*((a + b)*cos(2*x)^2 - 2*a*cos(2*x) + a - b)*
sqrt(a + b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x
)^2 - 2*cos(2*x) + 1)) - (a^2 - b^2)*cos(2*x)) - 4*((2*a^4 + a^3*b - 5*a^2*
b^2 - 5*a*b^3 - b^4)*cos(2*x)^4 + 2*a^4 + 7*a^3*b + 9*a^2*b^2 + 5*a*b^3 + b
^4 - 2*(4*a^4 + 2*a^3*b - a^2*b^2 + 2*a*b^3 + b^4)*cos(2*x)^3 + 12*(a^4 + a
^3*b)*cos(2*x)^2 - 2*(4*a^4 + 8*a^3*b + 3*a^2*b^2 - 2*a*b^3 - b^4)*cos(2*x)
)*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*co
s(2*x) + 1)))/(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^
5 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*cos(2*x
)^4 - 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*cos(2
*x)^3 + 2*(3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)
*cos(2*x)^2 - 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^
5)*cos(2*x)), -1/6*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(2*x)^4 + a^4 + 2*a^3*b
 + a^2*b^2 - 4*(a^4 - a^2*b^2)*cos(2*x)^3 + 2*(3*a^4 - 2*a^3*b + 3*a^2*b^2)
*cos(2*x)^2 - 4*(a^4 - a^2*b^2)*cos(2*x))*sqrt(-a - b)*arctan(((a + b)*cos(
2*x)^2 - 2*a*cos(2*x) + a - b)*sqrt(-a - b)*sqrt(((a + b)*cos(2*x)^2 - 2*(a
 - b)*cos(2*x) + a + b)/(cos(2*x)^2 - 2*cos(2*x) + 1))/((a^2 + 2*a*b + b^2)
*cos(2*x)^2 + a^2 + 2*a*b + b^2 - 2*(a^2 - b^2)*cos(2*x))) + 2*((2*a^4 + a^
3*b - 5*a^2*b^2 - 5*a*b^3 - b^4)*cos(2*x)^4 + 2*a^4 + 7*a^3*b + 9*a^2*b^2 +
 5*a*b^3 + b^4 - 2*(4*a^4 + 2*a^3*b - a^2*b^2 + 2*a*b^3 + b^4)*cos(2*x)^3 +
 12*(a^4 + a^3*b)*cos(2*x)^2 - 2*(4*a^4 + 8*a^3*b + 3*a^2*b^2 - 2*a*b^3 - b
^4)*cos(2*x))*sqrt(((a + b)*cos(2*x)^2 - 2*(a - b)*cos(2*x) + a + b)/(cos(2
*x)^2 - 2*cos(2*x) + 1)))/(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*
b^4 + a^2*b^5 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*
b^5)*cos(2*x)^4 - 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^
2*b^5)*cos(2*x)^3 + 2*(3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4
+ 3*a^2*b^5)*cos(2*x)^2 - 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*
b^4 - a^2*b^5)*cos(2*x))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\left(a + b\cot^4(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)**4)**(5/2),x)
```

```
[Out] Integral(cot(x)/(a + b*cot(x)**4)**(5/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 276 vs. 2(103) = 206.
time = 0.46, size = 276, normalized size = 2.36

$$-\frac{\left(2\left(\frac{(2a^3b-a^2b^2-4ab^3-b^4)\sin(x)^2}{a^4b+2a^3b^2+a^2b^3}+\frac{3(3ab^3+b^4)}{a^4b+2a^3b^2+a^2b^3}\right)\sin(x)^2+\frac{3(a^2b^2-5ab^3-2b^4)}{a^4b+2a^3b^2+a^2b^3}\right)\sin(x)^2+\frac{5ab^3+2b^4}{a^4b+2a^3b^2+a^2b^3}}{6\left(a\sin(x)^4+b\sin(x)^4-2b\sin(x)^2+b\right)^{\frac{3}{2}}}-\frac{\log\left(\left|-\left(\sqrt{a+b}\sin(x)^2-\sqrt{a\sin(x)^4+b\sin(x)^4-2b\sin(x)^2+b}\right)\sqrt{a+b}+b\right|\right)}{2(a^2+2ab+b^2)\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cot(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] -1/6*((2*((2*a^3*b - a^2*b^2 - 4*a*b^3 - b^4)*sin(x)^2/(a^4*b + 2*a^3*b^2 +
 a^2*b^3) + 3*(3*a*b^3 + b^4)/(a^4*b + 2*a^3*b^2 + a^2*b^3))*sin(x)^2 + 3*(
a^2*b^2 - 5*a*b^3 - 2*b^4)/(a^4*b + 2*a^3*b^2 + a^2*b^3))*sin(x)^2 + (5*a*b
^3 + 2*b^4)/(a^4*b + 2*a^3*b^2 + a^2*b^3))/(a*sin(x)^4 + b*sin(x)^4 - 2*b*s
in(x)^2 + b)^(3/2) - 1/2*log(abs(-(sqrt(a + b)*sin(x)^2 - sqrt(a*sin(x)^4 +
 b*sin(x)^4 - 2*b*sin(x)^2 + b))*sqrt(a + b) + b))/((a^2 + 2*a*b + b^2)*sqr
t(a + b))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(x)}{\left(b\cot(x)^4+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a + b*cot(x)^4)^(5/2),x)
```

```
[Out] int(cot(x)/(a + b*cot(x)^4)^(5/2), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#            Albert Rich to use with Sagemath. This is used to
#            grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#            'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#            issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```